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NON-ADIABATIC PERTURBATIONS AND NON-GAUSSIANITY DURING INFLATION: AFFINE GRAVITY APPROACH

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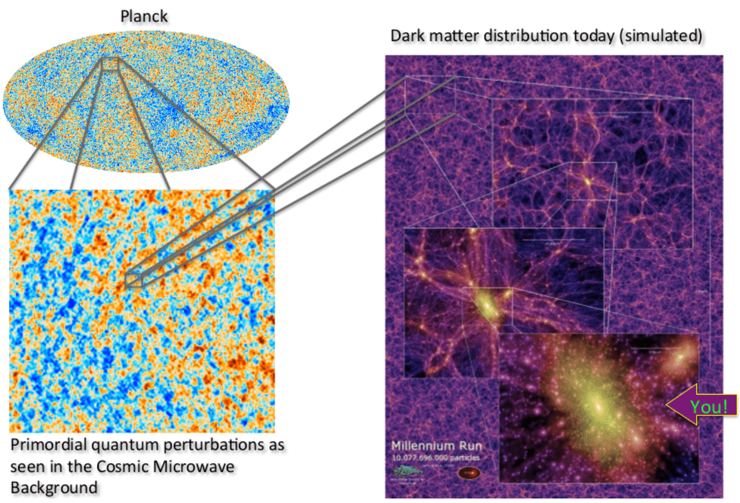

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Cosmology: From theory to data, from data to theory - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/The-big-picture-in-cosmology-Primordial-perturbations-as-seen-in-the-Cosmic-Microwave_fig1_260604134

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I, Isaac Bamwidhi, the undersigned, a graduate student at the United Arab Emirates University (UAEU), and the author of this thesis entitled "*Non-adiabatic Perturbations and Non-gaussianity during Inflation*", hereby, solemnly declare that this thesis is my own original research work that has been done and prepared by me under the supervision of Professor Salah Nasri, in the College of Science at UAEU. This work has not previously formed the basis for the award of any academic degree, diploma or a similar title at this or any other university. Any materials borrowed from other sources (whether published or unpublished) and relied upon or included in my thesis have been properly cited and acknowledged in accordance with appropriate academic conventions. I further declare that there is no potential conflict of interest with respect to the research, data collection, authorship, presentation and/or publication of this thesis.

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Approval of the Master Thesis

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Abstract

This thesis is concerned with providing a complete study of non-Gaussianity and entropy perturbations that are sourced by multiple fields nonminimally coupled to gravity. The study will be performed in the framework of the two important formulations of gravity, namely: purely metric (general relativity) and purely affine formulation – where the metrical structure results from the dynamics of the spacetime affine connection. We shall employ a covariant formalism in our framework and demonstrate that it leads to a curved field space which can produce conspicuous departure from the purely metric gravity. This work is expected, not only to derive the main quantities such as non-adiabatic pressure and curvature perturbations in each formulation, but also to shed light on the frame (in) dependent character of the primordial perturbations. The approach will stand on a generic affine spacetime that supports scalar fields and requires (by its nature) nonzero potentials. Simply put, this thesis covers a comprehensive and systematic study of inflation based on a completely different approach to gravity: the purely affine gravity. Primordial perturbations are the most important factor in inflationary cosmology and this work will certainly bring out novelty to the field at the theoretical and observational levels since it aims at covering the topic in the framework of various formulations of gravity which is at the heart of inflation.

Keywords: Inflation, Non-adiabatic Perturbations, Non-gaussianity, Isocurvature modes, Minimal and Non-minimal coupling, Anisotropy, Metric formulation, Affine gravity.

Title and Abstract (in Arabic)

الاضطرابات البدائية خلال نظرية التضخم الكوني

الملخص

تهتم هذه الأطروحة بتقديم دراسة كاملة عن الاضطرابات غير الغوسية والانتروبيا التي يتم الحصول عليها من عدة مجالات غير مقترنة بشكل محدود بالجاذبية. سيتم إجراء الدراسة في إطار الصيغتين المهمتين للجاذبية، وهما: القياس المتري البحت (النسبية العامة) والصياغة الأفينية البحتة ـ حيث ينتج الهيكل المتري عن ديناميكيات اتصال الزمكان. سنستخدم شكليات متغيرة في إطار عملنا ونوضح أنها تؤدي إلى مساحة مجال منحنية يمكن أن تنتج خروجًا واضحًا عن الجاذبية المترية البحتة. هذا العمل متوقع، ليس فقط لاشتقاق الكميات الرئيسية مثل الضغط غير ثابت الحرارة واضطرابات الانحناء في كل صيغة ، ولكن أيضًا لإلقاء الضوء على الإطار (في) الطابع التابع للاضطرابات البدائية. سوف يقف النهج على الزمكان التبادلي العام الذي يدعم الحقول العددية ويتطلب (بطبيعته) إمكانات غير صفرية. ببساطة، تغطي هذه الأطروحة دراسة شاملة ومنهجية للتضخم بناءً على نهج مختلف تمامًا للجاذبية: جاذبية التقارب البحت. الاضطرابات البدائية أهم عامل في علم الكونيات التضخمية وهذا العمل سيظهر بالتأكيد الجديد في المجال على المستويين النظري والمراقبة لأنه يهدف إلى تغطية الموضوع في إطار صيغ مختلفة للجاذبية التي هي في قلب التضخم.

مفاهيم البحث الرئيسية: التضخم، الاضطرابات غير الكظرية، عدم الانحراف، أوضاع التقوس المتساوي، الاقتران الأدنى وغير الأدنى، تباين الخواص، الصياغة المترية، الجاذبية التقريبية.

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Next, I would like to thank the academic committee who provided oversight on my thesis. Their advice and support have been crucial to this study, and I would like to appreciate their input. I will give special mention to Dr. Ehab Malawi through whom I learnt Mathematical methods of Physics and Analytical Physics – subjects I enjoyed immensely. I cannot forget Prof. Amrane Noureddine who was not only my instructor in solid state physics but was also the coordinator of the programme and was fundamental to helping me finish and defend my thesis in time. Next, I would like to appreciate the efforts of Dr. Mohammad Abdul Latif who introduced me to the fundamentals of Electrodynamics, Prof. Saleh Thaker who was not only very helpful in helping me register my courses from the very beginning but also taught me in Seminar course. It would be remiss of me to forget Prof. Ahmad Hasan, who helped through one of the toughest times of my life and helped me stand strong and finish this course, and Dr. Naslim Neelamkodan who taught me one of the most exciting courses – Astronomy and Astrophysics.

Lastly, but certainly not least, I would like to offer special thanks go to my mother, brothers, sisters, and all my friends – especially my classmates Mohammed Ahli, Khulood Alshehhi and Husam AlTakroori who supported me along the way and never lost hope in me.

Dedication

To my beloved mother, family and friends

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List of Abbreviations

CMB	Cosmic Microwave Background
GR	General Relativity
ΛCDM	Lambda Cold Dark Matter
Mpc	Megaparsec
3PCF	Three-point Correlation Function
2PCF	Two-point Correlation Function

Chapter 1

Chapter 1: Introduction

1.1 Overview

Ever since the proposal of inflation by Alan Guth (1981) as he was trying to solve the flatness and horizon problems, and its transformation into a working model by Linde and Albrecht, and Steinhardt (Albrecht & Steinhardt, 1982; Linde, 1982), it has been known as the most plausible scenario for the early universe – where it serves as a relevant mechanism for the origin of structure. Not only does it explain the observed spatial flatness and large-scale homogeneity of the universe at the time of recombination and photon decoupling, but it also predicts with a high accuracy the Gaussian statistics of the tiny anisotropies in the Cosmic Microwave Background (CMB) (Pinol, 2021). Generating primordial perturbations, which can be probed directly from the CMB anisotropies, is considered one of the important and interesting predictions of the inflationary models currently available (Akrami et al., 2019). However, several models exist that fit the current data and it is therefore necessary to determine a much more plausible inflationary paradigm. In the simplest model, we have a single scalar field called inflaton coupled to Einstein's gravity whose potential energy drives inflation leading to an adiabatic perturbation. In an effort to bridge the currently available models, and motivated by Elementary particle and High energy physics, various inflationary scenarios with multiple scalar fields have gained much attention in the last few decades (Lyth & Riotto, 1999). In general, these multiple fields interact with gravity nonminimally, a fact that necessitates studying the predictions in both Einstein and Jordan frames (Kaiser, 2016; Kaiser et al., 2013; White et al., 2013).

Furthermore, the cosmological perturbation in standard cosmology is known to be adiabatic and nearly Gaussian. However, measurement of the power spectrum of the temperature anisotropies in the CMB radiation expose a deficiency of power in low multipoles compared to the predictions from Lambda Cold Dark Matter (ΛCDM) cosmology. These deviations might be accounted for by the possibility of isocurvature modes (or non Gaussianity) (Schutz et al., 2014). Also, in single field models, isocurvature modes are completely suppressed in the long wavelength limits. In contrast, in multiple field models, these isocurvature modes can – in principle – amplify the curvature perturbations and alter their evolution well after they have crossed outside the horizon. All this, added to the fact that multiple field inflation produces non-adiabatic (Isocurvature) perturbations that could survive on superhorizon scales (Bassett et al., 2006; Langlois & Tent, 2012; Langlois & van Tent, 2011; Malik & Wands, 2005; Weinberg, 2004), as well as the non-Gaussian distribution of these perturbations, lead to a conclusion that Multiple field models should be brought to the forefront in the study of early inflation.

1.2 Statement of the Problem

Several studies have been dedicated to investigating multi-field inflation (Bardeen et al., 1983; Kaiser & Todhunter, 2010; Senatore & Zaldarriaga, 2012; Sfakianakis, 2014) and more recently (Carrilho et al., 2018; Martin & Pinol, 2021) have comprehensively demonstrated the physics of the early universe driven by multiple fields using a purely metric theory of gravity. Furthermore, numerous studies have also gone into studying the features of multifield inflation using extensions of GR and also Palatini formalism (Antoniadis et al., 2019; Carrilho et al., 2018; Tenkanen, 2020).

Since these studies are based only on purely metric gravity (i.e., GR in the case of minimal couplings) then one must treat the above features in different theories of gravity. In fact, besides being successful as a relativistic theory of gravity and accounting for various astrophysics phenomena, there is no reason to consider GR or even its modifications as the most viable theory for the early universe. The goal of this thesis is to study the evolution of the primordial perturbations in affine gravity which has not been exhaustively carried out previously. Affine gravity with scalar fields has been proposed to explain various phenomena (Azri, 2019; Azri et al., 2020; Azri & Nasri, 2021a).

1.3 Research Objectives

The objective of this thesis is to study inflation driven by multiple fields purely in the context of affine gravity. We shall study the entropy perturbations and discuss how they differ from the metric formulation and probe any deviations from Gaussianity that will result from this treatment.

It will be interesting to see the predictions that are obtained from this treatment with special focus being placed on both minimal and non-minimal coupling. From this, we shall obtain the tensor to scalar ratio r and compare it to that predicted by general relativity.

The results will then be used to make new predictions in the standard model.

Chapter 2

Chapter 2: Big Bang Cosmology

2.1 Introduction

Perhaps the best theory that has survived the test of time in attempting to explain the very beginnings of the universe is the Big Bang theory. The Big Bang theory relies on the cosmological principle (i.e., the universe is isotropic and homogeneous on large scales) (Stoeger et al., 1995). This implies that the metric of the universe must be of the form.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{1}$$

$$= -dt^2 + a^2(t)\left(\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right). \tag{2}$$

This metric is called the Friedmann-Lemaître-Robertson-Walker FLRW metric for flat space; $a = a(t)$ is the scale factor which depends on time; κ is the global curvature of the universe; r, θ, ϕ are co-moving coordinates and the Greek indices represent the space time components (0,1,2,3). The use of Latin indices will be to denote spatial components (1,2,3).

The Big Bang is supported by observations that will be discussed in a later subsection. First proposed by Georges Lemaître, it advances that about 12 to 14 billion years ago, the universe was only a few millimetres across and was in a hot dense state (Soter & Tyson, 2001). Since then, it has been expanding according to Einstein's field Equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}. \tag{3}$$

$\mu, \nu = 0, 1, 2, 3$; $R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu}$ is the Ricci tensor; $R = R^{\mu}_{\mu}$ is the Ricci scalar; and $T_{\mu\nu}$ is the stress-energy tensor. We shall use natural units where $c = \hbar = 1$ in addition to adopting the metric signature of $(-,+,+,+)$. These Equations can be solved for a homogeneous and isotropic universe to obtain the Friedmann Equations which will be represented in Figure 1.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \tag{4}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{\kappa}{a^2} \tag{5}$$

From the background metric in Equation (1),

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{6}$$

This is called the Energy conservation Equation.

ρ is the energy density; p is the pressure; $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

- $\kappa = -1$ represents a hyperbolic space (open space).
- $\kappa = +1$ represents a spherical space (closed space).

The universe is flat if $\kappa = 0$ or if it has a critical density of (Dodelson & Schmidt, 2020)

$$\rho_{crit} = \frac{3H^2}{8\pi G_N} \tag{7}$$

It is also easy to derive these equations using Newtonian Physics.

The graph below shows the evolution of the scale factor with time for a radiation and a matter dominated universe.

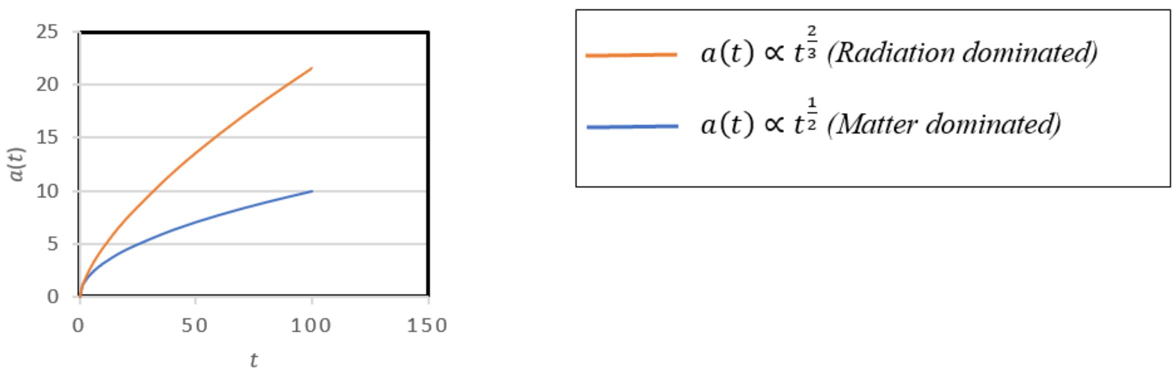


Figure 1: The evolution of the scale factor with time

2.2 Equation of State

The equation of state for a perfect fluid is characterised by a dimensionless number w, called the Equation of state parameter (Tamayo, 2020).

$$w = \frac{p}{\rho} \tag{8}$$

$w = 0$ represents a matter dominated universe.

$w = \frac{1}{3}$ represents a radiation or relativistic matter dominated universe.

$w = -1$ represents a cosmological constant dominated universe.

Using Equation (6), we can obtain a relationship between the energy density and the scale factor (Tamayo, 2020)

$$\rho \propto a^{-3(1+w)}. \quad (9)$$

This implies that:

$\rho \propto \frac{1}{a^3}$ for a matter dominated universe.

$\rho \propto \frac{1}{a^4}$ for a radiation dominated universe.

$\rho =$ constant for a cosmological constant dominated universe.

2.3 The Cosmological Constant

Allusion has been made of the cosmological constant in subsection 2.2 and we should formally state its relation to the energy density (Carroll, 2001).

$$\rho_0 = \frac{3}{8\pi G_N}\Lambda \quad (10)$$

ρ_0 is the energy density of empty space (or vacuum energy).

Λ is the cosmological constant. It does not change with changing a since it is a property of space.

For a flat universe with a positive cosmological constant,

$$a(t) \propto e^{\sqrt{\Lambda}t} = e^{Ht} \quad (11)$$

This is a spacetime called de sitter space.

For a closed universe with a positive cosmological constant,

$$a(t) \propto \frac{1}{\sqrt{\Lambda}} \cosh \sqrt{\Lambda}t \quad (12)$$

Equation (12) describes a universe which contracts, bounces and expands. We are now able to write the full Einstein's field Equations (3) as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu} \quad (13)$$

We can also rewrite the Friedmann Equation (5) to include matter (both baryonic matter and dark matter), radiation (and relativistic matter), and vacuum energy (García-Bellido, 2015).

$$H^2 = \frac{C_M}{a^3} + \frac{C_R}{a^4} + \Lambda - \frac{\kappa}{a^2} \qquad (14)$$

C_M and C_R are constants associated with matter and radiation, respectively. This can also be written as

$$H^2 = H_0^2\left(\frac{\Omega_M}{a^3} + \frac{\Omega_R}{a^4} + \frac{\Omega_\kappa}{a^2} + \Omega_\Lambda\right) \qquad (15)$$

$$\Omega_{\text{total}} = \Omega_M + \Omega_R + \Omega_\kappa + \Omega_\Lambda = 1 \qquad (16)$$

Today, $\Omega_M \sim 0.3089$, $\Omega_R \sim 0$, $\Omega_\kappa \sim 0$, $\Omega_\Lambda \sim 0.6911$ (Aghanim et al., 2020)

The interpretation of this is that when the universe was young, radiation dominated and the scale factor, $a(t)$ increased as $t^{\frac{1}{2}}$. However, over a period, it transitioned into a matter dominated universe and $a(t)$ increased as $t^{\frac{2}{3}}$. It is only recently, on cosmological timescales that it has this time transitioned into vacuum energy dominated, with $a(t)$ evolving as $e^{H_0 t}$ where H_0 is the Hubble parameter at the present time and it is approximately 70 km/s/Mpc (Bahcall, 2015).

2.4 Successes of the Big Bang Model

Firstly, the Big Bang theory enlightens us that the universe had a beginning (Dodelson & Schmidt, 2020).

Secondly the universe has been expanding from the beginning according to Hubble's law (i.e., $v = H_0 D$) – where H_0 is the Hubble parameter at the time of measurement (Mohapatra, 2021). This demonstrates that the universe was once compact.

Furthermore, the Big Bang predicts that the universe was initially very dense and hot. In 1964, radio astronomers Ronald Wilson and Arno Penzias detected the CMB which pervades the observable universe and is the remnant of the heat that existed at the beginning of the universe (Gawiser & Silk, 2000).

Lastly, the Big Bang predicts the abundance of light elements like Hydrogen and Helium in the early universe, a process called "Big Bang Nucleosynthesis" (BBN). The prediction accords with data from observation (Burles et al., 2001; Copi et al., 1995).

2.5 Shortcomings of the Big Bang Model

The first limitation of the Big Bang model of cosmology is the Flatness-Oldness problem. To study this problem, we define a density parameter Ω_0 (Coles & Ellis, 1997).

$$\Omega_0 = \frac{Measured\ density\ of\ the\ universe}{Critical\ density\ of\ the\ universe} = \frac{\rho}{\rho_{crit}} \quad (17)$$

From Equations (5), (7) and (16),

$$1 - \Omega_0 = -\frac{\kappa}{(aH)^2} \quad (18)$$

$\Omega_0 > 1$ implies that the universe is "closed" and will eventually collapse. If Ω_0 was above unity in the beginning, it would have collapsed early in its evolution before the formation of galaxies.

$\Omega_0 < 1$ implies that the universe is "open" and will expand forever. If $\Omega_0$was below unity in the beginning, it would have expanded so rapidly that structures would not have formed.

$\Omega_0 = 1$ implies that the universe is "flat" and has critical density.

From observations by (Bennett et al., 2003), $\Omega_0 \sim 1$, since $\rho \sim 9 \times 10^{-27} kgm^{-3}$. This is strange since $\Omega_0 = 1$ is an unstable equilibrium point and the implication is that within 10^{-43} seconds of the Big Bang, the density of the universe was within 1 part in 57 of the critical density for the curvature to remain this flat after the 13.4 billion years that the Big Bang predicts the age of the universe to be.

The second limitation is the so-called Horizon problem. Observations from the CMB radiation exhibit a marked degree of large-scale homogeneity and thermal equilibrium which is at odds with the standard Big Bang model. The presence of the cosmological horizon precludes any two points – whose distance of separation exceeds the horizon size – from reaching thermal equilibrium. This is because they cannot have ever been in causal contact (Kinney, 2004). Given that $H_0 \sim 6 \times 10^{-61} M_{pl}$ and $T_0 \sim 5 \times 10^{-31} M_{pl}$, then

$$\frac{d_{pl}}{d_c} \sim \frac{T_0}{H_0} \sim 10^{30} \quad (19)$$

d_{pl} is the size of the universe at Planck scales; d_c is the size of the causal regions; The current temperature of the universe $T_0 = (2.7 \pm 10^{-5})K \cong 2.3 \times 10^{-13}$GeV.

This implies that at Planck scales, there are 10^{90} disconnected regions. If ordinary expansion cannot iron out inhomogeneities, it is striking that the universe currently is uniform and that the CMB radiation temperature is the same in all directions, considering that there were so many regions that were never in causal contact at Planck scales (de Haro & Elizalde, 2022).

The last limitation that we shall discuss is the Magnetic monopole problem (also called the Exotic-relics problem). The Grand Unified theories predict that the very high temperature of the Big Bang should have produced magnetic monopoles. Yet all observations have failed to detect their existence (Acharya et al., 2021; Dirac, 1976; Acharya et al., 2019). According to the theories, the strong force, the weak force and the electromagnetic force only became fundamental forces 10^{-11}s after the big bang, due to spontaneous symmetry breaking.

Chapter 3

Chapter 3: Inflationary Cosmology

3.1 Introduction

The idea of inflationary cosmology was advanced to solve problems such as those discussed in suction (2.5). The shortcomings of the big bang arise from assuming that $\ddot{a} < 0$, implying that Ω_0 will tend to always shift away from 1. However, inflationary cosmology proposes that $\ddot{a} > 0$. Inserting this in Equation (4) predicts

$$\rho + 3p < 0 \Leftrightarrow \frac{d}{dt}(aH)^{-1} < 0 \qquad (20)$$

Before proceeding further, it will be necessary to define a few terms.

Firstly, the proper distance is defined as the distance between two simultaneous events A and B in an inertial reference frame in which they occur at $t_A = t_B$ (Hogg, 1999). The homogeneity and isotropy of the metric in Equation (2) allows us to set $d\phi = d\theta = r = dt = 0$ in Equation and obtain

$$d_p = s(t) = \int_0^s ds' = a(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}} \qquad (21)$$

This yields the following solutions depending on whether k is +ve, 0, or -ve:

$$d_p = a(t) = \begin{cases} \frac{1}{\sqrt{\kappa}}\sin^{-1}\left(r\sqrt{\kappa}\right) & \text{for } \kappa > 0 \\ r & \text{for } \kappa = 0 \\ \frac{1}{\sqrt{|\kappa|}}\sinh^{-1}\left(r\sqrt{|\kappa|}\right) & \text{for } \kappa < 0 \end{cases}$$

However, if we consider the worldline of a light ray connecting the two events, $ds = 0$. Which implies

$$\frac{dt}{da} = \frac{dr}{\sqrt{1-kr^2}} \longrightarrow \int_{t_e}^{t_0} \frac{dt'}{a(t')} = \int_0^r \frac{dr}{\sqrt{1-kr^2}} = \frac{d_p(t_0)}{a(t_0)} \qquad (22)$$

$d_p(t_0)$ is the proper distance between two co-moving observers at a time $t = t_0$.

$$d_p(t_0) = a(t_0)\int_{t_e}^{t_0} \frac{dt'}{a(t')} \qquad (23)$$

When $t_e \to 0$, $d_p(t_0) = d_H$.

d_H is called the Particle Horizon.

It is more useful for us to eliminate t in the expression by using the expression $dt = \frac{da}{(aH)}$. We end up with

$$d_H = a(t_0)\int_0^a \frac{da}{a^2H(a)} = a(t_o)\int_0^a \frac{d(\ln a)}{(aH)} \tag{24}$$

We define the Particle horizon as the proper distance between the observer that receives the light signal at present and the comoving particle that emitted this light at the very beginning of the Universe (Bolotin & Tanatarov, 2014).

Next, we shall define the Hubble radius R_H as the distance from the observer, beyond which objects recede at a rate greater than the speed of light due to the expansion of the universe. (Seshavatharam & Lakshminarayana, 2012). From Hubble's law,

$$R_H = \frac{1}{H} \tag{25}$$

3.2 Solving the Flatness Problem

During inflation, the Hubble parameter, H remains constant and therefore we can see from Equation (18)

$$\Omega_0 - 1 = \frac{\kappa}{(aH)^2} \propto \frac{1}{a^2} \tag{26}$$

As a^2 in Equation (26) increases rapidly (i.e., $a(t) = e^{Ht}$), it is obvious that Ω_0 will tend rapidly towards 1 at which point, the Big Bang cosmology can take over. (Tsujikawa, 2003)

3.3 Solving the Horizon Problem

Let us assume that during inflation, the scale factor rose as Equation (11). In that case, we can fine tune the number of e-folds, N necessary to solve the Horizon problem. Equation (22) becomes

$$d_H = e^{Ht}\int_{t_i}^{t_0} \frac{dt'}{e^{Ht}} \approx H^{-1}e^{H(t_0 - t_i)} \tag{27}$$

Since $t_0 - t_i \gg H^{-1}$ (Hubble radius), d_H grows as fast as $a(t)$ as shown in the Figure 2.

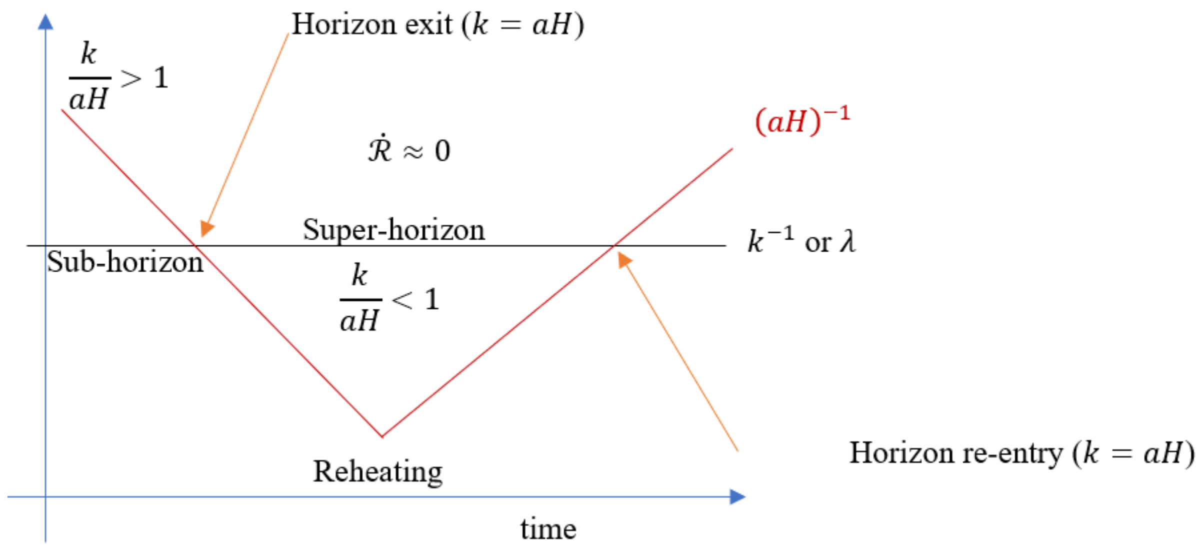


Figure 2: The evolution of the comoving curvature perturbation with time. Beyond the Hubble radius, it remains invariant. Figure from (Baumann, 2012).

During inflation the causally connected region must have become much smaller than it was at the onset as shown in Figure 2 (Tsujikawa, 2003).

$$d_H \frac{a_{end}}{a(t_0)} < d_{H_i} \frac{a_{end}}{a_i} = d_{H_i}(t_i) e^N \tag{28}$$

a_{end} is the scale factor at the end of inflation; a_i at the beginning of inflation; d_{H_i} is the particle horizon at the beginning of inflation; t_i the time at the beginning of inflation and t_0 is the time at the end of inflation. According to (Remmen & Carroll, 2014), we need $N \geq 60$ to solve the horizon problem.

3.4 Solving the Monopole Problem

Inflation very easily resolves the Magnetic monopole problem since for $N \geq 55$ the magnetic monopoles will be severely diluted to be present in any considerable concentration after the inflationary period (Lazarides, 2006).

3.5 Dynamics of Inflation

In the simplest single field inflation, the associated field satisfies the so-called slow-roll conditions where it evolves slowly along its nearly flat potential. The potential energy of the inflaton depends, in general, on various physical quantities like its mass and self-coupling parameter, and one can show that to produce an amplitude of density perturbations compatible with observation, one must severely fine-tune some of these physical parameters in most models.

This has opened the possibility to models of inflation with nonminimal couplings such as (standard model) Higgs-inflation (Bauer & Demir, 2008; Bezrukov & Shaposhnikov, 2008). In these models, the nonminimal coupling parameters play an important role in getting a tiny density perturbation that fits the data without adjusting any physical parameter. It is also known that nonminimal couplings to gravity are essential at the quantum level where they gain nonzero values even if they are absent at the tree-level (Birrell et al., 1984; Buchbinder et al., 1992).

Various classes of inflation with nonminimal coupling to gravity have been thoroughly performed in the context of purely metric gravity where predictions are studied in both Einstein and Jordan frames (Kaiser, 2016; Kaiser & Sfakianakis, 2014; Kaiser & Todhunter, 2010; Schutz et al., 2014).

Let us consider the minimally coupled action below (Azri & Demir, 2017; Riotto, 2017)

$$\mathcal{S}[g_{\mu\nu}, \phi\,] = \int \sqrt{-g}\left(\frac{1}{2}M_p^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)d^4x \qquad (29)$$

The first term represents the gravitational (also called the Albert-Hilbert) term, $\mathcal{S}_g$; the second and third term represent the action of the scalar field, $\mathcal{S}_\phi$ and $V(\phi)$ is the potential energy.

Variation of $\mathcal{S}[g_{\mu\nu}, \phi]$ with respect to the metric, $g_{\mu\nu}$ yields the Einstein's Equation (3) in which the energy momentum tensor

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{S}_\phi\right)}{\delta g_{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right) \qquad (30)$$

Lowering the indices, and calling up upon Equation (2), we obtain

$$T_{00} = \rho = \frac{1}{2}\dot{\phi}^2 + V + \frac{1}{2}\frac{(\nabla\phi)^2}{a^2} \qquad (31)$$

$$T_{ii} = p = \frac{1}{2}\dot{\phi}^2 - V - \frac{1}{6}\frac{(\nabla\phi)^2}{a^2} \qquad (32)$$

For a homogeneous background, $\frac{\nabla\phi}{a} \to 0$ and we end up with

$$\rho = \tfrac{1}{2}\dot{\varphi}^2 + V \text{ and } p = \tfrac{1}{2}\dot{\varphi}^2 - V \qquad (33)$$

Where φ is the background field with only a time dependence. We shall deal with the spatial dependence in a later section.

From Equation (8), the equation of state becomes

$$w = \frac{\frac{1}{2}\dot{\varphi}^2 - V(\varphi)}{\frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \quad (34)$$

Which implies that if $V \gg \dot{\varphi}^2$, then $w \approx -1 < -\frac{1}{3}$ (i.e., $p \approx -\rho = -V$) which drives inflation the same way the cosmological constant does as seen in section (2.2.); and the scale factor evolves as $a \sim e^{Ht}$ as seen in Equation (11). However, if the field dominates the energy, then its energy density will dominate both the energy density of radiation and that of matter (Senatore, 2017).

Variation of $\mathcal{S}[g_{\mu\nu}, \varphi]$ with respect to the field, φ gives us the Klein-Gordon Equation

$$\Box\phi - V_{,\phi} = 0; \qquad \Box \equiv \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}g^{\mu\nu}\partial_\mu\right) \quad (35)$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + V_{,\phi} = 0 \quad (36)$$

During expansion, the scalar field rolls down its potential with a gradient $V_{,\phi} \equiv V' \equiv \frac{dV}{d\phi}$. $3H\dot{\phi}$ term is the damping term.

For a homogeneous universe, $\frac{\nabla\phi}{a} \to 0$ and therefore,

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0 \quad (37)$$

3.6 Slow-roll Inflation

In slow-roll inflation, we assume the following conditions (Pozdeeva, 2021)

$$\left|\frac{\dot{\varphi}}{2}\right| \ll |V| \text{ and } |\ddot{\varphi}| \ll |3H\dot{\varphi}|, \left|V_{,\varphi}\right| \quad (38)$$

Combining Equations (5), (33), (37) and (38) in a zero curvature Friedmann universe, we obtain

$$H^2 \approx \frac{8\pi}{3m_p^2}V = \frac{1}{3M_p^2}V$$
$$3H\dot{\varphi} \approx -V_{,\varphi} \quad (39)$$

$M_p^2 = m_p^2/8\pi$ is the reduced Planck mass and $m_p^2 = 1/G_N$

From condition (1) of Equation (38), H is nearly constant and $\dot{H} < H^2$. This implies that $\frac{\dot{H}}{H^2} < 1$. From this, we shall define the first slow-roll parameter ϵ

$$\epsilon = -\frac{\dot{H}}{H^2} \tag{40}$$

Using Equations (39) and (40), we derive the expression

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2 \tag{41}$$

We can define our second slow roll parameter as (Liddle, 1999)

$$\eta = M_p^2\left(\frac{V_{,\varphi\varphi}}{V}\right) \tag{42}$$

We define $V_{,\varphi\varphi} \equiv (\frac{d^2V}{d\varphi^2})$

And the last one

$$r = 16\epsilon \tag{43}$$

For slow-roll inflation, $|\epsilon|, |\eta| \ll 1$

Inflation ends when $\epsilon = 1$ and the number of e-folds required for it to end is obtained from Equation (11) and is given by the expression (Kinney, 2004).

$$N = \int_{t_i}^{t_f} H dt = \int_{\varphi_i}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi \approx -\frac{1}{M_p^2}\int_{\varphi_i}^{\varphi_f} \frac{V}{V_\varphi} d\varphi \geq 50 - 60 \tag{44}$$

Another important parameter that ought not to be left out is the spectral index n_s. For a universe without extreme mass fluctuations, the power spectrum should be a power law $\mathcal{P}(k) = Ak^{n_s}$ where n_s is the spectral index obtained from

$$n_s = 1 - 6\epsilon + 2\eta + \mathcal{O}(\epsilon^2, \eta^2) \tag{45}$$

3.7 Comment about the End of Inflation

It has been alluded to that inflation ends when $\epsilon = 1$ and at this point $\ddot{\varphi}$ from equation (37) can no longer be ignored. The field will roll down its potential until it begins to oscillate about its minimum.

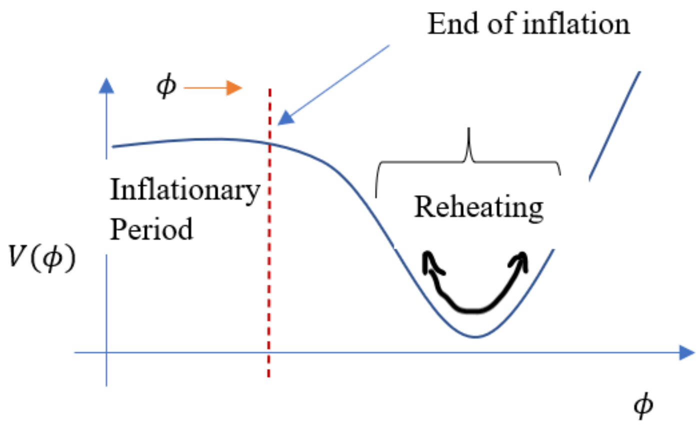


Figure 3: The field evolves along a flat potential but rolls down quite dramatically

From Equation (33) and (37), we get the rate of the energy density loss which is given by the equation

$$\dot{\rho} = -3H\dot{\varphi}^2 \tag{46}$$

During the oscillation of the field around its minimum potential, the energy density will decay in the order of $\rho \sim a^{-3}$. This is like the matter dominated universe seen in section (2.2.). And since the field is acting like pressure-less matter, we can re-write Equation (46) as

$$\dot{\rho}_\phi = -3H\rho_\phi \tag{47}$$

The field will decay into lighter particles which eventually thermalize to a temperature, T_R in a process known as reheating. It should be here noted that during the short period of inflation, the universe must expand adiabatically. It is only at the end of inflation, during the transition from an inflaton dominated universe to a radiation dominated universe that non-adiabaticity is considered. However, let us calculate the decay rate, Γ_ϕ of the inflaton field as shown below (Riotto, 2017).

$$\Gamma_\phi = \frac{1}{\tau_\phi} = H = \frac{1}{\sqrt{3}M_p}\rho_\phi = \frac{1}{M_p}\sqrt{\frac{\rho_R}{3}} \tag{48}$$

We consider that the energy of the inflaton is converted into the energy of the radiation.

If the reheating temperature is T_{RH}, then (Cook et al., 2015)

$$\rho_R = \frac{\pi^2}{30} g_* T_{RH}^4. \tag{49}$$

Where g_* is the radiation degrees of freedom. Combining (48) and (49) we get

$$T_{RH} = \left(\frac{90}{\pi^2 g_*}\right)^{\frac{1}{4}} \sqrt{\Gamma_\phi M_p} \tag{50}$$

This is the temperature at which the radiation dominated epoch begins and the standard big bang cosmology takes over.

Chapter 4

Chapter 4: Adiabatic and Non-adiabatic Perturbations during Inflation

In the previous chapter, we have dealt with inflation in a homogeneous and isotropic universe. However, one of the unexpected successes (possibly the greatest success) of the inflationary theory is that it explains how large-scale expansion, in effect, "sowed the seeds" of structure formation. By structure we mean galaxies and galactic clusters. During the inflationary phase, the quantum fluctuations that are initially present in the cosmic soup are amplified to super-Hubble scales ending up as density perturbations which are exhibited in the anisotropy of the CMB radiation[1].

For a single scalar field, the fluctuations produced are adiabatic, as shall be seen later, and the density perturbations produced are gaussian with $n_s \approx 1$.

Let us study the perturbations in the Klein Gordon Equation (36). We begin by writing our field in component parts as shown below (Kaiser & Todhunter, 2010).

$$\phi = \phi(x^\mu) = \varphi(t) + \delta\phi(x^\mu) \qquad (51)$$

$\delta\phi(x^\mu)$ is the small linear perturbation term around the homogeneous background, $\varphi(t)$.

We use first order perturbations because the density fluctuations in the early universe are too small for higher order terms to be significant. The quantum fluctuation of the inflaton is $\delta\phi = T_H = \frac{H}{2\pi} = Hawking\ temperature$ (Bunch & Davies, 1978; Lazarides, 2006; Vilenkin & Ford, 1982) and it in turn induces density perturbations given by $\delta\rho = V_{,\phi}\,\delta\phi$.

The density perturbations eventually grow with inflation going beyond super-Hubble scales.

Considering the case with no interactions, inserting Equation (47) into (36), and separating the background and first order equations, we obtain Equation (37) and

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{1}{a^2}\nabla^2\delta\phi + V_{,\phi\phi}\,\delta\phi = 0 \qquad (52)$$

[1] In the early stages of the hot Big Bang, radiation was undergoing Compton scattering due to interaction with electrons. However, as stable nuclei began to form, photons were able to escape, cooling adiabatically as the universe continued to expand while retaining a black body spectrum. It is the picture of this last scattering that we observe in the CMB radiation.

Decomposing the field perturbation using Fourier transform, $\nabla^2\delta\phi = -k^2\delta\phi$. k is the comoving wavenumber.

For a case of a scalar field and metric with interactions, we must construct a perturbed FLRW metric. To do that, we shall employ the formalism employed in (Bardeen, 1980; Riotto, 2017; Uggla & Wainwright, 2011). First, we do a Scalar, Vector, Tensor (SVT) decomposition. It involves the spontaneous breakdown of the symmetries in GR by the FLRW background to $SO(3)$ group of global spatial rotations, performing a Fourier transform to single out a vector $\vec{k}$, and then decompose the states into the helicity eigenstates with respect to the $SO(2)$ rotations. For an arbitrary scalar field χ under rotation θ, $\chi_{\vec{k}} \to e^{im\theta}\chi_{\vec{k}}$. θ determines the helicity of the state.

$m = 0$ is a scalar, $m = \pm 1$ is a vector and $m = \pm 2$ is a tensor.

The idea of SVT decomposition can best be demonstrated by decomposing the metric tensor, $g_{\mu\nu}$ (Bassett et al., 2006)

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{0i} \\ g_{i0} & g_{ij} \end{pmatrix};\ g_{00} = -(1 + 2\Phi) \tag{53}$$

$$g_{i0} = g_{0i} = 0 + 2a(\partial_i B - S_i);\ \partial^i S_i = 0 \tag{54}$$

$$g_{ij} = a^2(1 - 2\Psi)\delta_{ij} + 2\partial_{ij}F + 2\partial_{(i,}F_{j)} + h_{ij};\ \partial^i F_i = h^i_i = \partial^i h_{ij} = 0 \tag{55}$$

Counting the components, we get a total of 4 scalars (Φ, Ψ, B, F); two vectors (S_i, F_j), and one tensor h_{ij}

The scalar component is responsible for structure formation, the tensor component is responsible for the production of primordial gravitational waves, and the vector perturbations can be ignored because they decay over time.

The scalar degrees of freedom can then be collected to write the perturbed line element as

$$\begin{aligned} ds^2 &= -(1 + 2\Phi)dt^2 + 2a(\partial_i B)dx^i dt \\ &+ a^2\{(1 - 2\Psi)\delta_{ij} + 2\partial_i\partial_j F\}dx^i dx^j \end{aligned} \tag{56}$$

Working in the longitudinal gauge, we set $B = F = 0$ and end up with

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j \tag{57}$$

This is known as the Newtonian Gauge and Φ, Ψ are the gauge invariant Bardeen potentials.

GR is invariant under diffeomorphisms. So, any physical quantity should be invariant under the transformation.

$$x^\mu \to x^{\mu'} = x^\mu + \xi^\mu \tag{58}$$

ξ^μ can also be decomposed as was done in (53 – 55) as $\xi^\mu = \xi^0, \partial_i A + B_i; \partial^i B_i = 0$ from which we can infer that there are 2 scalars (ξ^0, A) and one vector B_i degree of freedom.

Using Equation (36) and (57) we obtain the gauge-dependent equation of motion

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + \frac{k^2}{a^2}\delta\phi + V_{,\phi\phi}\delta\phi = -2V_\phi\Phi + \dot{\varphi}(\dot{\Phi} + 3\dot{\Psi}) \tag{59}$$

and the first-order perturbed Einstein Equations give (Brax et al., 2009)

$$3H\left(\dot{\Psi} + H\Phi\right) + \frac{1}{a^2}k^2\Psi = -\frac{1}{2M_p^2}\delta\rho, \tag{60}$$

$$\dot{\Psi} + \mathrm{H}\Phi = -\frac{1}{2M_p^2}\delta q, \tag{61}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + H\dot{\Phi} + \left(3H^2 + 2\dot{H}\right)\Phi = \frac{1}{2M_p^2}\delta p + \frac{1}{3a^2}k^2(\Phi - \Psi), \tag{62}$$

$$\frac{1}{a^2}k^2(\Phi - \Psi) = 0. \tag{63}$$

Where $\delta\rho$ is the density perturbation obtained from the 00 component of the Einstein's Equations, δq is the momentum flow term obtained from the $0i$ component and δp is the isotropic pressure perturbation obtained from the ij component. They are defined below as

$$\delta\rho = \dot{\varphi}\delta\dot{\phi} - \dot{\varphi}^2\Phi + V_\phi\delta\phi \tag{64}$$

$$\delta q = -\dot{\varphi}\delta\phi \tag{65}$$

$$\delta p = \dot{\varphi}\delta\dot{\phi} - \dot{\varphi}^2\Phi - V_\phi\delta\phi \tag{66}$$

From Equation (63) it can be observed that for the canonical action in Equation (29) the anisotropic pressure is absent and therefore, $\Phi = \Psi$

However, it should be noted that for non-minimal coupling (Kaiser & Todhunter, 2010),

$$\partial_i \partial_j (\Phi - \Psi) = -\frac{1}{f} \partial_i \partial_j \delta f; i \neq j. \quad (67)$$

Where f is the coupling function.

Next, we employ the spatially flat gauge-invariant Mukhanov-Sasaki variable which has the definition (Kaiser et al., 2013; Mukhanov, 1988; Sasaki, 1986)

$$Q \equiv \delta\phi + \frac{\dot{\varphi}}{H}\psi \quad (68)$$

which is directly related to the curvature perturbation in the comoving gauge and according to the lectures (Baumann, 2011), the momentum density of the comoving gauge will vanish and therefore $\delta T_{0i} \equiv 0$. This implies that from Equation (65), $\delta q = -\dot{\varphi}\delta\phi$.

(Lyth, 1985) defines the comoving curvature perturbation as

$$\mathcal{R} \equiv \psi - \frac{H}{\rho + p}\delta q \quad (69)$$

Which from Equations (33), (65) and (68) yields

$$\mathcal{R} = \psi + \frac{H}{\dot{\varphi}}\delta\phi = \frac{H}{\dot{\varphi}}Q. \quad (70)$$

It will now be necessary to write down some definitions that will help us in our study

1. An adiabatic fluid is one where (Kodama & Sasaki, 1984),

$$\frac{\delta p_{ad}}{\delta\rho} = \frac{\dot{p}}{\dot{\rho}} \quad (71)$$

   Where p_{ad} is the adiabatic pressure perturbation.

2. For non-adiabatic fluids, especially as will be seen in multi-field inflation (Huston & Christopherson, 2012),

$$\delta p = \delta p_{nad} + c_s^2 \delta\rho \quad (72)$$

   Where δp_{nad} is the non-adiabatic pressure perturbation; $c_s^2 = \frac{\dot{p}}{\dot{\rho}}$ is the sound speed for the fluid.

3. Total entropy perturbation is defined as

$$\mathcal{S} = H\left(\frac{\delta p}{\dot{p}} - \frac{\delta\rho}{\dot{\rho}}\right) \tag{73}$$

From this expression, if the entropy perturbation, $\mathcal{S} = 0$, then we obtain the expression for adiabatic perturbation in Equation (71), and Equation (72) becomes $\delta p = c_s^2 \delta\rho$.

Using Equation (73) we can substitute δp, $\delta\rho$, $\dot{\rho}$ and $\dot{p}$ to obtain

$$\mathcal{S} = \frac{2V_\phi\left(\dot{\varphi}\delta\dot{\phi} - \ddot{\varphi}\delta\phi - \dot{\varphi}^2\Phi\right)}{9H\dot{\varphi}^3 + 6\dot{\varphi}^2 V_\phi} \tag{74}$$

However, we can construct a gauge-invariant density perturbation from (Bardeen, 1980)

$$\delta\rho_m \equiv \delta\rho - 3H\delta q \tag{75}$$

$$= \dot{\varphi}\delta\dot{\phi} - \ddot{\varphi}\delta\phi - \dot{\varphi}^2\Phi \tag{76}$$

Substituting this into Equation (74) we obtain

$$\mathcal{S} = \frac{2V_\phi \delta\rho_m}{9H\dot{\varphi}^3 + 6\dot{\varphi}^2 V_\phi} \tag{77}$$

But also combining Equations (60) and (61) yields

$$\delta\rho_m = -\frac{2M_p^2}{a^2} k^2 \Psi \tag{78}$$

This implies that our entropy perturbation is given by

$$\mathcal{S} = -\frac{4V_\phi M_p^2}{a^2\left(9H\dot{\varphi}^3 + 6\dot{\varphi}^2 V_\phi\right)} k^2\Psi = -\frac{4V_\phi \rho}{3\left(9H\dot{\varphi}^3 + 6\dot{\varphi}^2 V_\phi\right)}\left(\frac{k}{aH}\right)^2 \Psi \tag{79}$$

Lastly, let us compute the non-adiabatic pressure from Equation (72)

$$\delta p_{nad} = \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho \tag{80}$$

$$= -2V_\phi \delta\phi - \frac{2V_\phi}{3H\dot{\varphi}}\delta\rho \tag{81}$$

$$= \frac{4V_\phi M_p^2}{3a^2 H\varphi^\cdot} k^2\Psi = \frac{4V_\phi \rho}{9H\dot{\varphi}} \left(\frac{k}{aH}\right)^2 \Psi \quad (82)$$

It can be concluded from this that on large scales, (i.e., $k \ll aH$) the non-adiabatic pressure becomes too small (Sasaki & Stewart, 1996). It can also be concluded from Equation (79) that the entropy perturbation is also suppressed on cosmologically large scales.

It would be inadequate to close this section without some mention of the evolution of the co-moving curvature perturbation (Baumann, 2012).

$$\dot{\mathcal{R}} = -\frac{H}{\dot{\varphi}^2} \delta p_{nad} + \mathcal{O}\left(\frac{k}{aH}\right)^2 \quad (83)$$

This implies that

$$\dot{\mathcal{R}} = -\frac{4V_\phi \rho}{9\dot{\varphi}^3} \left(\frac{k}{aH}\right)^2 \Psi + \mathcal{O}\left(\frac{k}{aH}\right)^2 \quad (84)$$

We can conclude that at super-Hubble scales, when $\frac{k}{aH} \ll 1$, $\mathcal{R}$ is conserved.

Chapter 5

Chapter 5: Non-gaussianity

5.1 Introduction

We have seen in chapter 3 that after the inflationary period, there was a period of reheating that the universe became so hot for the photons and electrons that had been formed to remain free. The photons were constantly being scattered – in a process called the Thompson scattering – by the electrons and therefore were not free to permeate through the dense plasma. However, as the temperature cooled, the universe became transparent to photons since electrons had started to combine with protons to form stable atoms. This process is called recombination (van Tent, 2021).

The photons released at this period are what we refer to as the CMB radiation and it seems to be issuing from a surface beyond which we can observe, which is referred to as the last scattering surface. Prodding through the CMB radiation has enabled us to make some interesting cosmological observations that enrich us with the knowledge of the conditions of the universe in its initial stages. One of the observations that have been made is that the fluctuations of the CMB radiation are nearly Gaussian. However, many theories predict some divergence from Gaussianity (Hahn et al., 2019; Yadav & Wandelt, 2010). This enables us to eliminate models of inflation that do not agree with the observed level of non-Gaussianity.

5.2 The Power Spectrum

Following from the Heisenberg uncertainty principle, quantum fluctuations are random (though chaotic is possibly the better description). As a result, we cannot – with precision – measure the distribution of temperature in the CMB fluctuations in all directions, or even galaxy positions. The best we can do is measure the statistical properties of the distribution. For Gaussian distribution, all we need is the two-point correlation function defined as

$$\langle \mathcal{R}_{\boldsymbol{k}} R_{\boldsymbol{k}'} \rangle \equiv \frac{H^2}{2k^3}(1 + k^2\tau^2), \tag{85}$$

Where $\tau = \int_0^t \frac{1}{a(t')} dt'$ is referred to as the conformal time. This is the power spectrum of the comoving curvature perturbation.

However, at super horizon scales, (i.e., $k \ll aH$), $k^2\tau^2 \ll 1$. This means that the power spectrum can be defined as

$$\mathcal{P}_{\mathcal{R}}(k) = \langle \mathcal{R}_k R_{k'} \rangle = \left(\frac{H^2}{2k^3}\right)_{|k \ll aH} \tag{86}$$

Since H and k have dimensions, it is more convenient to define the dimensionless power spectrum as (Baumann, 2012)

$$\Delta_s^2 = \Delta_{\mathcal{R}}^2 = \frac{k^3 \mathcal{P}_{\mathcal{R}}(k)}{2\pi^2}. \tag{87}$$

$$= \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\varphi}}\right)^2_{|k=aH}. \tag{88}$$

It can also be expressed as a power law as

$$\frac{k^3 \mathcal{P}_{\mathcal{R}}(k)}{2\pi^2} = A_s k^{n_s - 1}, \tag{89}$$

Where A_s is the amplitude of density perturbations which we shall make use of in our results.

For tensors,

$$\Delta_{\mathcal{T}}^2 = 2 \times \frac{k^3 \mathcal{P}_{\mathcal{T}}(k)}{2\pi^2}. \tag{90}$$

The 2 is simply to account for the two polarization modes of the spectrum.

We then end up with

$$\Delta_{\mathcal{T}}^2 = \frac{8}{M_p^2} \left(\frac{H}{2\pi}\right)^2 \tag{91}$$

From these two, we can define a useful quantity called the tensor-to-scalar ratio r, which incidentally we mentioned in Equation (43). We now formally define it as

$$r \equiv \frac{\Delta_{\mathcal{T}}^2(k)}{\Delta_s^2(k)} = \frac{8}{M_p^2} \left(\frac{d\phi}{dN}\right)^2 ; dN \equiv Hdt \tag{92}$$

If we integrate this expression over the whole period of inflation, we obtain

$$\frac{\Delta\phi}{M_p} = \int_{N_{end}}^{N_{CMB}} \sqrt{\frac{r}{8}}\, dN. \tag{93}$$

For a single field inflation, since the curvature perturbation $\mathcal{R}$ is Gaussian, then all we need to calculate is the power spectrum – since it contains all the statistical information required (Maldacena, 2003). However, to determine non-Gaussianity, the 3PCF must be determined. In k-space, it is also called the Bispectrum.

5.3 The Bispectrum

We define the 3PCF as

$$\langle \mathcal{R}_{\boldsymbol{k}_1} \mathcal{R}_{\boldsymbol{k}_2} \mathcal{R}_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \mathcal{B}_{\mathcal{R}}(k_1, k_2, k_3) \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) \tag{94}$$

The amplitude of non-Gaussianity can be calculated from

$$\mathcal{R}(x) = \mathcal{R}_g(x) + \frac{3}{5} f_{NL}^{local} \{\mathcal{R}_g(x)\}^2, \tag{95}$$

where the first term represents the linear Gaussian part, and the second term represents the non-linear Gaussian correction at some fixed point x. f_{NL}^{local} is the amplitude of non-Gaussianity. Plugging Equation (95) into (94) we obtain

$$\begin{aligned} \mathcal{B}_{\mathcal{R}}(k_1, k_2, k_3) = \frac{6}{5} f_{NL}^{local} \{ & \mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_2) + \mathcal{P}_{\mathcal{R}}(k_2)\mathcal{P}_{\mathcal{R}}(k_3) \\ & + \mathcal{P}_{\mathcal{R}}(k_3)\mathcal{P}_{\mathcal{R}}(k_1)\} \end{aligned} \tag{96}$$

In (Kaiser et al., 2013), the authors go through the details of the calculation of primordial Bispectrum from multifield inflation with non-minimal couplings in the context of GR. We should take away from this that multifield inflation always produces some non-Gaussianity though the level depends on the model of inflation being studied.

We are now in position to present the new features of inflation in the context of purely affine gravity.

Chapter 6

Chapter 6: Entropy Perturbations in Affine Gravity

6.1 Introduction

Having gone through the formalism of the previous chapters, we can now introduce inflationary dynamics of two-field inflation in the context of purely affine gravity. We shall find that in this instance, the affine connection does not depend on the metric but rather the metrical structure results from the affine connection. We shall use specific non-canonical field kinetic terms to flatten the curved manifold.[2] In this way, new predictions will be made by virtue of the coupling function being solely coupled to the potential.

Two important cases will be studied. The first one will be when the kinetic terms are canonical. In this instance, we shall observe that the induced field space metric will tend to be conformal to flat. Next, we shall study what happens if the same coupling function is set on $\boldsymbol{R}_{\mu\nu}(\Gamma)$ and $\delta_{ab}\nabla_{\mu}\phi^{a}\nabla\phi^{b}$ and will analyse what happens in the single-field limit using a quartic potential and derive the spectral index and the gravitational waves produced. Gravitational waves are characterised by a small tensor-to-scalar ratio of the order $r \sim 10^{-6}$ for a strong curvature coupling.

It should be noted that we shall not be using analytical methods in solving the background equations as we did in section 3. We shall instead employ the PyTransport package (Mulryne & Ronayne, 2017) to obtain the analytical solutions of the background equations, the spectrum of the perturbation, the 3PCF, and also observe the behaviour of the reduced bi-spectrum f_{NL}.

6.2 Dynamics of Multiple Field Inflation in Affine Gravity with Non-minimal Coupling

From the definition of the curvature tensor,

$$\boldsymbol{R}^{\alpha}_{\mu\beta\nu} = \partial_{\beta}\Gamma^{\alpha}_{\mu\nu} - \partial_{\nu}\Gamma^{\alpha}_{\mu\beta} + \Gamma^{\delta}_{\mu\nu}\Gamma^{\alpha}_{\delta\beta} - \Gamma^{\delta}_{\mu\beta}\Gamma^{\alpha}_{\delta\nu}, \tag{97}$$

from which the Ricci curvature tensor $\boldsymbol{R}_{\mu\nu}$ and the Ricci scalar $\boldsymbol{R}$ are derived, it is based on the spacetime connection, Γ which provides us with the rule for parallel displacements and defines geodesics for freely falling bodies through a curved manifold. So affine gravity depends on the affine connection with no prior notion of the spacetime metric and this symmetric connection defines the symmetric Ricci tensor, $\boldsymbol{R}_{\mu\nu}(\Gamma) = \boldsymbol{R}_{\nu\mu}(\Gamma)$.

[2] For an action with canonical fields, the field manifold has a conformally flat shape.

Let us consider the action (Azri & Nasri, 2020)

$$\mathcal{S}[\Gamma,\phi] = \int \frac{\sqrt{|f(\phi)\boldsymbol{R}_{\mu\nu}(\Gamma) - \nabla_\mu\phi^a\nabla_\nu\phi^a|}}{V(\phi)} d^4x \tag{98}$$

Where $V(\phi) \neq 0$; $\phi^a(x)$ are the scalar fields with $a = 1, \dots, N$; N represents the number of scalar fields and $f(\phi)$ is the nonminimal coupling function which reduces to M_p^2 in the case of minimal coupling to gravity. We shall discover later that this function will take on the generic form $f(\phi) = M_p^2 + \xi\phi^2$.

We can now vary the action infinitesimally with respect to the affine connection. Since the scalar fields and the coupling function do not depend on the connection, we obtain

$$\delta\mathcal{S}[\Gamma,\phi] = \frac{1}{2}\int \frac{f(\phi)K^{-\frac{1}{2}}\delta\boldsymbol{R}_{\mu\nu}}{V(\phi)} d^4x \tag{99}$$

Where $$K_{\mu\nu}(\Gamma,\phi) = f(\phi)\boldsymbol{R}_{\mu\nu}(\Gamma) - \nabla_\mu\phi^a\nabla_\nu\phi^a \tag{100}$$

Using the palatini identity (Guarnizo et al., 2010) which states that

$$\boldsymbol{R}_{\mu\nu}(\Gamma) = \nabla_\gamma\left(\delta\Gamma^\gamma_{\nu\mu}\right) - \nabla_\nu(\delta\Gamma^\gamma_{\gamma\mu}), \tag{101}$$

we obtain

$$\delta\mathcal{S}[\Gamma,\phi] = \frac{1}{2}\int \Bigg(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}\nabla_\gamma\left(\delta\Gamma^\gamma_{\nu\mu}\right) - f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}\nabla_\nu(\delta\Gamma^\gamma_{\gamma\mu}) \Bigg) d^4x. \tag{102}$$

Integrating both terms by parts, eliminating the full derivative, and relabelling indices, we obtain

$$\delta\mathcal{S}[\Gamma,\phi] = \frac{1}{2}\int \Bigg\{ \nabla_\nu\left(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\alpha\mu}\right)\delta^\sigma_\gamma - \nabla_\gamma\left(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\alpha\sigma}\right)\Bigg\}\delta\Gamma^\gamma_{\alpha\sigma}d^4x \tag{103}$$

Using the principle of stationary action,

$$\nabla_\nu\left(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\alpha\mu}\right)\delta^\sigma_\gamma - \nabla_\gamma\left(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\alpha\sigma}\right) = 0 \tag{104}$$

Taking the trace of the equation, we end up with

$$\nabla_\gamma\left(f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}\right)=0. \qquad (105)$$

What is interesting in this result is that in solving it, the affine connection reduces to the Levi-Civita connection and – in essence – we generate the metric tensor $g_{\mu\nu}$. In other words, unlike the metric gravity theories where $g_{\mu\nu}$ is fundamental to gravity, in our study it can be seen clearly that it is simply a solution to Equation (105) as shown below

$$f(\phi)\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}=M_p^2\sqrt{|g|}(g^{-1})^{\mu\nu} \qquad (106)$$

Where M_p is arbitrary. Using the compatibility condition $\nabla_\lambda g_{\mu\nu}=0,$ we express the equation above as (Azri et al., 2021).

$$K_{\mu\nu}(g,\phi)=\frac{M_p^2V(\phi)}{f(\phi)}g_{\mu\nu} \qquad (107)$$

Making use of Equation (100),

$$f(\phi)\boldsymbol{R}_{\mu\nu}-\nabla_\mu\phi^a\nabla_\nu\phi^a=\frac{M_p^2V(\phi)}{f(\phi)}g_{\mu\nu} \qquad (108)$$

$$f(\phi)\boldsymbol{R}_{\mu\nu}=\frac{M_p^2V(\phi)}{f(\phi)}g_{\mu\nu}+\nabla_\mu\phi^a\nabla_\nu\phi^a \qquad (109)$$

Multiplying through by $g^{\mu\nu}$ and then $g_{\mu\nu}$ we obtain

$$\frac{1}{2}f(\phi)\boldsymbol{R}g_{\mu\nu}=\frac{2M_p^2V(\phi)}{f(\phi)}g_{\mu\nu}+\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\phi^a\nabla_\beta\phi^a g_{\mu\nu} \qquad (110)$$

Now we can construct the Einstein's Equations from (109) and (110)

$$f(\phi)\boldsymbol{G}_{\mu\nu}=\nabla_\mu\phi^a\nabla_\nu\phi^a-\frac{1}{2}\nabla^\beta\phi^a\nabla_\beta\phi^a g_{\mu\nu}-\frac{M_p^2V(\phi)}{f(\phi)}g_{\mu\nu} \qquad (111)$$

Comparing Equation (111) to Equation (3) we do realise that we have recovered Einstein's field Equations where $f(\phi)=M_p^2$ and the energy momentum tensor from which spacetime curvature is sourced is given by

$$T_{\mu\nu}=\frac{1}{f(\phi)}\left(\nabla_\mu\phi^a\nabla_\nu\phi^a-\frac{1}{2}\nabla^\beta\phi^a\nabla_\beta\phi^a g_{\mu\nu}-\frac{M_p^2V(\phi)}{f(\phi)}g_{\mu\nu}\right) \qquad (112)$$

Varying the action in Equation (98) with respect to the fields gives us

$$\delta\mathcal{S}[\Gamma,\phi] = \int \Big(-\frac{1}{V(\phi)^2}V_{,a}\delta\phi^a + \frac{1}{2V(\phi)}\sqrt{|K(\Gamma,\phi)|}(K^{-1})^{\mu\nu}\{f_{,a}\boldsymbol{R}_{\mu\nu}(\Gamma)\delta\phi^a - \delta(\nabla_\mu\phi^a\nabla_\nu\phi^a)\}\Big)d^4x \quad (113)$$

Where for any function $f(\phi)$, $f_{,a} = \frac{\partial f(\phi)}{\partial\phi^a}$.

Using integration by parts on the last term and using the principle of least action, we get

$$\partial_\alpha\left(\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}\partial_\beta\phi^a\right) + \frac{1}{2}f_{,a}\frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)}(K^{-1})^{\mu\nu}\boldsymbol{R}_{\mu\nu}(\Gamma) - \frac{\sqrt{|K(\Gamma,\phi)|}}{V(\phi)^2}V_{,a} = 0 \quad (114)$$

Which by using Equation (106) transforms to

$$\partial_\alpha\left(\frac{M_p^2\sqrt{|g|}}{f(\phi)}(g^{-1})^{\mu\nu}\partial_\beta\phi^a\right) + \frac{1}{2}f_{,a}\left(\frac{M_p^2\sqrt{|g|}}{f(\phi)}(g^{-1})^{\mu\nu}\boldsymbol{R}_{\mu\nu}(g)\right) - V_{,a}\frac{M_p^4\sqrt{|g|}}{f(\phi)^2} = 0 \quad (115)$$

The equation ultimately takes the form

$$\Box\phi^a - V_{,a} + \frac{1}{2}f_{,a}\boldsymbol{R}(g) + \left(1 - \frac{M_p^2}{f(\phi)}\right)V_{,a} - \frac{f_{,a}}{f(\phi)}\nabla^\alpha\phi^b\nabla_\alpha\phi^b = 0 \quad (116)$$

Looking at this equation, we observe that only in the case of minimal coupling where we set $f(\phi) = M_p^2$, would the last two terms vanish. This is contrary to what would be observed in the metric treatment of gravity where the 2 last terms are absent even in the case of nonminimal coupling. To proceed, we expand the fields $\phi^a(x^\mu)$ around a homogeneous background $\varphi^a(t)$ as was done in Equation (51).

$$\phi^a(x^\mu) = \varphi^a(t) + \delta\phi^a(x^\mu) \quad (117)$$

And for first order expansion,

$$f(\phi^a) = f(\varphi^a) + f_{,b}(\varphi^a)\delta\phi^b; \quad V(\phi^a) = V(\varphi^a) + V_{,b}(\varphi^a)\delta\phi^b. \quad (118)$$

In what follows, we may – for simplicity – set $M_p^2 = 1$. We shall derive the background equations for the flat FLRW metric in a homogeneous universe.

The background evolution equation obtained from Equation (116) is

$$\ddot{\varphi}^a + 3H\dot{\varphi}^a + \frac{1}{f}V_{,a} - 3(\dot{H} + 2H^2)f_{,a} - \frac{1}{f}(\dot{\varphi}^b)^2 f_{,a} = 0 \tag{119}$$

Let us again make an observation. Firstly, if $f = 1$, the fourth and fifth terms will disappear, and we shall end up with $\ddot{\varphi}^a + 3H\dot{\varphi}^a + V_{,a} = 0$. Note that this is the background equation we obtained in Equation (37) where we had minimal coupling for a single field. In the case of non-minimal coupling for metric gravity, we would have ended up with $\ddot{\varphi}^a + 3H\dot{\varphi}^a + V_{,a} - f_a, \boldsymbol{R} = 0$.

From Equation (112),

$$T_{00} = \rho = \frac{1}{f(\varphi)}\left(\frac{1}{2}(\dot{\varphi}^a)^2 + \frac{V(\varphi)}{f(\varphi)}\right) \tag{120}$$

$$T_{ii} = p = \frac{1}{f(\varphi)}\left(\frac{1}{2}(\dot{\varphi}^a)^2 - \frac{V(\varphi)}{f(\varphi)}\right) \tag{121}$$

We use Equation (111) to derive the gravitational field equations

$$3H^2 = \rho = \frac{1}{f(\varphi)}\left(\frac{1}{2}(\dot{\varphi}^a)^2 + \frac{V(\varphi)}{f(\varphi)}\right) \tag{122}$$

$$2\dot{H} + 3H^2 = -p = -\frac{1}{f(\varphi)}\left(\frac{1}{2}(\dot{\varphi}^a)^2 - \frac{V(\varphi)}{f(\varphi)}\right) \tag{123}$$

In what follows, we study the case of a scalar fields and metric with interactions. We revert to the metric in Equation (57) to obtain the first-order perturbed energy-momentum equations below (Azri & Nasri, 2020)

$$\delta\rho = \frac{1}{f(\phi)}\left(\dot{\varphi}^a\delta\dot{\phi}^a - (\dot{\varphi}^a)^2\Phi + \frac{1}{f(\phi)}V_{,a}\delta\phi^a\right) - \frac{f_{,a}}{f(\phi)^2}\left((\dot{\varphi}^b)^2 + \frac{2V(\phi)}{f(\phi)}\right)\delta\phi^a \tag{124}$$

$$\delta p = \frac{1}{f(\phi)}\left(\dot{\varphi}^a\delta\dot{\phi}^a - (\dot{\varphi}^a)^2\Phi - \frac{1}{f(\phi)}V_{,a}\delta\phi^a\right) - \frac{f_{,a}}{f(\phi)^2}\left((\dot{\varphi}^b)^2 - \frac{2V(\phi)}{f(\phi)}\right)\delta\phi^a \tag{125}$$

$$\delta q = -\frac{1}{f}(\dot{\varphi}^a \delta\phi^a) \tag{126}$$

Inserting Equation (124) into (60), and (126) into (61) we obtain

$$3H(\dot{\Psi} + H\Phi) + \frac{1}{a^2}k^2\Psi = -\frac{1}{2}\{\delta\rho\}.$$

$$= -\frac{1}{2}\left\{\frac{1}{f(\phi)}\left(\dot{\varphi}^a\delta\dot{\phi}^a - (\dot{\varphi}^a)^2\Phi + \frac{1}{f(\phi)}V_{,a}\delta\phi^a\right)\right. \tag{127}$$

$$\left. - \frac{f_{,a}}{f(\phi)^2}\left((\dot{\varphi}^b)^2 + \frac{2V(\phi)}{f(\phi)}\right)\delta\phi^a\right\}.$$

$$\dot{\Psi} + \mathrm{H}\Phi = \frac{1}{2f}\ (\dot{\varphi}^a\delta\phi^a). \tag{128}$$

The $i \neq j$ term issuing from the anisotropic pressure in Equation (111) becomes

$$\partial_i\partial_j(\Phi - \Psi) = 0; i \neq j \tag{129}$$

Combining Equation (127) and (128), we can get the compact equation

$$\delta\rho + \frac{3H\dot{\varphi}^b\delta\phi^b}{f(\phi)} = -\frac{2}{a^2}k^2\Psi \tag{130}$$

Let us compare this result with Equation (67). We notice that in this case even if we have non-minimal coupling, the anisotropic pressure is non-existent and therefore $\Phi = \Psi$ just like we had for a single field with minimal coupling.

6.3 Entropy Perturbations

In Equation (72), we defined the source of pressure perturbations, and it included the adiabatic and non-adiabatic contribution. This culminated in the definition of non-adiabatic pressure in Equation (80) as $\delta p_{nad} \equiv \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho$

Since δp, $\dot{p}$, $\rho^{\cdot}$ and $\delta\rho$ are known, we can substitute them into the equation and obtain

$$\delta p_{nad} = \left(\frac{2\dot{\varphi}^a\delta\rho}{3H(\dot{\varphi}^b)^2 f(\phi)} + \frac{2\delta\phi^a}{f(\phi)^2}\right)\left(\frac{2Vf_{,a}}{f(\phi)} - V_{,a}\right). \tag{131}$$

Substituting Equation (130) into (131) we obtain

$$\delta p_{nad} = \left(\frac{4H\dot{\varphi}^a}{3(\dot{\varphi}^b)^2 f(\phi)}\right)\left(V_{,a} - \frac{2Vf_{,a}}{f(\phi)}\right)\left(\frac{k}{aH}\right)^2 \Psi$$
$$- \frac{2V_{,a}}{f(\phi)^2}\left(\delta\phi^a - \frac{\dot{\varphi}^a}{(\dot{\varphi}^c)^2}\dot{\varphi}^b\delta\phi^b\right) \qquad (132)$$
$$+ \frac{4Vf_{,a}}{f(\phi)^3}\left(\delta\phi^a - \frac{\dot{\varphi}^a}{(\dot{\varphi}^c)^2}\dot{\varphi}^b\delta\phi^b\right).$$

It is interesting to see the implication of Equation (132). We saw the case of metric gravity with minimal coupling of a single field in chapter 4 and concluded that when $k \ll aH$, the $\delta p_{nad} \sim 0$ (i.e., non-adiabatic pressure is suppressed at super-Hubble scales). However, for nonminimal coupling, in our affine case, we still retain two terms which are sources of our nonadiabatic pressure perturbation despite the first term being supressed. We can say that in the unsuppressed terms, one term represents the non-adiabatic pressure perturbation component due to the presence of multiple fields, $\delta p_{nad}^{multifield}$ and the other is the non-adiabatic term arising out of non-minimal coupling, $\delta p_{nad}^{nonminimal}$. Let us define them as

$$\delta p_{nad}^{multifield} = -\frac{2V_{,a}}{f(\phi)^2}\left(\delta\phi^a - \frac{\dot{\varphi}^a}{(\dot{\varphi}^c)^2}\dot{\varphi}^b\delta\phi^b\right)$$
$$\delta p_{nad}^{nonminimal} = \frac{4Vf_{,a}}{f(\phi)^3}\left(\delta\phi^a - \frac{\dot{\varphi}^a}{(\dot{\varphi}^c)^2}\dot{\varphi}^b\delta\phi^b\right) \qquad (133)$$

Let us now limit our case to two fields where $\phi^a = (\phi, \chi)$ Equation (111) can be written transforms to

$$\boldsymbol{G}_{\mu\nu} = \frac{1}{f(\phi)}\Bigg(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}\nabla^\beta\phi\nabla_\beta\phi g_{\mu\nu} + \nabla_\mu\chi\nabla_\nu\chi - \frac{1}{2}\nabla^\beta\chi\nabla_\beta\chi g_{\mu\nu}$$
$$- \frac{M_p^2 V(\phi,\chi)}{f(\phi,\chi)} g_{\mu\nu}\Bigg) \qquad (134)$$

We now adapt these equations to a flat FLRW spacetime. By using the 00 components of the Einstein's Equations, we can see that the time evolution of the background fields $\phi = \phi(t)$ and $\chi = \chi(t)$ obey the equation

$$3H^2 = \frac{1}{f(\phi,\chi)}\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} + \frac{V(\phi,\chi)}{f(\phi,\chi)}\right) \qquad (135)$$

And using the ii components of the Einstein's Equations, we obtain

$$2\dot{H} + 3H^2 = -\frac{1}{f(\phi,\chi)}\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} - \frac{V(\phi,\chi)}{f(\phi,\chi)}\right). \qquad (136)$$

Our intention is to derive an expression for non-adiabatic pressure perturbation for the two fields, ϕ and χ. Since we have already established that the non-adiabatic perturbation will have two components in the case of multiple fields that are nonminimally coupled, we simply use our definitions in Equation (133) to come up with

$$\delta p_{nad}^{multifield} = \frac{2\dot{\phi}\dot{\chi}\left(\dot{\chi}V_{,\phi} - \dot{\phi}V_{,\chi}\right)}{\left(\dot{\phi}^2 + \dot{\chi}^2\right)f(\phi,\chi)}\left(\frac{\delta\phi}{\phi^{\cdot}} - \frac{\delta\chi}{\dot{\chi}}\right) \tag{137}$$

$$\delta p_{nad}^{nonminimal} = \frac{4V\dot{\phi}\dot{\chi}\left(\dot{\chi}f_{,\phi} - \dot{\phi}f_{,\chi}\right)}{\left(\dot{\phi}^2 + \dot{\chi}^2\right)f(\phi,\chi)^3}\left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\chi}{\dot{\chi}}\right) \tag{138}$$

We can make some observations from these two components. Firstly, if we set $f(\phi,\chi) = 1$ (i.e., for minimal coupling), the first component becomes

$$\delta p_{nad}^{multifield} = \frac{2\dot{\phi}\dot{\chi}\left(\dot{\chi}V_{,\phi} - \dot{\phi}V_{,\chi}\right)}{\left(\dot{\phi}^2 + \dot{\chi}^2\right)}\left(\frac{\delta\phi}{\phi^{\cdot}} - \frac{\delta\chi}{\dot{\chi}}\right) \tag{139}$$

This is what would be expected in the metric treatment with minimal coupling of both fields to gravity. Furthermore, the second component would completely disappear (i.e., $\delta p_{nad}^{nonminimal} = 0.$ leaving us with only one source of non-adiabatic perturbation. Next, if we have one field, both the components are going to vanish, which agrees with the metric treatment we covered in chapter 4. Of course, there are no surprises in this case since we can always transform a non-minimally coupled action to a minimally coupled action and therefore lose any adiabatic perturbation contribution. This is a case in which the affine treatment shines. One should expect the same results to be obtained despite a change in frames for example from the Jordan to the Einstein frame. If the results are not the same, then we ought to ask ourselves whether it is in order for us to talk about frame equivalence. There is no confusion with the affine treatment as we shall see in the following subsection as we transition from the non-minimal coupling to the minimal coupling case. For more on this, reference must be made to (Faraoni et al., 1998; Maeda, 1989; White et al., 2013).

6.4 Transition from Non-minimal Coupling to Minimal Coupling

Another way in which affine treatment of gravity shines brighter than the metric gravity is in the transition from non-minimal coupling to minimal coupling. In the metric formalism, we go through the tedious process of conformal transformations (Kamenshchik et al., 2016) in order to move from the Jordan to the Einstein frame. In our case, however, we perform the transition by simply redefining the potential as we shall see presently.

First, let us restate our action in Equation (98)

$$\mathcal{S}[\Gamma,\phi] = \int \frac{\sqrt{|f(\phi)\boldsymbol{R}_{\mu\nu}(\Gamma) - \boldsymbol{k}_{ab}(\phi)\nabla_\mu\phi^a\nabla_\nu\phi^b|}}{V(\phi)} d^4x \tag{140}$$

Where the fields are non-canonical and are coupled to $\boldsymbol{k}_{ab}(\phi)$ which we refer to as the non-Euclidean field space metric.

The Einstein field Equations have already been derived in Equation (111). Next, we shall redefine the potential as

$$V(\phi) \to U(\phi) = \frac{M_p^4}{f(\phi)^2} V(\phi) \tag{141}$$

This naturally gives rise to a new curved metric defined as

$$\boldsymbol{\mathcal{G}}_{ab}(\phi) = \frac{M_p^2}{f(\phi)} \boldsymbol{k}_{ab}(\phi) \tag{142}$$

Our action then transitions to

$$\mathcal{S}[\Gamma,\phi] = \int \frac{\sqrt{|M_p^2\boldsymbol{R}_{\mu\nu}(\Gamma) - \boldsymbol{\mathcal{G}}_{ab}(\phi)\nabla_\mu\phi^a\nabla_\nu\phi^b|}}{U(\phi)} d^4x \tag{143}$$

We vary the action with respect to the spacetime connection as was done in Equations (99) to (105) and obtain the simple form of the action shown below.

$$\nabla_\lambda \left(\frac{M_p^2\sqrt{|\overline{K}(\Gamma,\phi)|}}{U(\phi)} (\overline{K}^{-1})^{\mu\nu} \right) = 0 \tag{144}$$

Where

$$\overline{K}(\Gamma,\phi) = M_p^2\boldsymbol{R}_{\mu\nu}(\Gamma) - \boldsymbol{\mathcal{G}}_{ab}(\phi)\nabla_\mu\phi^a\nabla_\nu\phi^b. \tag{145}$$

We have already discovered that in this instance, we do not have a generic spacetime metric. The metric will be a solution to our equations derived from Equation (144). (i.e., our spacetime depends on the affine connection). So, solving (144) yields,

$$\frac{M_p^2\sqrt{|\overline{K}(\Gamma,\phi)|}}{U(\phi)} (\overline{K}^{-1})^{\mu\nu} = M_p^2\sqrt{|g|}(g^{-1})^{\mu\nu}. \tag{146}$$

And since $\nabla_\lambda g_{\mu\nu} = 0$, we get

$$\overline{K}_{\mu\nu}(g,\phi) = U(\phi) g_{\mu\nu}, \tag{147}$$

from which we derive the Einstein Equations below by employing Equation (145) and following the steps in Equations (108) to (111).

$$M_p^2 \boldsymbol{G}_{\mu\nu} = \boldsymbol{\mathcal{G}}_{ab}\nabla_\mu\phi^a\nabla_\nu\phi^b - \frac{1}{2}\boldsymbol{\mathcal{G}}_{ab}\nabla^\beta\phi^a\nabla_\beta\phi^b g_{\mu\nu} - U(\phi)g_{\mu\nu} \tag{148}$$

By switching back to $V(\phi)$ and $K_{\mu\nu}(\Gamma,\phi)$, it can be observed that the metric generated in Equation (147) coincides with that we generated in Equation (107) for the case of non-minimal coupling. We here confirm what we stated at the beginning of this subsection. The case of non-minimal coupling and minimal coupling are related to each other simply by the transformation of the potential.

This is in stark contrast to the metric formulation where the rigorous process of conformal transformation must be performed to transition from one frame to the other. Moreover, the potential and field redefinitions we made in Equations (141) and (142) to transition from (111) to (148) have no effect on the spacetime metric and consequently, the Hubble parameter. This is interesting because both the gauge invariant curvature perturbation $\mathcal{R}$ in (69) and the number of e-folds N in (44) are intrinsically dependent on H. So, the inflationary dynamics are not changed by the change of frame in the purely affine treatment – unlike the metric gravity treatment.

Varying the action (143) and following the steps gone through in Equations (113) to (116), we arrive at the equation

$$\partial_\mu\left(\frac{\sqrt{|\bar{K}(\Gamma,\phi)|}}{U(\phi)}(\bar{K}^{-1})^{\mu\nu}\partial_\nu\phi^b\boldsymbol{\mathcal{G}}_{ab}(\phi)\right) - \frac{1}{2}\frac{\sqrt{|\bar{K}(\Gamma,\phi)|}}{U(\phi)}(\bar{K}^{-1})^{\mu\nu}\nabla_\mu\phi^b\nabla_\nu\phi^c\boldsymbol{\mathcal{G}}_{bc,a} - \frac{\sqrt{|\bar{K}(\Gamma,\phi)|}}{U(\phi)^2}U_{,a} = 0, \tag{149}$$

Where $\boldsymbol{\mathcal{G}}_{bc,a} = \frac{\partial\boldsymbol{\mathcal{G}}_{bc}(\phi)}{\partial\phi^a}$

Making use of Equations (146) and (147), we obtain

$$\partial_\mu\left(\frac{\sqrt{|g|}}{U(\phi)}(g^{-1})^{\mu\nu}\partial_\nu\phi^b\boldsymbol{\mathcal{G}}_{ab}(\phi)\right) - \frac{\sqrt{|g|}}{2}(g^{-1})^{\mu\nu}\nabla_\mu\phi^b\nabla_\nu\phi^c\boldsymbol{\mathcal{G}}_{bc,a} - \sqrt{|g|}U_{,a} = 0. \tag{150}$$

$$\boldsymbol{\mathcal{G}}_{ab}\Box\phi^b + \left(\boldsymbol{\mathcal{G}}_{ab,c} - \frac{1}{2}\boldsymbol{\mathcal{G}}_{bc,a}\right) g^{\mu\nu}\nabla_\mu\phi^b\nabla_\nu\phi^c - U_{,a} = 0 \tag{151}$$

The second term can be manipulated to produce the Levi-Civita field space connection, and the final equation will be

$$\Box\phi^a + \Gamma^a_{bc} g^{\mu\nu}\nabla_\mu\phi^b\nabla_\nu\phi^c - \boldsymbol{\mathcal{G}}^{ab}U_{,b} = 0 \tag{152}$$

This is the evolution equation of motion in (Kaiser et al., 2013) for metric gravity in the Einstein frame. However, we should take note of the manifold's metric in Equation (142). It is quite different from that obtained for the metric case which is defined as

$$\boldsymbol{\mathcal{G}}_{ab} = \frac{M_p^2}{2f(\phi)}\left(\boldsymbol{k}_{ab}(\phi) + \frac{3}{f(\phi)} f_{,a}f_{,b}\right) \tag{153}$$

The first term is like our manifold's metric; however, the second term has additional terms which consist of derivatives of the non-minimal coupling function which bring about kinetic terms for the scalar fields in the Einstein frame which are absent in the purely affine formulation.

We shall now employ the covariant formalism to study the perturbations present during inflation and the dynamics of (143). In what follows, we shall use the approach developed in (Di Marco et al., 2003; Easther & Giblin Jr, 2005; Kaiser et al., 2013; Langlois & Renaux-Petel, 2008; Nibbelink & van Tent, 2002).

We first employ Equation (117) to expand each scalar field around its classical background $\varphi^a(t)$ and use Equation (56) to expand the spacetime metric to first order perturbation in a spatially flat FLRW metric. The field displacements and derivatives of the fields will appear as vectors in the manifold (Kaiser, 2016). We define the covariant derivative as

$$\boldsymbol{D}_c\delta\phi^a = \partial_c\delta\phi^a + \Gamma^a_{bc}\delta\phi^b \tag{154}$$

From (Peterson & Tegmark, 2011a, 2011b, 2012) we use the covariant derivative with respect to cosmic time as

$$\boldsymbol{D}_t\delta\phi^a \equiv \dot{\varphi}^c\boldsymbol{D}_c\delta\phi^a = \delta\dot{\phi}^a + \Gamma^a_{bc}\delta\phi^b\dot{\varphi}^c \tag{155}$$

Where Γ^a_{bc} are the Christoffel symbols obtained from the metric $\boldsymbol{\mathcal{G}}_{ab}$

The background part of the Einstein Equations (148) yield

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\boldsymbol{\mathcal{G}}_{ab}\dot{\varphi}^a\dot{\varphi}^b + U(\phi)\right) \tag{156}$$

$$\dot{H} = -\frac{1}{2M_p^2}\boldsymbol{\mathcal{G}}_{ab}\dot{\varphi}^a\dot{\varphi}^b \tag{157}$$

We re define the Mukhanov-Sasaki variable in Equation (68) for multiple fields as

$$Q^a \equiv \delta\phi^a + \frac{\dot{\varphi}}{H}\psi \tag{158}$$

Using Equations (154), (155) and (158), Equation (152) separates into the background and first order expressions

$$\boldsymbol{D}_t\dot{\varphi}^a + 3H\dot{\varphi}^a + \boldsymbol{\mathcal{G}}^{ab}U_{,b} = 0 \tag{159}$$

And

$$\boldsymbol{D}_tQ^a + 3H\boldsymbol{D}_tQ^a + \left\{\frac{k^2}{a^2}\delta_b^a + \mathcal{M}_b^a - \frac{1}{M_p^2a^3}\boldsymbol{D}_t\left(\frac{a^3}{H}\dot{\varphi}^a\dot{\varphi}_b\right)\right\}Q^b = 0, \tag{160}$$

Where

$$\mathcal{M}_b^a \equiv \boldsymbol{\mathcal{G}}^{ac}\boldsymbol{D_b}\boldsymbol{D_c}U(\phi) - \boldsymbol{R}_{cdb}^a\dot{\varphi}^c\dot{\varphi}^d \tag{161}$$

is the effective mass-squared matrix written in terms of the Riemann tensor of the curved manifold.

It is now necessary to simplify our equations by introducing the following quantities. First, we have the length of the background fields' vector,

$$|\dot{\varphi}|^2 \equiv \dot{\sigma}^2 = \boldsymbol{\mathcal{G}}_{ab}\dot{\varphi}^a\dot{\varphi}^b, \tag{162}$$

with a unit vector defined as

$$\hat{\sigma}^2 \equiv \frac{\dot{\varphi}^a}{\dot{\sigma}}. \tag{163}$$

Next, we introduce the turn-rate of the background field defined as

$$\omega^a \equiv \boldsymbol{D}_t\sigma^a, \tag{164}$$

and has a magnitude $\omega = |\omega^a|$. Lastly, we introduce the vector perpendicular to the motion of the fields, $\hat{s} \equiv \omega^a = \frac{\omega^a}{\omega}$. We are now in position to decompose the vector fluctuations into the adiabatic and non-adiabatic perturbation components which will be given respectively by

$$\begin{aligned} Q_\sigma &= \hat{\sigma}_aQ^a \\ Q_s &= \hat{s}_aQ^a \end{aligned} \tag{165}$$

We now split Equation (160) into an adiabatic and non-adiabatic component, respectively as shown below

$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left\{\frac{k^2}{a^2} + \mathcal{M}_{\sigma\sigma} - \omega^2 - \frac{a^{-3}}{M_p^2}\frac{d}{dt}\left(\frac{a^3}{H}\dot{\sigma}^2\right)\right\} Q_\sigma = 2\frac{d}{dt}(\omega Q_s) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega Q_s \tag{166}$$

$$\ddot{Q}_s + 3H\dot{Q}_s + \left\{\frac{k^2}{a^2} + \mathcal{M}_{ss} - 3\omega^2\right\} Q_s = \frac{4M_p^2\omega}{\dot{\sigma}}\frac{k^2}{a^2}\Psi. \tag{167}$$

Where

$$\mathcal{M}_{\sigma\sigma} = \hat{\sigma}_a\hat{\sigma}^b\mathcal{M}_b^a. \qquad \mathcal{M}_{ss} = \hat{s}_a\hat{s}^b\mathcal{M}_b^a. \tag{168}$$

6.5 Canonical Kinetic Terms

To transform the action from non-canonical to canonical form is done by simply transforming the field space metric $\boldsymbol{k}_{ab}$ to a factor of $\boldsymbol{\delta}_{ab}$. In other words, we end up with

$$\boldsymbol{\mathcal{G}}_{ab}(\phi) = \frac{M_p^2}{f(\phi)}\boldsymbol{\delta}_{ab}(\phi) \tag{169}$$

We say that the metric is conformally flat. This is a very simple process and does not require the manipulation that one has to inevitably go through to transform Equation (153) into a conformally flat field space metric. Restricting ourselves to two fields (i.e., $\phi^a = (\phi, \chi)$, we calculate the non-zero components of the connection from Equation (142) and obtain

$$\Gamma^\phi_{\phi\phi} = \Gamma^\chi_{\chi\phi} = \Gamma^\chi_{\phi\chi} = -\Gamma^\phi_{\chi\chi} = -\frac{f_{,\phi}}{2f(\phi,\chi)} \qquad \Gamma^\chi_{\chi\chi} = \Gamma^\phi_{\phi\chi} = \Gamma^\phi_{\chi\phi} = -\Gamma^\chi_{\phi\phi} = -\frac{f_{,\chi}}{2f(\phi,\chi)} \tag{170}$$

Generally, for a field space to be flat, it is necessary that $\boldsymbol{R}^a_{cdb}(\boldsymbol{\mathcal{G}}) = 0$. However, since we have limited ourselves to two fields, it is sufficient that $\boldsymbol{R}(\boldsymbol{\mathcal{G}}) = 0$. And from the definition of the Ricci scalar, $\boldsymbol{R}$

$$\boldsymbol{R}_{cb} = \boldsymbol{R}^a_{cab} = \partial_a\Gamma^a_{cb} - \partial_b\Gamma^a_{ca} + \Gamma^d_{cb}\Gamma^a_{da} - \Gamma^d_{ca}\Gamma^a_{db} \qquad \boldsymbol{R} = \boldsymbol{R}^b_b = \boldsymbol{\mathcal{G}}^{cb}\boldsymbol{R}_{cb}, \tag{171}$$

We obtain

$$\boldsymbol{R}(\boldsymbol{\mathcal{G}}) = \frac{1}{M_p^2}\left(f_{,\phi\phi} + f_{,\chi\chi} - \frac{f_{,\phi}^2 + f_{,\chi}^2}{f(\phi,\chi)}\right). \quad (172)$$

It is obvious from this expression that generally Equation (172) cannot be zero except if restrictions are placed on the coupling parameters. This is because we cannot transform our fields in such a way as to change their kinetic terms from non-canonical to canonical form simultaneously. In the following discussion, we put the restrictions on the coupling function so that $\boldsymbol{R}$ vanishes and therefore transform the field space metric $\boldsymbol{\mathcal{G}}$ to Euclidean form.

6.6 Flattening the Field Space and the Consequences to the Inflationary Dynamics

To do this, we let

$$\boldsymbol{k}_{ab}(\phi) = \frac{f(\phi)}{M_p^2}\boldsymbol{\delta}_{ab} \quad (173)$$

It can be seen from Equation (169) that this implies that $\boldsymbol{\mathcal{G}}_{ab} = \boldsymbol{\delta}_{ab}$ and the potential remains the same as in (141). We shall restrict ourselves to the single field case with non-minimal coupling and then study the multiple field case (restricting ourselves to two fields).

6.7 Non-minimal Coupling with a Single Field

The non-minimal coupling function we shall use is

$$f(\phi) = M_p^2 + \xi_\phi \phi^2. \quad (174)$$

The non-canonical coupling term from Equation (173) is

$$\boldsymbol{k}(\phi) = \frac{f(\phi)}{M_p^2}, \quad (175)$$

And the potential will be

$$V(\phi) = \frac{\lambda_\phi \phi^4}{4} \quad (176)$$

The background evolution equation will take the form

$$\ddot{\phi} + 3H\dot{\phi} + U_{,\phi} = 0 \quad (177)$$

The slow-roll conditions from Equation (38) are restated as

$$\left|\frac{\dot{\phi}}{2}\right| \ll |U(\phi)|; \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|, |U_{,\phi}| \quad (178)$$

Which implies that

$$3H\dot{\phi} \simeq -U_{,\phi} \quad (179)$$

And since

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + U(\phi)\right), \tag{180}$$

We obtain

$$3M_p^2 H^2 \simeq U(\phi), \tag{181}$$

Where the new potential in this case – following from Equation (141) and (176), is

$$U(\phi) = \frac{M_p^4 \lambda_\phi \phi^4}{4\left(M_p^2 + \xi_\phi \phi^2\right)^2}. \tag{182}$$

To calculate the number of e-folds, N, we restate Equation (44)

$$N = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\varphi}} d\varphi \approx -\frac{1}{M_p^2}\int_{\phi_i}^{\phi_f} \frac{U(\phi)}{U_{,\phi}} d\phi\,. \tag{183}$$

But

$$U_{,\phi} = \frac{m_p^6 \lambda_\phi \phi^3}{\left(M_p^2 + \xi_\phi \phi^2\right)^3}. \tag{184}$$

This means that if we use the limit $\phi^2 \gg \frac{M_p^2}{\xi}$,

$$N \simeq -\frac{\xi_\phi}{4M_p^4}\int_{\phi_i}^{\phi_f} \phi^3 d\phi. \tag{185}$$

Since the field at the beginning of inflation is much higher than at the end of inflation (i.e., $\phi_f \gg \phi_i$), then

$$N = \frac{\xi_\phi}{16M_p^4}\phi^4. \tag{186}$$

From this, the value of the field at the horizon crossing is

$$\frac{\phi_*}{M_p} = \left(\frac{16N_*}{\xi_\phi}\right)^{\frac{1}{4}}. \tag{187}$$

The first order slow-roll parameters are calculated from Equations (41) and (42).

$$\epsilon = \frac{M_p^2}{2}\left(\frac{U_{,\phi}}{U(\phi)}\right)^2 = \frac{8}{(16N_*)^{\frac{3}{2}}\sqrt{\xi_\phi}}. \tag{188}$$

To calculate η, we must first get $U_{,\phi\phi}$ by differentiating $U_{,\phi}$.

$$U_{,\phi\phi} = \frac{3M_p^6(M_p^2 - \xi_\phi \phi^2)\phi^2 \lambda_\phi}{\left(M_p^2 + \xi_\phi \phi^2\right)^4}. \tag{189}$$

Substituting this into (42) yields,

$$\eta = M_p^2 \left(\frac{U_{,\phi\phi}}{U(\phi)}\right) = -\frac{3}{4N_*}. \tag{190}$$

The spectral index, n_s, can then be calculated by using Equation (45) and it gives us

$$n_s \simeq 1 - \frac{3}{2N_*} - \frac{48}{(16N_*)^{\frac{3}{2}}\sqrt{\xi_\phi}}, \tag{191}$$

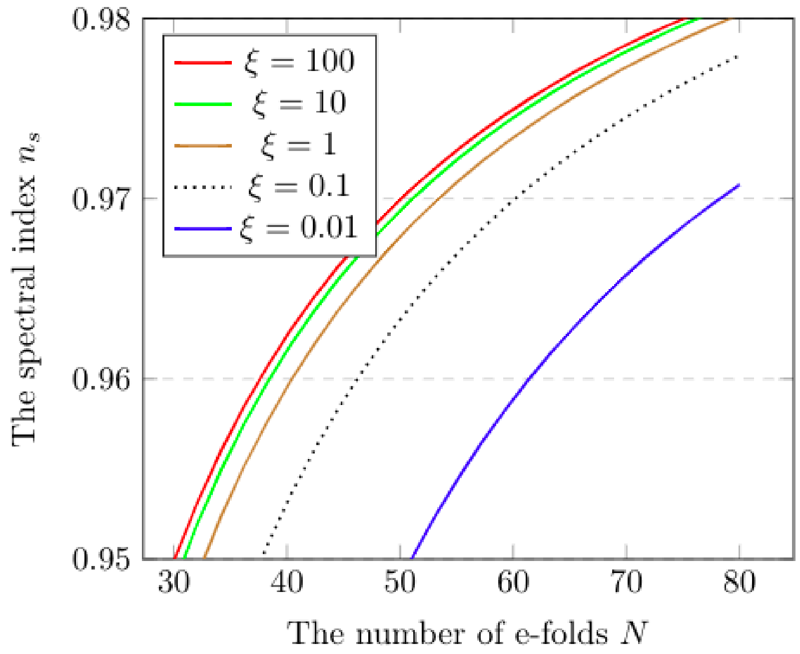


Figure 4: The larger the ξ, the lower the N that is required for n_s to fall within the observed range. A smaller ξ coincides with the usual $N = 50 - 60$.

The tensor-to-scalar ratio which is defined as $r = 16\epsilon$ becomes,

$$r = \frac{128}{(16N_*)^{\frac{3}{2}}\sqrt{\xi_\phi}}. \tag{192}$$

We can constrain ξ_ϕ by using the definition of the amplitude of scalar density perturbation $\boldsymbol{A}_s$,

$$\boldsymbol{A}_s \equiv \frac{U_*}{24\pi^2 \epsilon_*}. \tag{193}$$

Substituting for U_* and ϵ_*, one gets

$$\boldsymbol{A}_s \simeq 8 \times 10^{-3} \lambda_\phi \left(\frac{N}{\xi_\phi}\right)^{\frac{3}{2}}. \tag{194}$$

However, from the CMB radiation observation (Alimi et al., 2010), $\boldsymbol{A}_s \simeq 2.1 \times 10^{-9}$. This fixes the ξ_ϕ at

$$\xi_\phi = 2.5 \times 10^4 \lambda_\phi^{\frac{2}{3}} N_*. \tag{195}$$

Remarks on these results:

Looking at the standard model Higgs potential, $\phi \to h$, the parameter $\lambda_h \sim \mathcal{O}(1)$, the observed $\boldsymbol{A}_s$ would require the coupling parameter $\xi_h \simeq 10^4 N_*$. This would mean $n_s \simeq 1 - \frac{3}{2N_*}$ since the third term becomes negligible. Substituting the observed value $n_s \simeq 0.9626$ would give us $N \simeq 40 < 50$. The tensor-to-scalar ratio $r \simeq 8 \times 10^{-6}$. According to (Tristram et al., 2021), the current CMB measurements give an upper bound of "$r < 0.069$ when Planck EE, BB, and EB power spectra are combined consistently, and it tightens further to $r < 0.056$ when the Planck TT power spectrum is included in the combination and finally combining Planck with BICEP2/Keck 2015 data yields an upper limit of $r < 0.044$".

6.8 Non-minimal Coupling with Multiple Fields

We restrict ourselves to two fields and consider a quartic potential as shown below

$$V(\phi, \chi) = \frac{1}{4}\left(\lambda_\phi \phi^4 + 2\lambda_{int}\phi^2\chi^2 + \lambda_\chi \chi^4\right), \tag{196}$$

Where λ_ϕ, λ_{int}, λ_χ are dimensionless coupling constants. We also use a non-minimal coupling function,

$$f(\phi, \chi) = M_p^2 + \xi_\phi \phi^2 + \xi_\chi \chi^2, \tag{197}$$

Where ξ_ϕ and ξ_χ are also dimensionless coupling constants.

Substituting this into Equation (141), we get the redefined potential,

$$U(\phi, \chi) = \frac{M_p^4}{4} \frac{\left(\lambda_\phi \phi^4 + 2\lambda_{int}\phi^2\chi^2 + \lambda_\chi \chi^4\right)}{\left(M_p^2 + \xi_\phi \phi^2 + \xi_\chi \chi^2\right)^2}. \tag{198}$$

The single field limit is the one we have studied in the previous subsection. It must be noted that there is no symmetry that imposes $\lambda_\phi = \xi_\phi$ or $\lambda_\chi = \xi_\chi$ and therefore, it becomes hard to write the potential in terms of a single radial field. For a strong field, the flatness of the potential can be observed in a particular direction as

$$U(\phi) \simeq \frac{M_p^4 \lambda_a}{4\xi^2}\left\{1 + \mathcal{O}\left(\frac{M_p^2}{\xi(\phi^a)^2}\right)\right\}, \tag{199}$$

Where $\phi^a = (\phi, \chi)$.

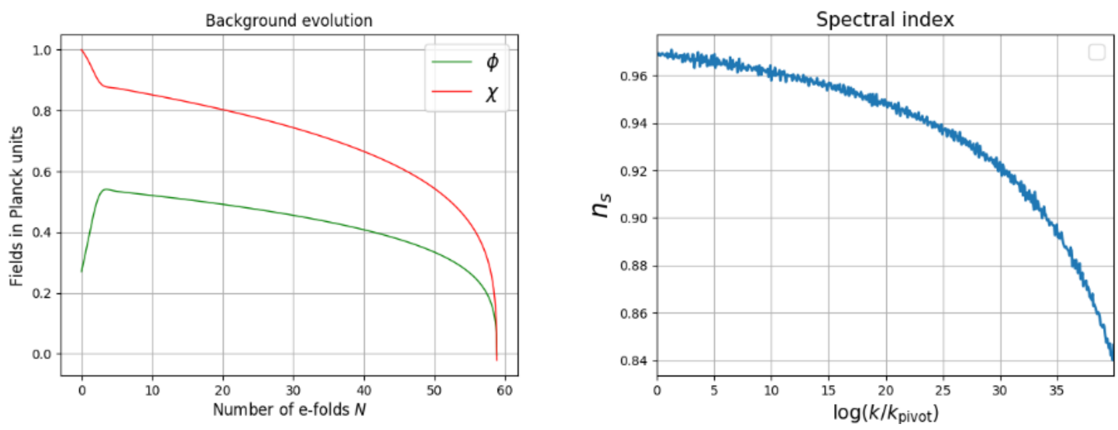
From Equations (156) and (159), we derive the background evolution equations below

$$3M_p^2 H^2 = \frac{1}{2}\{\dot{\phi}^2(t) + \dot{\chi}^2(t)\} + U(\phi, \chi) \tag{200}$$

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + U_{,\phi} &= 0 \\ \ddot{\chi} + 3H\dot{\chi} + U_{,\chi} &= 0. \end{aligned} \tag{201}$$

Notice that we have reduced the field space metric to a flat metric leading to the connection coefficients vanishing. It should also be noted that again, only the potential has been redefined. At this point, we cannot proceed with solving these equations along with Equation (160) without applying some numerical technique. It is for this reason that we make use of the open-source PyTransport code (Mulryne & Ronayne, 2017) to compute the necessary predictions.

The PyTransport code uses the δN formalism and uses the method described in (Dias et al., 2016; Ronayne & Mulryne, 2018) to solve the background evolution and displays the evolution of the fields in terms of N, and also the spectral index as shown in the Figures below.



a b

Figure 5: Evolution of the background fields in terms of ***N*** and the the numerical solution for the scalar tilt for the same field's initial values and model parameters used in the background evolution.

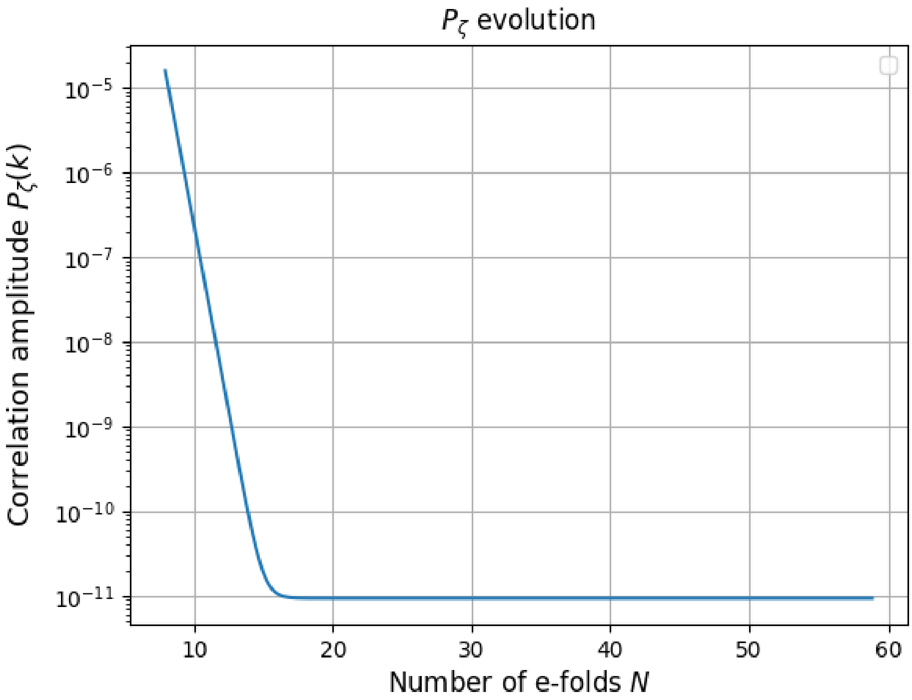


Figure 6: The evolution of the power spectrum of the curvature perturbation ζ

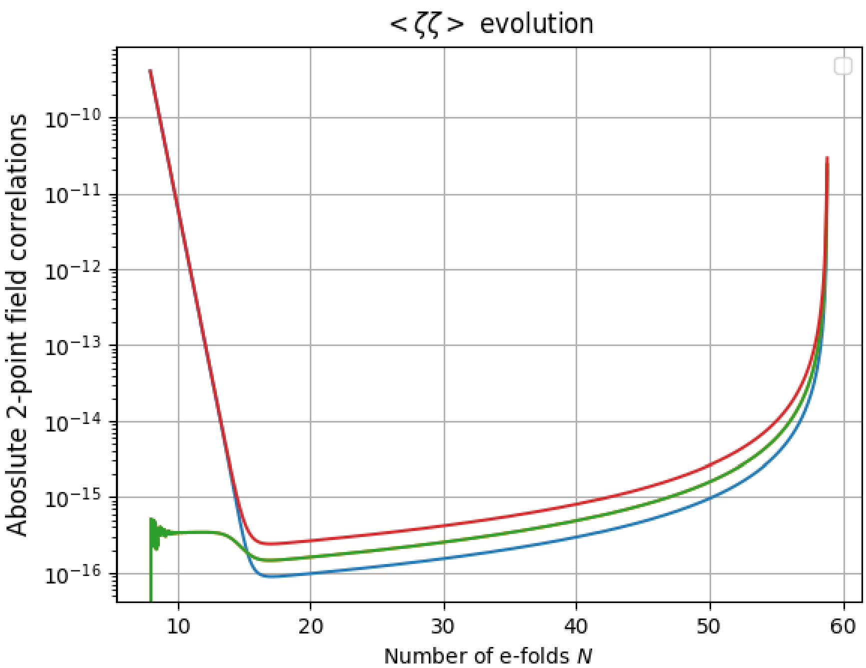


Figure 7: The two-point correlation function

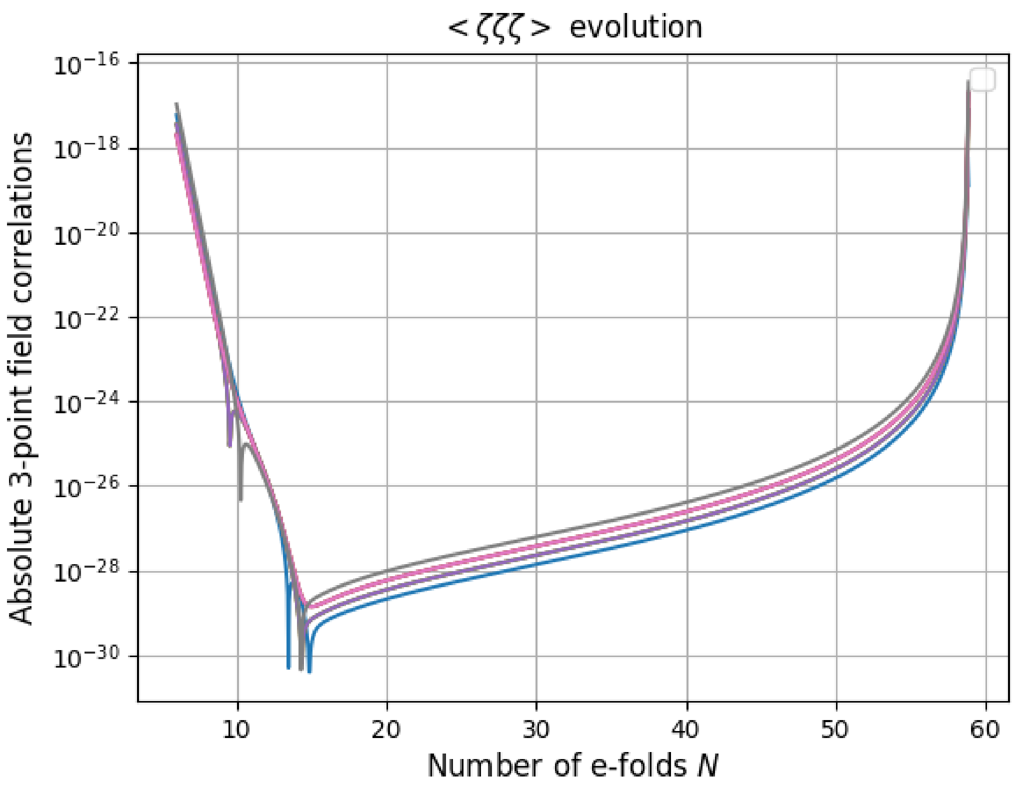


Figure 8: The evolution of the three-point function

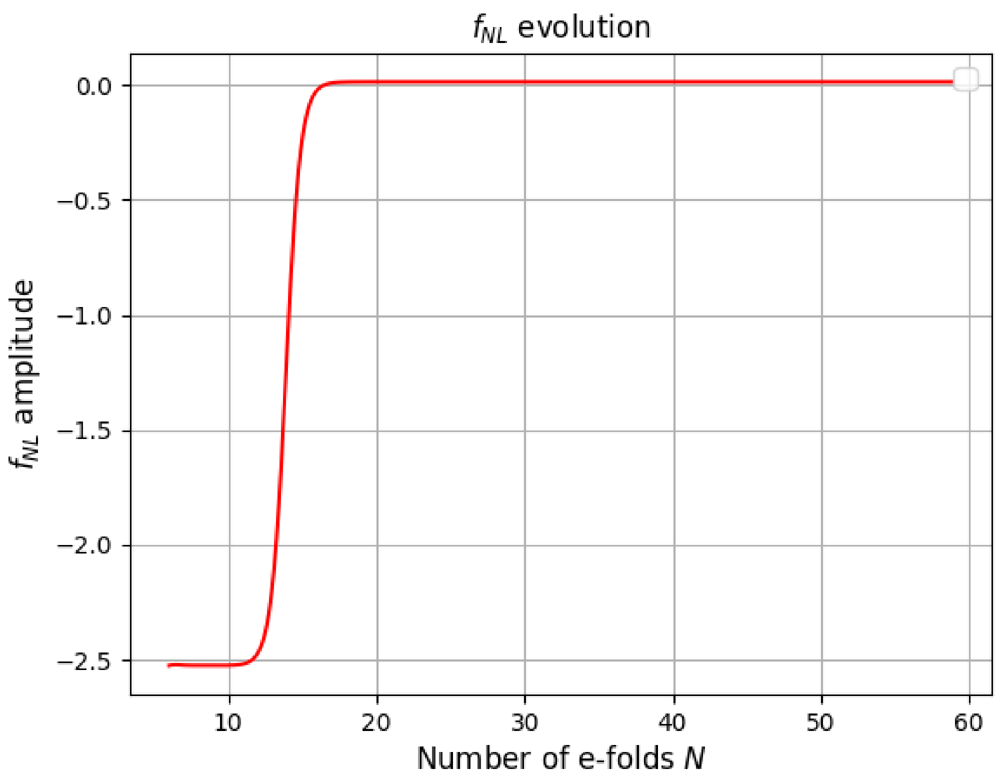


Figure 9: The reduced bispectrum f_{NL}

The values used for the couplings are $\lambda_{\phi} = 2.4 \times 10^{-3}$, $\lambda_{int} = 2 \times 10^{-4}$ and $\lambda_{\chi} = 3 \times 10^{-2}$ for the potential in Equation (186), and the non-minimal coupling constants are $\xi_{\phi} = 1.8 \times 10^{5}$ and $\xi_{\chi} = 10^{3}$. The f_{NL} indicates non-gaussianity for low values of N.

The potential parameters λ_a and the non-minimal coupling parameters ξ_a are set in such a way that the predicted spectral indes falls within the bounds of the measured value $n_s \simeq 0.9626$. These settings induce a tensor-to-scalar ratio $r \simeq 2.5 \times 10^{-4}$. For now, the observed temperature distribution of the CMB follows a nearly Gaussian distribution.

In Figures 6 and 7, the plots show the evolution of the power spectrum, P_{ζ} and the correlation function $\langle\zeta\zeta\rangle$ with respect to the number of e-folds N. It should be noted that we have set the same pameter constants for the Figures 5, 6, 7, 8 and 9. The power spectrum contains all the characteristics of the perturbations.

In general, multifield inflation predicts a non-Gaussian distribution of the primordial pertubations – including the two field case – to which we have restricted ourselves. We observe from the plots of Figure 7 that deviation from non-Gaussianity is observed in low values of N. In order to determine deviations from Gaussianity, we observe the possible non-zero three point field correlation function and the f_{NL} amplitude. For large N, $f_{NL} \sim 0$ and the distribution is largely Gaussian. From (Acharya et al., 2019), the $f_{NL}^{local} = -0.9 \pm 5.1$; $f_{NL}^{equil} = -26 \pm 47$; and $f_{NL}^{ortho} = -38 \pm 24$ (68%, CL, statistical). The results are obtained by combining temperature and polarisation analysis and include low multiploe $(4 \leq \ell < 40)$ polarisation data.

Comments about the Two-field Higgs Inflation in the Affine Treatment

The model we have used could be improved upon by requiring that all the minimal coupling parameters be equal. (i.e., $\xi_{\phi} = \xi_{\chi} = \xi$ for both scalar fields. In this way, the potential will be redefined to

$$U(h,\chi) = \frac{\lambda M_p^4 (h^2 + \chi^2 - v^2)}{4\left(M_p^2 + \xi(h^2 + \chi^2)\right)} \tag{202}$$

Where h is the standard model scalar Higgs boson, χ is the single Goldstone mode and v is the vacuum expectation value whose numerical value is $v \simeq 246$ GeV (Rajantie, 2018). With the standard model self coupling term $\lambda \sim \mathcal{O}(1)$, the $\boldsymbol{A}_s$ will require a big value of ξ and this would shift the predicted n_s from the observed value. We may therefore require that $\boldsymbol{\mathcal{G}}_{ab} \neq \boldsymbol{\delta}_{ab}$ (i.e.,We need a curved field space). We could achieve that by using $\boldsymbol{k}_{ab} = \boldsymbol{\delta}_{ab}$. In this way, both fields will be canonical while non-minimally couples to gravity as the action in Equation (98). Of course this will affect the slow-roll assumptions and the predictions will deviate from those of the case we have studied.

Chapter 7

Chapter 7: Conclusion

The Big Bang theory makes good predictions which are consistent with observations from the CMB radiation – which confirms that the universe is largely homogeneous on cosmologically large scales – and the abundance of elements. However, we have observed that it suffers from some drawbacks which necessitate the introduction of the theory of inflation. What makes the theory of inflation interesting is that it overcomes the problems of the big bang model, namely, the horizon and flatness problems, while it leads to the small fluctuations which are at the origin of the structure in the universe.

In this thesis, we have been interested in inflation driven by multiple fields and the associated features, namely, the isocurvature (entropy) perturbations and the deviation from Gaussianity.

First, we have reviewed the theory of inflation in its standard form where only a single field is considered. We have used the single field inflation paradigm to explain the dynamics of inflation, and derived the slow-roll parameters, (ϵ, η) that govern how many e-folds, N are required for inflation to stop so that the Big Bang cosmology can take over. Some mention has been made about reheating and the however, some parameters are still unknown which would help us pin down the exact temperature at which reheating occurs.

Next, we reviewed the cosmological perturbations that are responsible for the anisotropies that are observed in the CMB radiation. The density perturbations are a result of quantum fluctuations that exist in the scalar fields before the beginning of inflation and are stretched beyond superhorizon limits leading to the formation of structure. We prodded through the single field limit and concluded that the entropy perturbations and non-adiabatic pressure perturbations are suppressed at $k \ll aH$ distances. The comoving curvature perturbation remains constant at these scales, (i.e., $\dot{\mathcal{R}} = 0$ for $k \ll aH$).

We extended our study to include multiple fields, with nonminimal coupling, but restricted ourselves to the two-field régime. We discovered that at superhorizon distances, we have two sources of non-adiabatic perturbations: one coming from the presence of multiple fields and the other coming from the non-minimal coupling. This implies that the entropy perturbations are not suppressed through the whole inflationary period. It should be noted that though our study was restricted to the purely affine theory of gravity, the presence of entropy perturbations is intrinsic in the metric theory and in the Palatini treatment (Kaiser & Todhunter, 2010). Furthermore, having concluded that for a single field we produce a Gaussian distribution in the density perturbations, we found that in multiple fields, we have deviation from Gaussianity is observed in the lower values of N.

Since gravity is an aspect of spacetime curvature, which depends on the spacetime connection, we have considered that the affine connection should be introduced first instead of having the concept of the metric appearing first from which to derive the connection. In our study, we have shown how the metric is a solution to the equations of motion. Moreover, we have gone on to exhibit the differences between this formulation and the standard metrical gravity in the following ways. Firstly, the transformation from non-minimal coupling to minimal coupling is performed by a simple redefinition of the potential – without the need for conformal transformations as is necessarily the case with the metrical treatment, or even in the Palatini case as one transforms from the Jordan to the Einstein frame (Gialamas et al., 2020). The metric in our case is kept unchanged and this feature enables us to overcome the frame ambiguity that is suffered by the other two formulations. As displayed in our study, the notion of adiabaticity is invariant as we transition from non-minimal to minimal coupling. Secondly, our treatment leads to a simple conformally flat field space metric, expressed only in terms of the non-minimal coupling function, when the fields are canonical. This is a result of the linearity of the curvature in the connection. Lastly, another useful feature is that we can impose equal interactions for the non-minimal coupling and the canonical kinetic terms. We let $\boldsymbol{k}_{ab}(\phi) = \left(\frac{f(\phi)}{M_p^2}\right)\boldsymbol{\delta}_{ab}$ and discover that the potential retains its redefinition. This feature is absent in the GR treatment and simplifies the inflationary dynamics while not affecting the effects of the non-minimal coupling on the potential.

In the application of multifield affine inflation, we restricted ourselves to the two field dynamics and have used a potential with fields having quartic powers. We have studied the inflationary dynamics by flattening the field manifold and have shown how the single field limit leads to observations that deviate from those predicted in Metric gravity. Our potential produced a tensor-to-scalar ratio $r \simeq 8 \times 10^{-6}$. This prediction falls between that made by the Metric formulation and the palatini formulation (Bezrukov & Shaposhnikov, 2008; Mulryne & Ronayne, 2017).

We have used the PyTransport code to numerically evaluate the solution for our two-field limit, choosing the initial conditions for the fields, as well as the potential parameters. With these parameters, the scalar tilt of the perturbations reads$r \simeq 2.5 \times 10^{-4} \ll 0.069$ which is the upper bound limit from the CMB radiation measurements. In addition, we have solved for the three-point correlation function using the code and tracked the possible deviation from Gaussianity through the reduced bi-spectrum.

In conclusion, more interesting features are expected to be predicted as this formulation is developed further than what has been achieved in this thesis – especially as more information gets gathered. Also, we have considered a symmetric part of the spacetime Ricci tensor, which is sufficient in describing our gravitational theory since only two fields were considered. However, it would be interesting to see what predictions can be made if the (symmetric) character of the spacetime Ricci curvature is relaxed (Azri & Nasri, 2021b).

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The thesis discusses the entropy perturbations and non-gaussianity during inflation. A comparison is made between the GR formulation with a single field, called inflaton, and the case of multiple fields coupled either minimally or non-minimally to gravity with interactions of redefined potential which is necessary in the context of affine gravity. The results help us to predict how our model accords with observation from the Cosmic microwave Background radiation.

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