BENDING-TORSION FLUTTER ANALYSIS OF A VISCOELASTIC TAPERED WING CARRYING AN ENGINE AND SUBJECTED TO A FOLLOWER THRUST FORCE

Youssef Shaaban Abdelfattah Matter

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BENDING-TORSION FLUTTER ANALYSIS OF A VISCOELASTIC TAPERED WING CARRYING AN ENGINE AND SUBJECTED TO A FOLLOWER THRUST FORCE

Youssef Shaaban Abdelfattah Matter

This thesis is submitted in partial fulfilment of the requirements for the degree of Master of Science in Mechanical Engineering

Under the Supervision of Dr. Tariq Darabseh

April 2021
Declaration of Original Work

I, Youssef Shaaban Abdelfattah Matter, the undersigned, a graduate student at the United Arab Emirates University (UAEU), and the author of this thesis entitled “Bending-Torsion Flutter Analysis of a Viscoelastic Tapered Wing Carrying an Engine and Subjected to a Follower Thrust Force”, hereby, solemnly declare that this thesis is my own original research work that has been done and prepared by me under the supervision of Dr. Tariq Darabseh, in the College of Engineering at UAEU. This work has not previously formed the basis for the award of any academic degree, diploma or a similar title at this or any other university. Any materials borrowed from other sources (whether published or unpublished) and relied upon or included in my thesis have been properly cited and acknowledged in accordance with appropriate academic conventions. I further declare that there is no potential conflict of interest with respect to the research, data collection, authorship, presentation and/or publication of this thesis.

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Copy ____ of ____
Flutter, a dynamic divergent instability, is one of the significant phenomena of Aeroelasticity. This dangerous aeroelastic phenomenon can occur to any flexible structure subjected to aerodynamic forces such as aircraft wings, bridges, buildings, etc. It is important to analyze the flutter in order to predict the speed and frequency at which it occurs so that structural damages and failures can be avoided. This thesis is concerned with non-dimensional parametric modelling of the flutter of a viscoelastic tapered wing with an attached engine. The main objectives of this thesis are to determine regions of stability and boundaries of flutter speed and frequency and to examine how various parameters, such as engine thrust and mass, engine location, taper ratio, and the viscoelastic damping, impact the flutter characteristics of the wing. The wing is considered as a cantilever tapered Euler-Bernoulli beam, made of a linear viscoelastic material where Kelvin-Voigt model is assumed to represent the viscoelastic behavior of the material. The wing is subjected to aerodynamic forces as well as a follower thrust force generated by the engine. Quasi-steady and unsteady assumptions are employed to model the aerodynamic forces. The governing equations of motion are derived through the extended Hamilton’s principle. The resulting partial differential equations are solved via Galerkin’s method along with the classical flutter investigation approach. The study reveals that a tapered wing would be more dynamically stable than a uniform wing. It is also observed that the viscoelastic damping provides wider stability region for the wing. The investigation shows that the engine thrust and mass have significant effects on the dynamic stability of the wing. The investigated system interactions induce aeroelastic instabilities as the system parameters exceed their certain critical values. The developed model could precisely predict the flutter condition. The obtained theoretical predictions are explained based on real-life cases to give a better understanding of the flutter phenomenon.

**Keywords:** Aeroelasticity, flutter, viscoelastic wing, follower force, Kelvin-Voigt model, Galerkin’s method, Theodorsen’s aerodynamic model.
تحليل الرفرفة الناتجة عن الانحناء والالتواء لجناح مدبب مصنوع من مادة لزجة مرنة

الملخص

الرفرفة (flutter)، حالة عدم الاستقرار الديناميكي المتباينة، هي إحدى الظواهر المهمة للمرونة الهوائية (Aeroelasticity). يمكن أن تحدث هذه الظاهرة الهوائية الخطيرة لأي هيكل مرنة يخضع لقوى هوائية مثل أجنحة الطائرات والجسور والمباني وما إلى ذلك. من المهم تحليل الرفرفة من أجل التنبؤ بالسرعة والتردد التي تحدث عنها بحيث يمكن تجنب الأضرار والأعمال الهيكلية. تتم هذه الدراسة بالنموذج البارامترية (parametric) لرفرفة الجناح المدبب اللزج مع وجود محرك. تتمثل الأهداف الرئيسية لهذه الدراسة في تحديد مناطق الاستقرار وحدود سرعة وتردد الرفرفة ودراسة كيفية تأثير العوامل المختلفة، مثل قوة دفع المحرك، وكثافة ومكان المحرك، ومقدار تدبب الجناح، والتخمود، على خصائص رفرفة الجناح.

يعتبر الجناح بمثابة عارضة أويلر برنولي (Euler-Bernoulli beam) مدبب ناتئ، مصنوع من مادة لزجة مرنة، يتم استخدام نموذج كلفن فويجت (Kelvin-Voigt) يمثل سلوك المادة اللزجة. يخضع الجناح لقوى ديناميكية هوائية بالإضافة إلى قوة دفع تابعة ناتجة عن المحرك. يتم استخدام افتراضات شبه ثابتة وغير ثابتة لنموذج القوى الديناميكية الهوائية. يتم استيفاء معادلات الحركة من خلال مبدأ هاملتون الموسع (extended Hamilton’s principle). ومن ثم يتم حل المعادلات التفاضلية الجزئية الناتجة عن طريق منهج جاليركن (Galerkin’s method) بالإضافة إلى الطريقة التقليدية لتحليل الرفرفة.

كشفت الدراسة أن الجناح المدبب سيكون أكثر استقرارًا من الناحية الديناميكية من الجناح المنتظم. ويجدر أيضًا أن التخمود اللزج المرنة يوفر منطقة استقرار أوسو للجناح. أظهر التحقق أن قوة دفع المحرك وكثافة لنجوته لهما تأثيرات كبيرة على الاستقرار الديناميكى للجناح. تؤدي تفاعلات النظام إلى عدم الاستقرار الديناميكي عندما تتجاوز المتغيرات البارامترية (parameters) قيمتها الحرجية. يمكن للنموذج المطور أن يتنبأ بدقة حالة الرفرفة. يتم شرح التنبؤات النظرية التي تم الحصول عليها بناءً على حالات واقعية لإعطاء أفضل لظاهرة الرفرفة.

مفهوم البحث الرئيسي: المرونة الهوائية، الرفرفة، جناح لج مرنة، قوة دفع تابعة، نموذج كلفن فويجت، منهج جاليركن، نموذج ثيودورسén للديناميكية الهوائية.
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Dedication

To my beloved father
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Chapter 1: Introduction

1.1 Background and Overview

Aeroelasticity is the field of study concerned with the interactions among aerodynamic, elastic, and inertial forces (see Figure 1). These interactions may result in aeroelastic phenomena which are usually classified as being either static (divergence) or dynamic (flutter). Divergence is a phenomenon that occurs when the moments resulting from aerodynamic forces overcome the elastic restoring forces due to structural stiffness. The aeroelastic flutter is defined as a dynamic lack of stability that occurs in a flexible structure subjected to aerodynamic loads, such as aircraft wings, bridges, buildings, etc. This instability happens at certain speed and frequency - called the flutter speed and the flutter frequency - which cause the structure to undergo divergent oscillations.

Figure 1: Concept of Aeroelasticity
1.2 Statement of the Problem

It is important to analyze the flutter in order to predict the speed and frequency at which it occurs so that structural damages and failures can be avoided. The flutter of a tapered viscoelastic wing carrying an engine and subjected to a follower thrust force is investigated. The mass and inertia of the engine are modeled in order to achieve more realistic behavior of the engine upon flutter characteristics of the system. By incorporating the effects of engine force and mass, engine location, taper ratio, viscoelastic damping of the wing structure, location of the elastic and inertial axes of the wing along with other parameters, better flutter predictions can be achieved.

1.3 Relevant Literature

Bending-torsion aeroelastic instabilities have been investigated by many researchers. Goland (1945, 1948) studied the flutter phenomenon of a uniform wing by analyzing a set of partial differential equations governing the motion of the wing. The use of quasi-steady aerodynamic theory for aeroelastic analysis of the lifting surfaces can be a good approximation for low frequency ranges (Fung, 2008; Dowell, 1967; Dowell & Voss 1965; Meirovitch, 1975). Moosavi et al. (2005) developed a systematic approach based on Galerkin’s method to investigate the flutter speed and frequency for a wing subjected to quasi-steady aerodynamic forces. The quasi-steady aerodynamic model can be used for low frequencies with acceptable results as investigated by Acum (1963). In addition, the nonlinear aeroelastic response of wings considering quasi-steady aerodynamic forces was investigated by Nayfeh et al. (2012b), Ghommem et al. (2010), and Abdelkefi et al. (2012, 2013). However, Haddadpour and Firouz-Abadi (2006) showed that the quasi-steady aerodynamic theory gives inaccurate flutter results for a lifting surface under incompressible flow.
compared to the unsteady aerodynamic model. In fact, the unsteady model provides more reliable results than the quasi-steady model which offers more conservative predictions.

1.3.1 Viscoelasticity

Viscoelastic materials, such as composite materials, are typically used for enhancing damping and reducing structural vibration. Jia-ju and Ke-hwa (1981) analyzed the dynamic response of Kelvin-Voigt viscoelastic simply supported beam. Baker et al. (1967) considered the air damping effect in addition to Kelvin-Voigt viscoelastic damping on a thin beam vibrating transversely in air to have a deeper physical insight into the damping process. Hilton and Vail (1993) formulated an analysis of subsonic and supersonic torsion-bending flutter of a viscoelastic cantilever wing using aerodynamic strip theory. They evaluated the effects of viscoelastic properties, temperature, rotary inertia, and shear. Including the influence of temperature, Martins et al. (2013) investigated the use of viscoelastic material in aeroelastic systems and examined the influence of the viscoelastic behavior on flutter speeds. It was shown that the flutter speed associated with the viscoelastic wing might be greater or lower than that associated with the elastic wing. In addition, it was pointed out that the use of viscoelastic materials has either stabilization or destabilization contributions on the response of the structure (Hilton, 1957, 1960, 1991; Yi et al., 1996; Ungar, 1971).

1.3.2 External Store / Engine

The engine’s thrust which acts as a non-conservative follower force on a mass attached to the wing may affect the behavior of the wing vibration. Many authors have
studied bending-torsional flutter of elastic systems exposed to such non-conservative follower forces. For elastic systems, Bolotin (1962, 1963) has presented two books discussing comprehensively the dynamic stability of conservative and non-conservative elastic systems. By applying a method of expansion of fractional power of parameters, Bolotin and Zhinzher (1969) used a cantilever viscoelastic bar subjected to a tip follower force to show that, for realistic damping behavior, there is a part of quasi-stability region should be added to the instability region. Feldt and Herrman (1974) worked on the study of flutter speed and frequency of a cantilever wing carrying external mass and subjected to a concentrated follower force at its tip in addition to the surrounding flowing fluid. Hodges (2001) and Hodges et al. (2002) examined the effects of a transverse follower force on stability regions of HALE wing; however, they did not account for the inertia properties of the engine and considered only one location along the wing. The dynamic stability of wings carrying external stores and subjected to a lateral follower force was examined by Fazelzadeh et al. (2009). The study observed that the engine mass, thrust, and location are of great influence on the dynamic stability of the aircraft wing. Firouz-Abadi et al. (2013) studied the effect of two engines on the dynamic stability of a composite tapered and swept wing in compressible subsonic airflow. The dynamic instability of a cantilever composite wing with an attached mass subjected to follower force representing the thrust of the engine was studied by Amoozgar et al. (2013). Therein, it was reported that the ply angle, engine location, the magnitude of engine mass and thrust have significant effects on the aeroelastic stability of the composite wings. Their work was followed up and extended to a time-dependent thrust by Mazidi et al. (2013). Farsadi et al. and Izadpanahi et al. (2019; 2019) also confirmed in their studies that the engine position highly influences the aeroelastic response of the wing-engine system. The effects of
the location and the spacing between the attachment points of external store on the flutter characteristics of simply supported and cantilever composite laminated plates are studied by Lin et al. (2018).

1.3.3 Nonlinear Aeroelasticity

The nonlinear aeroelasticity has been addressed by many authors. Vasconcellos et al. (2012) investigated the discontinuous, polynomial, and hyperbolic tangent representations of a control surface free-play nonlinearity in a three degree of freedom aeroelastic system. A nonlinear analysis was carried out by Abdelkefi et al. (2012) to identify the pitch free-play nonlinearity along with its effect on the bifurcation type of a two degree of freedom aeroelastic system where the databases were generated experimentally. Vasconcellos and Abdelkefi (2015) studied the multi-segmented nonlinearity in the pitch degree of freedom and its effects on the dynamic stability of a two degree of freedom aeroelastic system. The effects of wing geometric properties and follower force on the flutter boundary of a nonlinear structural wing model were investigated by Zafari et al. (2019). The study revealed that the system will become unstable as the wing chord increases.

1.4 Potential Contributions and Limitations of the Study

Several solution methodologies have been developed to analyze the dynamic instability problem, among which the $k$ and $p-k$ methods are the most well-known and commonly used by engineers and researchers. Although these methods have strong merits in the determination of the flutter conditions, they require high computational time since they involve iterative algorithms. Hence, they become computationally ineffective when a parametric study is carried out. In addition, the solution techniques
associated with the $k$ and $p$-$k$ methods are problematic to be modified in order to include and investigate the viscoelastic damping (Patil et al., 2004).

In literature, many parametric studies have been carried out on 2-Dimensional typical wing section. In addition, some researchers have analyzed the flutter of a 3-Dimensional uniform wing and some have studied the flutter of 3-Dimensional tapered wing. Most of the work in literature have considered either the effect of the taper ratio or the viscoelastic damping on the flutter, individually. However, the flutter of a 3D tapered wing that is made of a viscoelastic material and carries an engine with a follower thrust force has not been intensively investigated yet.

This work introduces the viscoelastic damping of the wing material and structure in addition to the taper ratio in order to obtain results that are much closer to the real case. Moreover, the flutter determinant method is modified and employed in this work to conduct a non-dimensional parametric study since it requires less computational time and can provide accurate results.

The objective of this thesis is to develop an aeroelastic model that can examine the dynamic response of and determine regions of stability for a viscoelastic wing carrying an engine and subjected to a follower thrust force modeled as a cantilever tapered Euler-Bernoulli beam.

The wing is also subjected to aerodynamic forces that can be represented by the quasi-steady model or the Theodorsen’s unsteady model. The governing equations of motion are developed using the extended Hamilton’s principle and solved using Galerkin method and classical flutter investigation procedure. Furthermore, the non-dimensional parametric study is conducted to investigate the effects of parameters such as engine thrust, engine mass, engine location, taper ratio, viscoelastic damping of
wing structure, wing’s elastic axis location, wing’s inertial axis location, and many other parameters on the flutter speed and frequency.

The limitation of this study is that it does not consider the wing sweep, wing dihedral, nor the geometric and/or aerodynamic twist of the wing. Also, only one engine is considered in this work. In the analysis done by Fazlzadeh et al. (2020), it is revealed that the wing sweep angle and pre-twist angle have considerable effects on the flutter behavior of a tapered wing. Furthermore, it is shown by Amoozgar et al. (2020) that the wing dihedral or the wing curvature influences the dynamic stability of the wing. Therefore, future research may include these parameters and can involve additional external stores/engines.
Chapter 2: Mathematical Formulation

2.1 Wing Model

A cantilever viscoelastic tapered wing of length $l$ carrying an engine is shown in Figure 2. The engine mass is considered as a concentrated point load exerted on the engine’s center of gravity. The engine location along the wing span is denoted by $x_e$. In addition to the aerodynamic loading, the wing is also subjected to a follower thrust force (denoted here by $P$) generated by the attached engine. This engine thrust is applied exactly on the engine’s center of gravity and directed along the chord-wise direction of the wing. This gives the ability to recognize the thrust of the engine as a transverse follower force. Furthermore, the structural link between the wing and the engine (known as pylon) is assumed to be rigid. The tip effects, such as downwash, of the finite-span wing are ignored.

![Figure 2: Wing Configuration](image)
The deformed typical section of the wing, at the engine location, is modeled in Figure 3. The points $EA$, $C_{gw}$ and $AC$ refer to the wing elastic axis, the center of gravity of the wing, and the aerodynamic center of the wing, respectively. Here, $y_0$ denotes the chord-wise distance from the wing’s leading edge to the elastic axis ($EA$) and $y_a$ is the chord-wise distance between the wing’s center of gravity ($C_{gw}$) and the elastic axis ($EA$).

The center of gravity of the engine is denoted by $C_{ge}$, the chord-wise distance between the engine’s center of gravity ($C_{ge}$) and the elastic axis of the wing ($EA$) is denoted by $y_e$, and the vertical distance from the wing’s elastic axis ($EA$) to the engine’s center of gravity ($C_{ge}$) is denoted by $z_e$.

![Figure 3: Wing Typical Section](image)

The coordinate axes $x$, $y$, and $z$ are fixed on the wing root in which the $x$-axis lies exactly on the elastic axis and directed along the length of the wing in the span-wise direction. The orthogonal axes $x'$, $y'$, and $z'$ are attached to represent the wing
after deformation. The external loads cause the wing to be deformed such that the elastic axis of the wing is moved by an amount of $h(x,t)$ in the $z$-direction (plunge). Moreover, the wing rotates about its elastic axis by an angle of $\theta(x,t)$ (pitch). Therefore, the aeroelastic system described herein has two degrees of freedom (2 DoF).

The cantilever wing model is considered to taper in one plane, namely the $xy$-plane. By introducing the taper ratio, the general equations of the quantities of a wing section will be functions of the distance $x$ from the wing root as given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c(x)$</td>
<td>Cord length</td>
<td>$c(x) = c_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$m(x)$</td>
<td>Wing mass per unit span</td>
<td>$m(x) = m_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$y_0(x)$</td>
<td>Elastic axis location</td>
<td>$y_0(x) = y_{0r} \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$y_a(x)$</td>
<td>Offset between $EA$ and $C_g$</td>
<td>$y_a(x) = y_{ar} \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$I(x)$</td>
<td>Moment of inertia of the wing</td>
<td>$I(x) = I_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$J(x)$</td>
<td>Polar moment of inertia of the wing</td>
<td>$J(x) = J_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$EI(x)$</td>
<td>Wing bending rigidity</td>
<td>$EI(x) = EI_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$GJ(x)$</td>
<td>Wing torsional rigidity</td>
<td>$GJ(x) = GJ_r \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
<tr>
<td>$I_{EA}(x)$</td>
<td>Wing mass moment of inertia per unit span</td>
<td>$I_{EA}(x) = I_{EA_r} \left(1 - c_t \frac{x}{l}\right)$</td>
</tr>
</tbody>
</table>
The subscript \( r \) refers to the value of the parameter at the wing root and \( c_t \) represents the taper ratio which is given by:

\[
c_t = 1 - \frac{c_{tip}}{c_r}
\]

(2.1)

where \( c_{tip} \) is the chord length at the wing tip. If the wing has the same chord length at the tip and the root, the taper ratio will be zero.

### 2.2 Structural Governing Equations

The wing is made of a linear viscoelastic material, where the stress is linearly proportional to the strain history. Different models were constructed to represent the viscoelastic behavior of the material. It was predicted that the spring–dashpot models are useful to conceive how viscoelastic behavior can arise. Kelvin-Voigt model is employed to describe the behavior of the viscoelastic material. The Kelvin–Voigt model consists of a spring and a dashpot connected in parallel. Figure 4 shows how the Kelvin-Voigt model is applied to the wing section.

![Figure 4: Wing Profile with Kelvin-Voigt Model](image-url)
Because the two elements are subjected to the same strain, the total stress is the sum of stress in each element, so:

\[
\frac{\sigma_n}{\varepsilon} = \{E + \eta_E \partial_t\} = E^*
\]  \hspace{1cm} (2.2)

Similarly,

\[
\frac{\sigma_s}{\gamma} = \{G + \eta_G \partial_t\} = G^*
\]  \hspace{1cm} (2.3)

where \(\partial_t = \frac{\partial}{\partial t}\), \(\sigma_n\) and \(\sigma_s\) are the normal and shear stresses, respectively; \(E\) and \(G\) are the elastic and shear moduli, respectively; \(\eta_E\), and \(\eta_G\) are the coefficients of viscous damping forces in bending and torsion, respectively; and \(\varepsilon\) and \(\gamma\) are the normal and shear strains, respectively.

In the following analysis, the operation \((\ )'\) denotes the derivative with respect to the span-wise location \(x\) and the operation \((\ )\) refers to the time derivative of the variable. The equations of motion are derived via the variational Hamilton’s principle that can be expressed as:

\[
\int_{t_1}^{t_2} \left[\delta PE - \delta KE_w - \delta KE_e - \delta W_A - \delta W_F\right] = 0
\]  \hspace{1cm} (2.4)

where \(PE\) is the potential energy of the system, \(KE_w\) is the kinetic energy of the wing, \(KE_e\) is the kinetic energy of the engine, \(W_A\) is the virtual work of the distributed aerodynamic loads, \(W_F\) is the virtual work of the concentrated engine thrust, and \(\delta\) is the variational operator.
The first variation of the potential energy of the system is given by:

$$\delta PE = \frac{1}{2} \int_0^l [2G J' \delta \theta' + 2E' h'' \delta h'' + 2P (x_e - x) H(x_e - x) \theta \delta h''] dx$$  \hspace{1cm} (2.5)

where $H(x_e - x)$ is the Heaviside function, which is used in order to account for the location of the engine thrust force. The first term in the integral represents the contribution of the torsional stiffness and damping of the wing in the potential energy of the system whereas the second term refers to the contribution of the bending stiffness and damping of the wing.

Substituting Equation (2.2) and Equation (2.3) into Equation (2.5),

$$\delta PE = \frac{1}{2} \int_0^l [2G J' \delta \theta' + 2\eta_c J \partial_t \theta' \delta \theta' + 2E' h'' \delta h'' + 2\eta_E I \partial_t h'' \delta h'' + 2P (x_e - x) H(x_e - x) \theta \delta h''] dx$$  \hspace{1cm} (2.6)

The wing gains its kinetic energy due to both motions, heaving and pitching. The first variation of the wing kinetic energy is given by:

$$\delta KE_w = \int_0^l \frac{1}{2} [2m \dot{h} \delta h + 2m_y \dot{a} \delta \theta + 2m \dot{y} \delta h + 2I_E \dot{\theta} \delta \dot{\theta}] dx$$  \hspace{1cm} (2.7)

The first variation of the engine kinetic energy is given by:

$$\delta KE_e = \int_0^l \left[ M_e (z_e^2 \dot{h}' \delta h' + z_e^2 \dot{\theta} \delta \dot{\theta} + \dot{h} \delta h + y_e \dot{h} \delta \theta + y_e \delta \dot{h} + y_e^2 \delta \dot{\theta}) + I_{Me} \dot{\theta} \delta \dot{\theta} \right] \delta_D (x_e - x) dx$$  \hspace{1cm} (2.8)

where $M_e$ is the engine mass, $I_{Me}$ is the engine moment of inertia, and $\delta_D (x_e - x)$ is the Dirac-Delta function, which is used in order to precisely account for the location of the engine mass along the wing span.
The variation of the virtual work of the distributed aerodynamic loads is given by:

$$\delta W_A = \int_0^l (-L\delta h + M\delta \theta) \, dx$$

(2.9)

where $L$ and $M$ are the aerodynamic lift force and twisting moment per unit span, respectively.

The variation of the virtual work of the concentrated engine thrust is given by:

$$\delta W_F = \int_0^l \left[-P\delta_D(x_e - x)\theta \delta h - [(P y_e \theta - P z_e)\delta_D(x_e - x)]\delta \theta \right] \, dx$$

(2.10)

Using the previous expressions for the variation of the potential and kinetic energies and the variation of the virtual work along with Kelvin-Voigt model to represent the viscoelastic behavior of the material, the equations of motion are obtained as:

$$m\ddot{h} + m y_e \ddot{\theta} + (E l \dddot{h})'' + \left(\eta_G I \dddot{\theta}\right)'' - P(x_e - x)H(x_e - x)\theta'' + 2PH(x_e - x)\theta'$$

$$+ [M_e \dddot{h} - M_e y_e \ddot{\theta} - M_e x_e^2 \dddot{h}'' + P\theta]\delta_D(x_e - x) = -L$$

(2.11)

$$I_{EA} \dddot{\theta} + m y_e \dddot{\theta} - (GJ \theta')' - (\eta_G J \dddot{\theta})' - P(x_e - x)H(x_e - x)\dddot{h}''$$

$$+ [M_e y_e \dddot{h} + (I_{Me} + M_e (x_e^2 + y_e^2))\ddot{\theta} - P z_e + P y_e \theta]\delta_D(x_e - x) = M$$

(2.12)

### 2.3 Aerodynamic Models

To represent the aerodynamic forces about the elastic axis, the quasi-steady and unsteady models for subsonic 2-dimensional flow are considered. The lift and moment equations based on the quasi-steady model are (Fung, 2008):
\[ L_{QS} = \frac{1}{2} \rho U^2 c \frac{dC_L}{d\theta} \left[ \theta + \frac{\dot{\theta}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \right] \]  
(2.13)

\[ M_{QS} = \frac{1}{2} \rho U^2 c^2 \left\{ -\frac{c \pi}{8U} \dot{\theta} + \left( \frac{y_0}{c} - \frac{1}{4} \right) \frac{dC_L}{d\theta} \left[ \theta + \frac{\dot{\theta}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \right] \right\} \]  
(2.14)

Here, \( \rho \) and \( U \) are the density and speed of air, respectively. The term \( \frac{dC_L}{d\theta} \) is considered to be constant, with an approximate value of \( 2\pi \) obtained theoretically for incompressible flow.

One of the most commonly used theories to represent the unsteady aerodynamic forces is the Theodorsen’s unsteady theory for subsonic 2-dimensional incompressible flow over a thin airfoil. Based on this theory, the lift and moment equations about the elastic axis are (Theodorsen, 1935; Hodges & Pierce, 2011):

\[ L = \frac{1}{4} \pi \rho c^2 \left[ \dot{h} + U \dot{\theta} - \left( y_0 - \frac{c}{2} \right) \dot{\theta} \right] + \frac{1}{2} \rho U^2 c \frac{dC_L}{d\theta} C(k) \left[ \theta + \frac{\dot{\theta}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \right] \]  
(2.15)

\[ M = \frac{1}{4} \pi \rho c^2 \left[ \left( y_0 - \frac{c}{2} \right) \dot{h} - U \left( \frac{3c}{4} - y_0 \right) \dot{\theta} - c^2 \left[ \frac{9}{32} + \frac{y_0}{c c} \left( \frac{y_0}{c} - 1 \right) \right] \right] \]
\[ + \frac{1}{2} \rho U^2 c^2 \frac{dC_L}{d\theta} C(k) \left( \frac{y_0}{c} - \frac{1}{4} \right) \left[ \theta + \frac{\dot{\theta}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \right] \]  
(2.16)

The derivation of Equations (2.15) and (2.16) is provided in Appendix A. Here, \( C(k) \) is the Theodorsen’s function, which is given as (Theodorsen, 1935):

\[ C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + iH_0^{(2)}(k)} \]  
(2.17)
where $H(k)$ is the Henkel function which involves $1^{\text{st}}$ and $2^{\text{nd}}$ kinds of Bessel functions. Here, $i$ is the imaginary unit and $k$ is the reduced frequency, given by:

$$k = \frac{\omega c}{2U}$$

(2.18)

where $\omega$ is the frequency of harmonic oscillations.

Equation (2.17) can be approximated by (Theodorsen, 1935):

$$C(k) = 1 - \frac{0.165}{1 - \frac{k}{i}} - \frac{0.335}{1 - \frac{0.30}{k}}i, \quad k \leq 0.5,$$

$$C(k) = 1 - \frac{0.165}{1 - \frac{k}{i}} - \frac{0.335}{1 - \frac{0.32}{k}}i, \quad k > 0.5$$

The approximate representation of the Theodorsen’s function is proven to provide accurate results, as discussed in Appendix B. Therefore, it is adopted throughout the analysis.

### 2.4 Final Governing Equations

The final governing equations of motion, for the tapered viscoelastic cantilever wing subjected to bending and torsion loading governed by the aerodynamic strip theory, can be obtained for both aerodynamic models. For the quasi-steady model, substitute Equation (2.13) in Equation (2.11) and Equation (2.14) in Equation (2.12),

$$m\ddot{h} + my\ddot{\theta} + (EI\dot{h})'' + (\eta x \dot{h}')'' - P(x_e - x)H(x_e - x)\theta'' + 2PH(x_e - x)\theta'$$

$$+ [M_{e\dot{h}} - M_{e\dot{y}}\dot{\theta} - M_{e\dot{z}}\dot{h}'' + P\theta]D(x_e - x)$$

$$+ \frac{1}{2} \rho U^2 c \frac{dC_L}{d\theta} \left[ \theta + \frac{\dot{h}}{U} + c \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right] = 0$$

(2.19)
\[ I_{EA} \ddot{\theta} + m y_a \ddot{h} - (G J \theta')' - (\eta_{ij} J \dot{\theta}') - P (x_e - x) H(x_e - x) h'' \\
+ [M_e y_e \ddot{h} + (L_{me} + M_e (z_e^2 + y_e^2)) \dot{\theta} - P z_e + P y_e \theta] \delta_D (x_e - x) \\
- \frac{1}{2} \rho U^2 c^2 \left\{ \frac{c \pi}{8 U} \theta + \left( \frac{y_0}{c} - \frac{1}{4} \right) \frac{dC_l}{d\theta} \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right] \right\} = 0 \] (2.20)

For the unsteady model, substitute Equation (2.15) in Equation (2.11) and Equation (2.16) in Equation (2.12),

\[ m \ddot{h} + m y_a \ddot{\theta} + (E I h'')'' + (\eta_{ij} I \ddot{h}'')'' - P (x_e - x) H(x_e - x) \theta'' + 2 P H(x_e - x) \theta' \\
+ [M_e \ddot{h} - M_e y_e \ddot{\theta} - M_e (z_e^2 + y_e^2) \ddot{h}'' + P \theta] \delta_D (x_e - x) \\
+ \frac{1}{4} \frac{\rho U c^2}{2} \left[ \ddot{h} + U \dot{\theta} - \left( \frac{y_0}{c} - \frac{1}{2} \right) \dot{\theta} \right] \\
+ \frac{1}{2} \frac{\rho U^2 c^2}{2} \frac{dC_l}{d\theta} C(k) \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right] = 0 \] (2.21)

\[ I_{EA} \ddot{\theta} + m y_a \ddot{h} - (G J \theta')' - (\eta_{ij} J \dot{\theta}') - P (x_e - x) H(x_e - x) h'' \\
+ [M_e y_e \ddot{h} + (L_{me} + M_e (z_e^2 + y_e^2)) \dot{\theta} - P z_e + P y_e \theta] \delta_D (x_e - x) \\
- \frac{1}{4} \frac{\rho U c^2}{2} \left[ \left( \frac{y_0}{c} - \frac{1}{2} \right) \ddot{h} - U \left( \frac{3 c}{4} - y_0 \right) \dot{\theta} - c^2 \left[ \frac{9}{32} + \frac{y_0}{c} \left( \frac{y_0}{c} - 1 \right) \right] \dot{\theta} \right] \\
- \frac{1}{2} \frac{\rho U^2 c^2}{2} \frac{dC_l}{d\theta} C(k) \left( \frac{y_0}{c} - \frac{1}{4} \right) \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right] = 0 \] (2.22)

### 2.5 Non-Dimensional Analysis

To strengthen the developed mathematical model and to extend its applicability to almost any aeroelastic system, a non-dimensional analysis is carried out. Several non-dimensional parameters are introduced as defined in Table 2. These parameters will form the foundation to build the non-dimensional equations of motion. It is worth mentioning that most of the non-dimensional parameters are obtained by normalizing the wing section quantities. For example, the dimensionless span-wise coordinate, \( \xi \), is defined as the span-wise location divided by the wing span. This is a common practice for normalization.
Table 2: Non-Dimensional Parameters Specification

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_h )</td>
<td>Uncoupled bending frequency</td>
</tr>
<tr>
<td>( \omega_\theta )</td>
<td>Uncoupled torsion frequency</td>
</tr>
<tr>
<td>( \sigma = \frac{\omega_h}{\omega_\theta} )</td>
<td>Uncoupled bending-to-torsion frequency ratio</td>
</tr>
<tr>
<td>( \omega^* = \frac{\omega}{\omega_\theta} )</td>
<td>Dimensionless frequency of oscillation</td>
</tr>
<tr>
<td>( \Omega = \frac{EI_r}{GJ_r} )</td>
<td>Bending-to-torsion rigidity ratio</td>
</tr>
<tr>
<td>( r_a = \sqrt{\frac{I_{EA_r}}{m_r c_r^2}} )</td>
<td>Dimensionless radius of gyration about the elastic axis</td>
</tr>
<tr>
<td>( \mu = \frac{m_r}{\rho c_r^2} )</td>
<td>Density ratio (also known as mass ratio)</td>
</tr>
<tr>
<td>( V = \frac{U}{c_\theta \omega_\theta} )</td>
<td>Dimensionless speed</td>
</tr>
<tr>
<td>( \eta^*_E = \frac{\eta_E I_r}{\sqrt{EI_r m_r l^4}} )</td>
<td>Dimensionless coefficient of viscous damping in bending</td>
</tr>
<tr>
<td>( \eta^*<em>G = \frac{\eta_G I_r}{\sqrt{GJ_r I</em>{EA_r} l^2}} )</td>
<td>Dimensionless coefficient of viscous damping in torsion</td>
</tr>
<tr>
<td>( AR = \frac{l}{c_r} )</td>
<td>Wing aspect ratio</td>
</tr>
<tr>
<td>( M^*_e = \frac{M_e}{m_r l} )</td>
<td>Dimensionless engine mass</td>
</tr>
<tr>
<td>( P^* = \frac{P l^2}{\sqrt{EI_r GJ_r}} )</td>
<td>Dimensionless engine thrust</td>
</tr>
<tr>
<td>( \xi = \frac{x}{l} )</td>
<td>Dimensionless span-wise coordinate</td>
</tr>
<tr>
<td>( X_e = \frac{x_e}{l} )</td>
<td>Dimensionless span-wise engine location</td>
</tr>
<tr>
<td>( Y_e = \frac{y_e}{c_r} )</td>
<td>Dimensionless chord-wise engine location</td>
</tr>
<tr>
<td>( Z_e = \frac{z_e}{l} )</td>
<td>Dimensionless vertical engine location</td>
</tr>
</tbody>
</table>
Chapter 3: Solution Procedure

Before proceeding with the solution of the developed equations of motion, it is important to define some basic terminologies for the wing configuration which are used throughout the flutter analysis. Table 3 lists the terminologies and their definition.

Table 3: Basic Wing Configuration Terminologies

<table>
<thead>
<tr>
<th>Terminology</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Wing</td>
<td>A wing that is not carrying an engine.</td>
</tr>
<tr>
<td>Uniform Wing</td>
<td>A wing that has a fixed chord length though the span. Opposite of tapered wing.</td>
</tr>
<tr>
<td>Elastic Wing</td>
<td>A wing that is not structurally damped.</td>
</tr>
</tbody>
</table>

Due to the complexity of the partial differential equations of motion, a closed form solution cannot be found. However, the solution can be investigated via Galerkin approximate-solution technique by selecting the eigenfunctions of a cantilever beam that satisfy all of the boundary conditions. Galerkin’s method is one of the Weighted Residual Methods which can be effectively applied to aeroelastic analysis because of its versatility (Moosavi et al., 2005). Here, the solutions of the wing deflection \( h \) and twist \( \theta \) are assumed to be in exponential form which can be expressed by:

\[
h(\xi, t) = \tilde{h}_n(\xi)e^{\lambda t}, \quad (n = 1, 2, 3, ...)
\]  

\[
\theta(\xi, t) = \tilde{\theta}_n(\xi)e^{\lambda t}, \quad (n = 1, 2, 3, ...)
\]  

(3.1)  

(3.2)
where \( \bar{h} \) and \( \bar{\theta} \) are the amplitudes of plunge and pitch motions, respectively, and are dimensionally the same as \( h \) and \( \theta \), respectively; \( \lambda \) represents the eigenvalues of the aeroelastic system, and the \( f_n(\xi) \) and \( \varphi_n(\xi) \) (given below) are the orthonormal uncoupled bending and torsion mode shapes of a cantilever beam, respectively (Hodges & Pierce, 2011; Nayfeh et al., 2012a).

\[
f_n(\xi) = \cosh(\kappa_n \xi) - \cos(\kappa_n \xi) - \frac{\cosh(\kappa_n) + \cos(\kappa_n)}{\sinh(\kappa_n) + \sin(\kappa_n)} [\sinh(\kappa_n \xi) - \sin(\kappa_n \xi)]
\]

\[
\varphi_n(\xi) = \sin\left(\frac{(2n - 1)\pi}{2} \xi\right)
\]

(3.3)

(3.4)

where \( \kappa_n \) are solutions of the characteristic equation \( 1 + \cos(\kappa) \cosh(\kappa) = 0 \).

Following Galerkin’s method, Equation (2.11) is multiplied by \( f_n \) and Equation (2.12) is multiplied by \( \varphi_n \) and both equations are integrated over the wing span in order to minimize the weighted average error that resulted from assuming a solution.

\[
\int_0^l \left[m \dddot{h} + m_y \dddot{\theta} + (EI \dddot{\theta})' + (\eta_G I \dddot{\theta})' - P(x_e - x)H(x_e - x)\theta'' + 2PH(x_e - x)\theta' + [M_e \dddot{h} - M_e y_\theta \dddot{\theta} - M_e z_e \dddot{\theta}^2 + P \theta] \delta_0 (x_e - x) + L \right] f_n \, dx = 0
\]

(3.5)

\[
\int_0^l \left[I_e \dddot{\theta} + m_y \dddot{\theta} - (G \theta')' - (\eta_G I \theta')' - P(x_e - x)H(x_e - x)\theta'' + [M_e y_\theta \dddot{\theta} + (I_m + M_e(z_e^2 + y_e^2)) \theta' + P z_e - P y_\theta \theta] \delta_0 (x_e - x) - M \right] \varphi_n \, dx = 0
\]

(3.6)

Substituting Equations (3.1) and (3.2) in Equations (3.5) and (3.6), using the non-dimensional parameters introduced in Table 2, and considering the expressions of the unsteady lift force and twisting moment, the following non-dimensional algebraic eigenvalue problem is obtained,
\[ [K + V^2D + \lambda VF + \lambda^2 G] [\ddot{\theta}] = 0 \]  

(3.7)

where

\[
K = \begin{bmatrix}
a_{11} & 0 \\
0 & b_{21}
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0 & 0 \\
C(k)b_{14} & C(k)b_{26}
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
C(k)a_{15} & C(k)a_{23} \\
b_{12} + C(k)b_{15} & b_{24} + C(k)b_{27}
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
a_{13} + a_{14} & -a_{21} + a_{22} \\
b_{11} - b_{13} & -b_{23} - b_{25}
\end{bmatrix}
\]

where the coefficients \(a_{ij}\) and \(b_{ij}\) are given in Appendix C.

For dimensionless air speed \(V \neq 0\), the eigenvalue \(\lambda\) is generally in the form \(\lambda = g + i\omega\), where the real part \((g)\) represents the damping ratio and the imaginary part \((\omega)\) is the frequency of harmonic oscillations.

### 3.1 Flutter Determination

In order to determine the critical speed \((V_f)\) at which the flutter occurs, Equation (3.7) is solved for the eigenvalues \(\lambda\) repeatedly by increasing the value of \(V\). The first four mode shapes were considered: 1\textsuperscript{st} bending mode, 1\textsuperscript{st} torsion mode, 2\textsuperscript{nd} bending mode, and 3\textsuperscript{rd} bending mode. The damping ratio \((g)\) is plotted versus the non-dimensional air speed \(V\) for all the four modes as shown in Figure 5. It is observed that for the system to be stable, all the eigenvalues should have negative real part. As the damping ratio becomes positive, the system will be unstable. Hence, the critical speed is at which the real part of the eigenvalue becomes zero \((g = 0)\). It is also observed that
there may be more than one critical speed. However, the lowest one is the most important which is associated with the second mode (1st torsion mode).

\[ \begin{align*}
\text{Figure 5: Damping Ratio vs. Non-Dimensional Speed} \\
\text{At the flutter boundary, only the imaginary part of the eigenvalue is nonzero, that is } \lambda = i \omega. \text{ Substituting this expression of the eigenvalue in Equations (3.1) and (3.2) and for } n = 1, \text{ the equations of wing deflection and twist will be:} \\
\quad h(\xi, t) &= \tilde{h}f_1(\xi)e^{i\omega t} \\
\quad \theta(\xi, t) &= \tilde{\theta}\varphi_1(\xi)e^{i\omega t} \\
\text{Now, Substituting Equations (3.8) and (3.9) into Equations (3.5) and (3.6), a set of two non-dimensional algebraic equations is obtained, which can be written in matrix form as:} \\
\begin{bmatrix} (A_1) & (B_1) \end{bmatrix} \begin{bmatrix} \tilde{h} \\ \tilde{\theta} \end{bmatrix} &= 0 \\
\end{align*} \]
The coefficients $A_i$ and $B_i$ depend on which aerodynamic model is used. Therefore, there are two sets of the coefficients $A_i$ and $B_i$ as given below.

- For the quasi-steady aerodynamic model:

\[
\begin{align*}
A_1 &= a_{11} + i \omega_f a_{12} - \omega_f^2 a_{13} - \omega_f^2 a_{14} \\
&\quad + \omega_f^2 a_{15} + i \omega_f V_f a_{16} \\
B_1 &= -\omega_f^2 b_{11} - b_{12} + b_{13} + \omega_f^2 b_{14} + b_{15} \\
&\quad + V_f^2 b_{16} + i \omega_f V_f b_{17} \\
A_2 &= \omega_f^2 a_{21} + a_{22} + \omega_f^2 a_{23} + i \omega_f V_f a_{24} \\
B_2 &= b_{21} + i \omega_f b_{22} + \omega_f^2 b_{23} + \omega_f^2 b_{24} - b_{25} \\
&\quad + V_f^2 b_{26} + i \omega_f V_f b_{27}
\end{align*}
\]

- For the unsteady aerodynamic model:

\[
\begin{align*}
A_1 &= a_{11} + i \omega_f a_{12} - \omega_f^2 a_{13} - \omega_f^2 a_{14} \\
&\quad + \omega_f^2 a_{15} + i \omega_f V_f C(k) a_{16} \\
&\quad - \omega_f^2 a_{17} \\
B_1 &= -\omega_f^2 b_{11} - b_{12} + b_{13} + \omega_f^2 b_{14} + b_{15} \\
&\quad + V_f^2 C(k) b_{16} + i \omega_f V_f C(k) b_{17} \\
&\quad + i \omega_f V_f b_{18} + \omega_f^2 b_{19} \\
A_2 &= \omega_f^2 a_{21} + a_{22} + \omega_f^2 a_{23} + i \omega_f V_f C(k) a_{24} \\
&\quad - \omega_f^2 a_{25} \\
B_2 &= b_{21} + i \omega_f b_{22} + \omega_f^2 b_{23} + \omega_f^2 b_{24} - b_{25} \\
&\quad + V_f^2 C(k) b_{26} + i \omega_f V_f C(k) b_{27} \\
&\quad - i \omega_f V_f b_{28} + \omega_f^2 b_{29}
\end{align*}
\]

where the coefficients $a_{ij}$ and $b_{ij}$ are given in Appendix C.

To obtain a nontrivial solution, the determinant of the coefficient matrix in Equation (3.10) is set to zero. With the determinant being complex in general, both its real and imaginary parts must vanish. This leads to two equations with two unknowns $V_f$ and $\omega_f^*$, which are the dimensionless flutter speed and dimensionless flutter frequency, respectively.

### 3.2 Flutter of a Clean Uniform Elastic Wing

In the absence of the engine, taper ratio, and the viscoelastic damping, the flutter condition for the wing considering the unsteady aerodynamic model can be obtained by employing different solution methods. For the purpose of comparison,
three solution methodologies are considered: the $k$ method, the $p-k$ method, and the determinant method. The coefficients $A_i$ and $B_i$ in Equation (3.10) depend on which method is employed. Therefore, there are three sets of the coefficients $A_i$ and $B_i$, and three techniques of solution as described in the following sections.

### 3.2.1 The $k$ Method

For the $k$ method, the coefficients $A_i$ and $B_i$ are reduced to:

$$A_1 = Z a_{11} - a_{13} + i \frac{c}{2k} C(k) a_{14} - a_{15}$$

$$B_1 = -b_{11} + \left( \frac{c}{2k} \right)^2 C(k) b_{12} + i \frac{c}{2k} C(k) b_{13} + i \frac{c}{2k} b_{14} + b_{15}$$

$$A_2 = a_{21} + i \frac{c}{2k} C(k) a_{22} - a_{23}$$

$$B_2 = Z b_{21} + b_{23} + \left( \frac{c}{2k} \right)^2 C(k) b_{24} + i \frac{c}{2k} C(k) b_{25} - i \frac{c}{2k} b_{26} + b_{27}$$

where

$$Z = \frac{1}{\omega^2} (1 + ig)$$  \hspace{1cm} (3.11)

Here, an artificial damping $g$ is introduced to the system. The flutter condition can be obtained by increasing the value of the reduced frequency $k$ and at each value, the Theodorsen function $C(k)$ is evaluated, and Equation (3.10) is solved for the roots $Z_{1,2}$. When the imaginary part of any of the roots becomes zero, this indicates the flutter boundary; and here the value of the reduced frequency will be $k_f$. The flutter frequency can be found by:

$$\omega_f = \frac{1}{\sqrt{\text{Re}(Z)}}$$  \hspace{1cm} (3.12)
and the corresponding flutter speed can be found by:

\[ U_f = \frac{\omega_f c}{2k_f} \]  

(3.13)

### 3.2.2 The p-k Method

For the p-k method, the coefficients \( A_i \) and \( B_i \) are given as:

\[ A_1 = a_{11} + p^2 a_{13} + i \frac{2k}{c} U^2 C(k)a_{14} - \left( \frac{2k}{c} \right)^2 U^2 a_{15} \]

\[ B_1 = p^2 b_{11} + U^2 C(k)b_{12} + i \frac{2k}{c} U^2 C(k)b_{13} + i \frac{2k}{c} U^2 b_{14} + \left( \frac{2k}{c} \right)^2 U^2 b_{15} \]

\[ A_2 = -p^2 a_{21} + i \frac{2k}{c} U^2 C(k)a_{22} - \left( \frac{2k}{c} \right)^2 U^2 a_{23} \]

\[ B_2 = b_{21} - p^2 b_{23} + U^2 C(k)b_{24} + i \frac{2k}{c} U^2 C(k)b_{25} - i \frac{2k}{c} U^2 b_{26} + \left( \frac{2k}{c} \right)^2 U^2 b_{27} \]

where

\[ p = \omega \left( \frac{g}{2} + i \right) \]  

(3.14)

In order to determine the flutter condition, a relatively longer algorithm is followed. First, a desirable range of the air speed \( U \) is defined starting from a small value but not zero to avoid the division by zero in the reduced frequency equation. At each air speed \( U_i \), an initial value of the reduced frequency is guessed call it \( k_i \). For this guessed value, the Theodorsen function \( C(k_i) \) is evaluated, and Equation (3.10) is solved for \( p_i \). Then, the frequency is obtained by:

\[ \omega_i = Im(p_i) \]  

(3.15)
and a new value of the reduced frequency, call it $k_n$, is obtained by:

$$k_n = \frac{\omega_i c}{2U_i}$$

(3.16)

If the difference between $k_i$ and $k_n$ is greater than a predefined acceptable error, the previous steps are repeated for the new value of the reduced frequency ($k_i = k_n$) to diminish the difference. When the difference becomes acceptable, or zero, the artificial damping $g_i$ is obtained by:

$$g_i = 2 \frac{\text{Re}(p_i)}{\text{Im}(p_i)}$$

(3.17)

The whole procedure is repeated for all values of the air speed. The flutter speed is then obtained from the plot of $g$ versus $U$ where the artificial damping is zero.

### 3.2.3 The Determinant Method

For the determinant method, the coefficients $A_i$ and $B_i$ will be:

$$A_1 = a_{11} - \omega_f^2 a_{13} + i \omega_f U_f C(k) a_{14} - \omega_f^2 a_{15}$$

$$B_1 = -\omega_f^2 b_{11} + U_f^2 C(k) b_{12} + i \omega_f U_f C(k) b_{13} + i \omega_f U_f b_{14} + \omega_f^2 b_{15}$$

$$A_2 = \omega_f^2 a_{21} + i \omega_f U_f C(k) a_{22} - \omega_f^2 a_{23}$$

$$B_2 = b_{21} + \omega_f^2 b_{23} + U_f^2 C(k) b_{24} + i \omega_f U_f C(k) b_{25} - i \omega_f U_f b_{26} + \omega_f^2 b_{27}$$

This method, unlike the $k$ and $p$-$k$ methods, does not involve an iterative algorithm. The flutter condition can be determined directly by solving Equation (3.10) for the flutter speed $U_f$ and the flutter frequency $\omega_f$. 
3.3 Validation of the Aeroelastic Model

To verify the accuracy of the aeroelastic model developed in this work, two test wing-models: Goland wing and HALE (High Altitude Long Endurance) wing, are considered. Each model represents a clean uniform elastic wing (i.e., the engine, the taper ratio, and the viscoelastic damping of the material are absent). The specifications of Goland and HALE wings are listed in Table 4 and Table 6, respectively. The non-dimensional parameters are calculated based on the properties of Goland and HALE wings and the values are given in Table 5 and Table 7, respectively.

Table 4: Properties of Goland Wing in SI Units (Goland, 1945)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing length (l)</td>
<td>m</td>
<td>6.096</td>
</tr>
<tr>
<td>Chord (c)</td>
<td>m</td>
<td>1.829</td>
</tr>
<tr>
<td>Bending rigidity (EI)</td>
<td>N.m²</td>
<td>9.75 x 10⁶</td>
</tr>
<tr>
<td>Torsional rigidity (GJ)</td>
<td>N.m²</td>
<td>0.985 x 10⁶</td>
</tr>
<tr>
<td>Mass of the wing per unit length (m)</td>
<td>kg/m</td>
<td>35.719</td>
</tr>
<tr>
<td>Mass moment of inertia about elastic axis per unit length (I_EA)</td>
<td>kg.m²/m</td>
<td>8.643</td>
</tr>
<tr>
<td>Elastic axis position from leading edge (y₀)</td>
<td>m</td>
<td>0.33 c</td>
</tr>
<tr>
<td>Inertial axis position from leading edge (y₀ + yₐ)</td>
<td>m</td>
<td>0.43 c</td>
</tr>
<tr>
<td>Air density (ρ)</td>
<td>kg/m³</td>
<td>1.225</td>
</tr>
</tbody>
</table>

Table 5: Values of Non-Dimensional Parameters for Goland Wing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>rₐ</td>
<td>0.269</td>
</tr>
<tr>
<td>σ</td>
<td>0.2538</td>
</tr>
<tr>
<td>μ</td>
<td>8.7141</td>
</tr>
<tr>
<td>Ω</td>
<td>9.895</td>
</tr>
<tr>
<td>AR</td>
<td>10/3</td>
</tr>
</tbody>
</table>
Table 6: Properties of HALE Wing (Patil et al., 2001)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing length ((l))</td>
<td>m</td>
<td>16</td>
</tr>
<tr>
<td>Chord ((c))</td>
<td>m</td>
<td>1</td>
</tr>
<tr>
<td>Bending rigidity ((EI))</td>
<td>N.m(^2)</td>
<td>(2 \times 10^4)</td>
</tr>
<tr>
<td>Torsional rigidity ((GJ))</td>
<td>N.m(^2)</td>
<td>(1 \times 10^4)</td>
</tr>
<tr>
<td>Mass of the wing per unit length ((m))</td>
<td>kg/m</td>
<td>0.75</td>
</tr>
<tr>
<td>Mass moment of inertia about elastic axis per unit length ((I_{EA}))</td>
<td>kg.m(^3)/m</td>
<td>0.1</td>
</tr>
<tr>
<td>Elastic axis position from leading edge ((y_0))</td>
<td>m</td>
<td>(0.5 \ c)</td>
</tr>
<tr>
<td>Inertial axis position from leading edge ((y_0 + y_a))</td>
<td>m</td>
<td>(0.5 \ c)</td>
</tr>
<tr>
<td>Air density ((\rho))</td>
<td>kg/m(^3)</td>
<td>0.0889</td>
</tr>
</tbody>
</table>

Table 7: Values of Non-Dimensional Parameters for HALE Wing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_a)</td>
<td>0.3651</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.032275</td>
</tr>
<tr>
<td>(\mu)</td>
<td>8.436</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>2</td>
</tr>
<tr>
<td>(AR)</td>
<td>16</td>
</tr>
</tbody>
</table>

Considering the unsteady aerodynamic model and using the determinant method, the flutter conditions (flutter speed and frequency) of Goland and HALE wings are determined and compared to those of Goland (1945) and Patil et al. (2001). The obtained results are given in Table 8 and excellent agreement is achieved.
Table 8: Validation of Flutter Condition for Goland and HALE Wings

<table>
<thead>
<tr>
<th>Wing Model</th>
<th>Goland Wing</th>
<th>HALE Wing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exact</td>
<td>Present Work</td>
</tr>
<tr>
<td></td>
<td>(Goland, 1945)</td>
<td>(%)</td>
</tr>
<tr>
<td>Flutter Speed (m/s)</td>
<td>137.16</td>
<td>136.45</td>
</tr>
<tr>
<td>Flutter Frequency (rad/s)</td>
<td>70.7</td>
<td>69.39</td>
</tr>
</tbody>
</table>

Furthermore, the results for the Goland wing are validated against those in Haddadpour and Firouz-Abadi (2006) for both aerodynamic models. The developed model gives very close results to those in the reference with an error less than 1.1% as seen in Table 9.

Table 9: Validation of Quasi-Steady and Unsteady Aerodynamic Models

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Steady</td>
<td>Speed (m/s)</td>
<td>110</td>
<td>110.36</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Frequency (rad/s)</td>
<td>93</td>
<td>94</td>
<td>1.08</td>
</tr>
<tr>
<td>Unsteady</td>
<td>Speed (m/s)</td>
<td>136.85</td>
<td>136.45</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>Frequency (rad/s)</td>
<td>70</td>
<td>69.39</td>
<td>0.87</td>
</tr>
</tbody>
</table>

3.4 Comparison of Solution Methods

To compare the solution methods ($k$, $p$-$k$, and determinant methods) explained in Section 3.2, the flutter conditions are evaluated using the three different solution
methodologies and compared to the exact values. The results obtained for Goland and HALE wings are presented in Table 10 and Table 11, respectively.

The results indicate that the determinant method leads to more accurate findings with an error that does not exceed 4%.

Table 10: Comparison of Solution Methods for Goland Wing

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>$k$ Method</th>
<th>$p-k$ Method</th>
<th>Determinant Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exact (Goland, 1945)</td>
<td>Exact (Goland, 1945)</td>
<td>Exact (Goland, 1945)</td>
</tr>
<tr>
<td>Flutter Speed (m/s)</td>
<td>137.16</td>
<td>136.38</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>136.37</td>
<td>0.58</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>136.45</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>Flutter Frequency (rad/s)</td>
<td>70.7</td>
<td>69.35</td>
<td>1.91</td>
</tr>
<tr>
<td></td>
<td>69.36</td>
<td>1.90</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>69.39</td>
<td>1.85</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Table 11: Comparison of Solution Methods for HALE Wing

<table>
<thead>
<tr>
<th>Solution Method</th>
<th>$k$ Method</th>
<th>$p-k$ Method</th>
<th>Determinant Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exact (Patil et al., 2001)</td>
<td>Exact (Patil et al., 2001)</td>
<td>Exact (Patil et al., 2001)</td>
</tr>
<tr>
<td>Flutter Speed (m/s)</td>
<td>32.51</td>
<td>33.58</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>33.56</td>
<td>3.23</td>
<td>3.23</td>
</tr>
<tr>
<td></td>
<td>33.46</td>
<td>2.92</td>
<td>2.92</td>
</tr>
<tr>
<td>Flutter Frequency (rad/s)</td>
<td>22.37</td>
<td>21.19</td>
<td>5.27</td>
</tr>
<tr>
<td></td>
<td>21.21</td>
<td>5.19</td>
<td>5.19</td>
</tr>
<tr>
<td></td>
<td>21.48</td>
<td>3.98</td>
<td>3.98</td>
</tr>
</tbody>
</table>
3.5 Flutter Analysis

As discussed in Section 2.1, the wing model investigated in this study is tapered and made of viscoelastic material. By introducing a viscoelastic damping to the system, the $k$ and $p$-$k$ methods become difficult to modify. Hence, the determinant method is adopted to carry out the rest of the analysis.

Due to the large number of parameters that are involved in the analysis, a good approach to perform the parametric study is to vary one parameter while fixing the values of the other parameters. The specifications, including dimensions and material mechanical properties, of Goland wing (given in Table 4) are used as reference to evaluate the dimensionless parameters (listed in Table 5) that are utilized throughout the non-dimensional parametric study. Whenever a non-dimensional parameter is being investigated, its value will vary depending on the study while the values of the other parameters will remain the same as in Table 5. In addition, the location of the engine’s center of gravity as well as the elastic and inertial axes’ locations used throughout the analysis are given in Table 12.

Table 12: Non-Dimensional Values of Locations of the Engine, Elastic Axis, and Inertial Axis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_e$</td>
<td>0.25</td>
</tr>
<tr>
<td>$Y_e$</td>
<td>0</td>
</tr>
<tr>
<td>$Z_e$</td>
<td>0</td>
</tr>
<tr>
<td>$y_0/c$</td>
<td>0.33</td>
</tr>
<tr>
<td>$y_a/c$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The non-dimensional parametric flutter analysis is conducted for both aerodynamic models, quasi-steady and unsteady. Galerkin’s method can be effectively
applied to the aeroelastic analysis because of its versatility. The implementation of Galerkin-method-based aeroelastic analysis is developed entirely within a numerical code to accelerate the calculations and to generate the data.
Chapter 4: Results and Discussion

In this chapter, the theoretical outcomes and findings of the non-dimensional parametric flutter analysis are thoroughly illustrated and discussed. The results involve the predictions of the quasi-steady and unsteady aerodynamic models with and without the presence of the engine. The flight condition, wing configuration, and value of the non-dimensional parameters considered for a specific study are stated for each case. Otherwise, the values are assumed to be the same as in Table 5 and Table 12.

The following results are presented in a technique to best show the effect of each parameter on the flutter conditions separately. However, it is important to keep in mind that all the parameters are corelated and the variation of one would affect some other parameters.

For all the figures presented in this section, plot (a) represents the non-dimensional flutter speed versus the parameter of interest and plot (b) represents the non-dimensional flutter frequency versus that parameter.

4.1 Taper Ratio

The non-dimensional flutter speed and frequency are sketched in Figure 6 versus the taper ratio ($\epsilon_t$) in the absence of the engine ($P=M_e=0$) and the viscoelastic damping of the material ($\eta_E=\eta_G=0$) which represents the response of a clean elastic tapered wing. It is observed that, for both aerodynamic models, increasing the taper ratio raises the flutter speed and flutter frequency, which means better stability characteristics. This indicates that a tapered wing would be more dynamically stable than a uniform one. Indeed, when the taper ratio increases, the surface area of the wing will decrease and hence, the aerodynamic forces will also decrease. Having less
aerodynamic loading acting on the wing, the flutter speed will be higher. This behavior is also observed in Mahran et al. (2015) for a plate wing and in Durmaz et al. (2007) for a beam wing. It is also observed that the quasi-steady aerodynamic model provides more conservative results than the unsteady model while the two models give the same behavior.

Figure 6: Effect of Taper Ratio on Flutter Condition
4.2 Viscous Damping

Figure 7 shows the plots of non-dimensional flutter speed and frequency for a clean uniform wing ($P=M_c=c_t=0$) versus the non-dimensional bending viscoelastic damping parameter ($\eta_E^*$) in the absence of the torsional viscoelastic damping parameter ($\eta_G=0$). It can be observed that increasing the value of $\eta_E^*$ increases the flutter speed and slightly decreases the flutter frequency. The effect is significant in the case of quasi-steady assumption.

Figure 7: Effect of Non-Dimensional Bending Viscoelastic Damping on Flutter Condition
Figure 8 illustrates the effect of the non-dimensional torsional viscoelastic damping parameter ($\eta_G^*$) on the non-dimensional flutter speed and frequency for a clean uniform wing in the absence of the bending viscoelastic damping parameter ($\eta_E=0$). The plot indicates that the effect of $\eta_G$ is the same as that of $\eta_E$.

Figure 8: Effect of Non-Dimensional Torsional Viscoelastic Damping on Flutter Condition

Figure 9 and Figure 10 show the plots of non-dimensional flutter speed and frequency for a clean wing ($P=M_c=0$) versus the viscoelastic damping parameter in
bending ($\eta E^*$) for the quasi-steady and unsteady aerodynamic models, respectively. The three curves represent three different values of taper ratio. It can be observed from the figures that if the wing material exerts a viscoelastic damping on the bending motion, the flutter speed will increase (i.e., the wing becomes more stable). It is also indicated that wings with higher taper ratios can be more dynamically stable if viscoelastic materials are used. On the other hand, the flutter frequency is slightly affected by the bending viscoelastic damping.

Figure 9: Effect of Non-Dimensional Bending Viscoelastic Damping on Flutter Condition for Different Taper Ratios for Quasi-Steady Model
Figure 10: Effect of Non-Dimensional Bending Viscoelastic Damping on Flutter Condition for Different Taper Ratios for Unsteady Model

Figure 11 and Figure 12 show the plots of non-dimensional flutter speed and frequency for a clean wing versus the viscoelastic damping parameter in torsion ($\eta_{G^*}$) for the quasi-steady and unsteady aerodynamic models, respectively, and for three different taper ratios. The figures indicate that introducing a viscoelastic damping in torsion raises the flutter speed and slightly reduces the flutter frequency. In addition, for the case of $c_t = 0.8$, a substantial increase in the flutter speed is observed as the
value of $\eta_G^*$ increases. It is important to note that the effect of the viscoelastic damping parameters is more significant in the case of quasi-steady assumption.

Overall, it is concluded that the viscous damping can improve the flutter characteristics of a tapered wing by up to 25%. Beheshtinia et al. (2017) obtained similar outcomes for uniform subsonic wings, where they found that the viscoelastic damping causes the flutter speed to increase.

Figure 11: Effect of Non-Dimensional Torsional Viscoelastic Damping on Flutter Condition for Different Taper Ratios for Quasi-Steady Model
Figure 12: Effect of Non-Dimensional Torsional Viscoelastic Damping on Flutter Condition for Different Taper Ratios for Unsteady Model

For a clean uniform wing, the plots of non-dimensional flutter speed and frequency versus bending viscoelastic damping ($\eta_E^*$) for different values of $\eta_G^*$ for the quasi-steady and unsteady models are illustrated in Figure 13 and Figure 14, respectively. The figures indicate that higher values of $\eta_G^*$ increase the flutter speed and decrease the flutter frequency. In addition, for the unsteady aerodynamic model, it is clear that the flutter behavior with respect to variation of $\eta_E^*$ is consistent for all the different values of $\eta_G^*$. 
Figure 13: Effect of Non-Dimensional Bending Viscoelastic Damping on Flutter Condition for Different Values of $\eta^*_G$ for Quasi-Steady Model
For a clean wing with taper ratio of $c_t = 0.8$, the non-dimensional flutter speed and frequency are plotted versus viscoelastic damping in bending and in torsion considering the unsteady aerodynamic model (see Figure 15). Both damping parameters increase the flutter speed of a tapered wing. However, the bending viscoelastic damping parameter slightly increases the flutter frequency while the torsional parameter decreases the flutter frequency. It is worth noting that the influence of $\eta_G^*$ on the flutter speed is more significant than that of $\eta_E^*$. Indeed, this outcome is
expected since the flutter mainly occurs when the torsional vibration mode becomes unstable as previously discussed in Section 3.1. Hence, any enhancement in the damping of the torsional motion would ultimately improve the dynamic stability of the wing. Better stability characteristics are achieved when the material of the wing has viscoelastic damping in both bending and torsion. This is in fact the situation of some materials, like composite materials for instance, as they are considered to be viscoelastic materials due to their content of resin (Lahellec & Suquet, 2007).

Figure 15: Effect of Non-Dimensional Viscoelastic Damping in Bending and Torsion on Flutter Condition for Taper Ratio of 0.8
4.3 Engine Mass and Thrust

Figure 16 shows the effect of engine mass on the non-dimensional flutter speed and frequency for a uniform elastic wing \((c_l=\eta_E=\eta_G=0)\) in the absence of the engine thrust \((P = 0)\). It is observed that, for both aerodynamic models, as the engine mass increases, the flutter speed as well as the flutter frequency decreases. This indicates that heavier engines tend to deteriorate the flutter characteristics of the wing.

Figure 16: Effect of Non-Dimensional Engine Mass on Flutter Condition
In the absence of engine thrust, the influence of engine mass on the non-dimensional flutter speed and frequency for an elastic wing ($\eta_E=\eta_G=0$) under unsteady aerodynamic loading is shown in Figure 17 for different values of taper ratio. It is observed that tapered wings are more sensitive to changes in the engine mass.

![Figure 17: Effect of Non-Dimensional Engine Mass on Flutter Condition of an Elastic Wing for Different Taper Ratios](image-url)
For a uniform wing under unsteady aerodynamic loading and in the absence of engine thrust, the non-dimensional flutter speed and frequency are plotted against the non-dimensional engine mass for different values of viscoelastic damping as shown in Figure 18. The results show that wings which are made of viscoelastic materials are slightly less sensitive to changes in the engine mass.

Figure 18: Effect of Non-Dimensional Engine Mass on Flutter Condition of a Uniform Wing for Different Values of Viscoelastic Damping
Figure 19 shows the effect of engine mass on the non-dimensional flutter speed and frequency of a uniform elastic wing under unsteady aerodynamic loading for different aspect ratios and in the absence of engine thrust. The results reveal that wings with higher aspect ratios are extremely sensitive to changes in the engine mass. In other words, the flutter speed and frequency drop dramatically as the engine mass increases for high aspect-ratio wings whereas the influence of engine mass on the flutter boundary is less for low aspect-ratio wings.

Figure 19: Effect of Non-Dimensional Engine Mass on Flutter Condition of a Uniform Elastic Wing for Different Aspect Ratios
The effect of engine mass on the non-dimensional flutter speed and frequency of a uniform elastic wing under unsteady aerodynamic loading for different non-dimensional engine thrusts is shown in Figure 20. It is clear that as the engine thrust increases, the flutter speed decreases while the flutter frequency increases. In addition, engine thrust has almost no influence on the behavior of the flutter with respect to engine mass. Referring to the wing model illustrated in Section 2.1, higher engine thrust will increase the twisting moment on the wing along the $x$-axis, which will make the wing less dynamically stable, and flutter occurs at lower air speeds.

![Figure 20: Effect of Non-Dimensional Engine Mass on Flutter Condition of a Uniform Elastic Wing for Different Values of Non-Dimensional Engine Thrust](image)
In the absence of engine mass, the influence of engine thrust on the non-dimensional flutter speed and frequency for an elastic wing under unsteady aerodynamic loading is shown in Figure 21 for different values of taper ratio. As explained previously, the increase in engine thrust reduces the flutter speed and raises the flutter frequency. It is observed from the behavior that tapered wings are vastly sensitive to changes in the engine thrust. This indicates that elastic wings with high taper ratio tend to lose their dynamic stability when the engine thrust increases.

Figure 21: Effect of Non-Dimensional Engine Thrust on Flutter Condition of an Elastic Wing for Different Taper Ratios
For a uniform wing under unsteady aerodynamic loading and in the absence of engine mass, the non-dimensional flutter speed and frequency are plotted against the non-dimensional engine thrust for different values of viscoelastic damping as shown in Figure 22. Although the viscoelastic damping enhances the dynamic stability of the wing, it has no effect on the behavior of the flutter with respect to engine thrust.

![Graph showing the effect of non-dimensional engine thrust on flutter condition of a uniform wing for different values of viscoelastic damping.](image)

Figure 22: Effect of Non-Dimensional Engine Thrust on Flutter Condition of a Uniform Wing for Different Values of Viscoelastic Damping
Figure 23 presents the effect of engine thrust on the non-dimensional flutter speed and frequency of a uniform elastic wing under unsteady aerodynamic loading for different aspect ratios and in the absence of engine mass. The results show that the flutter speed increases, and the flutter frequency decreases as the aspect ratio of the wing gets higher. It is also observed that wings with higher aspect ratios are less sensitive to changes in the engine thrust. Hence, the influence of engine thrust on the flutter characteristics is less for high aspect-ratio wings.

Figure 23: Effect of Non-Dimensional Engine Thrust on Flutter Condition of a Uniform Elastic Wing for Different Aspect Ratios
In Figure 24, the effect of engine thrust on the non-dimensional flutter speed and frequency of a uniform elastic wing under unsteady aerodynamic loading for different values of non-dimensional engine mass is shown. It is observed from the figure that the behavior of the flutter with respect to engine thrust is the same for the different values of engine mass.

Figure 24: Effect of Non-Dimensional Engine Thrust on Flutter Condition of a Uniform Elastic Wing for Different Values of Non-Dimensional Engine Mass

Figure 25 shows the effect of engine thrust on the non-dimensional flutter speed and frequency for a uniform elastic wing subjected to unsteady aerodynamic
loading and in the absence of engine mass for different bending-to-torsion rigidity ratios. It is observed that the bending-to-torsion rigidity ratio significantly affects the flutter behavior with respect to engine thrust. The flutter speed drops rapidly when the engine thrust increases for wings with high bending-to-torsion rigidity ratio. This observation aligns with the fact that the flutter occurs when the torsional vibration mode becomes unstable. Therefore, to enhance the dynamic stability of the wing, the torsional rigidity must be higher (i.e., low bending-to-torsion rigidity ratio). Fazelzadeh et al. (2020) obtained the same behavior.

Figure 25: Effect of Non-Dimensional Engine Thrust on Flutter Condition of a Uniform Elastic Wing for Different Bending-to-Torsion Rigidity Ratios
4.4 Engine Location

The effect of the engine location in the three directions on the flutter boundary of the wing is investigated. The results show that the position of the engine is of great importance as it influences the flutter condition of the wing considerably.

4.4.1 Span-Wise Engine Location

The engine location along the span \((X_e)\) is illustrated in Figure 26 where higher values of \(X_e\) indicate that the engine is moving towards the wing tip. The influence of the non-dimensional span-wise engine location on the non-dimensional flutter speed and frequency for an elastic wing under unsteady aerodynamic loading is shown in Figure 27 for different taper ratios. The non-dimensional engine thrust is \(P^* = 1\) and the non-dimensional engine mass is \(M_{e^*} = 0.1\). Figure 27 reveals that moving the engine from the wing root to almost 40\% of the wing span \((X_e = 0.4)\) slightly increases the flutter speed and decreases the flutter frequency. As the engine slides further towards the wing tip, the flutter speed increases dramatically. These results show agreement with those obtained by Amoozgar et al. (2013).

Moving the engine mass towards the tip of the wing will make the wing harder to twist and therefore, the flutter speed will be higher since flutter occurs in the tortional motion.

![Figure 26: Demonstration of the Span-Wise Engine Location](image)
It is also observed that this behavior is highly affected by the taper ratio. Tapered wings become more dynamically stable as the engine moves away towards the wing tip. However, due to the structural limitations and to avoid unnecessary increase in the roll moment of inertia, aircraft designers normally keep the engines closer to the fuselage. Nevertheless, some fighter jets carry external stores that are installed at the wing tip.

Figure 27: Effect of Non-Dimensional Span-Wise Engine Location on Flutter Condition of an Elastic Wing for Different Taper Ratios
For a uniform wing under unsteady aerodynamic loading, the non-dimensional flutter speed and frequency are plotted against the non-dimensional span-wise engine location for different values of viscoelastic damping as shown in Figure 28. The non-dimensional engine thrust is $P^* = 1$ and the non-dimensional engine mass is $M_e^* = 0.1$. The results show that wings which are made of viscoelastic materials experience the same flutter behavior with respect to the engine location along the span.

Figure 28: Effect of Non-Dimensional Span-Wise Engine Location on Flutter Condition of a Uniform Wing for Different Values of Viscoelastic Damping
4.4.2 Chord-Wise Engine Location

Figure 29 demonstrates the chord-wise engine location \((Y_e)\), where the negative value of \(Y_e\) indicates that the engine is located front the wing’s elastic axis.

![Figure 29: Demonstration of the Chord-Wise Engine Location](image)

The effect of the non-dimensional chord-wise location of the engine on the non-dimensional flutter speed and frequency for a uniform elastic wing with \(P^* = 1\) and \(M_{e^*} = 1\) is shown in Figure 30. The plot shows that, for the unsteady aerodynamic model, the flutter speed slightly increases as the engine slides from the wing leading edge up to 10% of the chord before the wing elastic axis \((Y_e = -0.1)\). Moving the engine further towards the wing trailing edge decreases the flutter speed. In addition, the quasi-steady aerodynamic model provides that the flutter speed decreases as the engine moves from the leading edge to the trailing edge. Indeed, similar behavior was obtained by Fazelzadeh et al. (2009) and Amoozgar et al. (2013), where it was pointed out that moving the engine from trailing edge to the leading edge in chord-wise direction makes the wing more stable.

This behavior can be explained by analyzing the moments about the elastic axis of the wing. When the engine’s center of gravity is located in front of the elastic axis, the engine mass will cause a restoring moment (in the negative x-axis direction)
counter affecting the aerodynamic twist moment. This will reduce the torsion on the wing and hence, the wing will flutter at a higher speed.

Figure 30: Effect of Non-Dimensional Chord-Wise Engine Location on Flutter Condition of a Uniform Elastic Wing

Figure 31 shows the plots of the non-dimensional flutter speed and frequency versus the non-dimensional chord-wise engine location for an elastic wing under unsteady aerodynamic loading for different taper ratios. The non-dimensional engine thrust is $P^* = 1$ and the non-dimensional engine mass is $M_{e}^* = 0.1$. The results indicate
that the flutter of a tapered wing is very sensitive to the engine location along the chord. As the engine moves from the leading edge towards the trailing edge, the flutter speed drops more rapidly for wings with higher taper ratio.

Figure 31: Effect of Non-Dimensional Chord-Wise Engine Location on Flutter Condition of an Elastic Wing for Different Taper Ratios

For a uniform wing under unsteady aerodynamic loading with non-dimensional engine thrust of $P^* = 1$ and the non-dimensional engine mass of $M_{e^*} = 0.1$, the non-dimensional flutter speed and frequency are plotted against the non-dimensional
chord-wise engine location for different values of viscoelastic damping as shown in Figure 32. The results show that wings which are made of viscoelastic materials experience the same flutter behavior with respect to the engine location along the chord.

Figure 32: Effect of Non-Dimensional Chord-Wise Engine Location on Flutter Condition of a Uniform Wing for Different Values of Viscoelastic Damping
4.4.3 Vertical Engine Location

The vertical engine location ($Z_e$) is demonstrated in Figure 33 where higher values of $Z_e$ means that the engine is placed further down away from the wing. Although moving the engine further below the wing is not realistic as there should be enough ground clearance for the runway, this analysis is carried out just to understand the effect of the vertical location of the engine on the flutter boundaries.

Figure 33: Demonstration of the Vertical Engine Location

The effect of the vertical location of the engine on the non-dimensional flutter speed and frequency for a uniform elastic wing is illustrated in Figure 34. The non-dimensional engine thrust is $P^* = 1$ and the non-dimensional engine mass is $M_e^* = 1$. The quasi-steady aerodynamic model provides that the wing becomes more dynamically stable as the engine goes further below the wing. However, different behavior is observed when the unsteady aerodynamic model is considered, where the wing becomes less stable as the engine goes further below the wing.

When the engine is placed far below the wing, the generated thrust will cause higher twisting moment about the elastic axis of the wing which will make the wing easier to become unstable. Therefore, the unsteady aerodynamic model predictions are more realistic.
For an elastic wing under unsteady aerodynamic loading, the non-dimensional flutter speed and frequency versus the non-dimensional vertical engine location are plotted for different taper ratios as shown in Figure 35. The non-dimensional engine thrust is \( P^* = 1 \) and the non-dimensional engine mass is \( M_e^* = 0.1 \). The results indicate that the flutter of a tapered wing is sensitive to the vertical location of the engine. Although the taper ratio enhances the flutter characteristics of the wing, it is observed...
that wings with higher taper ratios are more quickly to become dynamically unstable when the engine is placed away below the wing.

Figure 35: Effect of Non-Dimensional Vertical Engine Location on Flutter Condition of an Elastic Wing for Different Taper Ratios

The non-dimensional flutter speed and frequency are plotted against the non-dimensional vertical engine location for different values of viscoelastic damping for a uniform wing under unsteady aerodynamic loading with non-dimensional engine thrust of $P^* = 1$ and the non-dimensional engine mass of $M_e^* = 0.1$ as shown in Figure
36. It is clear from the figure that the viscoelastic damping does not have an impact on the flutter behavior with respect to the vertical location of the engine.

Figure 36: Effect of Non-Dimensional Vertical Engine Location on Flutter Condition of a Uniform Wing for Different Values of Viscoelastic Damping
4.5 Elastic Axis Location

The influence of the elastic axis location with respect to the leading edge ($y_0/c$) on the non-dimensional flutter speed and frequency for a clean elastic wing under unsteady aerodynamic loading is shown in Figure 37 for different taper ratios. It is revealed that as the elastic axis moves from 20% to about 35% of the chord, the flutter speed slightly decreases or remains unaffected depending on the value of the taper ratio. Moving the elastic axis further towards the trailing edge of the wing raises the flutter speed significantly. In fact, it can be observed that if the wing elastic axis is located at 55% of the chord from the leading edge, the wing might never experience dynamic instability as the flutter speed approaches infinity. However, due to the airfoil geometry, most aircraft wings have elastic axis that is located before the mid-chord axis ($y_0/c < 50\%$). Therefore, flutter would still occur but at higher speeds. Regarding the flutter frequency, it drops as the elastic axis moves from 20% of the chord towards the wing trailing edge. Excluding the region beyond 35% of the chord, it can be concluded that shifting the elastic axis away from the leading edge towards the trailing edge would cause the wing to be less stable. If the region from 35% to 50% of the chord is considered, the wing will become more dynamically stable if the elastic axis is located more closely towards the mid-chord axis.
Figure 37: Effect of Elastic Axis Location on Flutter Condition of a Clean Elastic Wing for Different Taper Ratios

For a clean uniform wing under unsteady aerodynamic loading, plots of the non-dimensional flutter speed and frequency versus the elastic axis location ($y_0/c$) for different values of viscoelastic damping are given in Figure 38. As detected from the figure, introducing the viscous damping to the wing tends to increase the flutter speed and decrease the flutter frequency. This effect is enormous when the elastic axis is located away from the leading edge of the wing. The figure also shows that if the
elastica is located at 42% of the chord, viscoelastic wings are not expected to undergo flutter.

Figure 38: Effect of Elastic Axis Location on Flutter Condition of a Clean Uniform Wing for Different Values of Viscoelastic Damping
4.6 Inertial Axis Location

The effect of the inertial axis location with respect to the elastic axis ($y_a/c$) on the non-dimensional flutter speed and frequency for a clean elastic wing under unsteady aerodynamic loading is shown in Figure 39. Note that $y_a/c = 0$ indicates that the inertial axis is located exactly at the elastic axis; and $y_a/c = 0.17$ indicates that the inertial axis is located right at the mid-chord of the wing. Since the wing mass is distributed more towards the leading edge because of the airfoil geometry, the wing center of mass will most likely be located in the front half of the chord. Therefore, locations of the inertial axis that is less than 53% of the chord ($y_a/c = 0.20$) are considered in this study. Any values of $y_a/c > 0.20$ would be meaningless.

It is observed from Figure 39 that shifting the inertial axis away from the elastic axis up to almost 45% of the chord ($y_a/c = 0.12$) reduces the flutter speed significantly. As the inertial axis moves further away from the elastic axis towards the trailing edge of the wing, the flutter speed either slightly increases, remains the same, or slightly decreases depending on the value of the taper ratio. Moreover, it is noticed that the flutter frequency rises as the inertial axis moves away from the elastic axis towards the wing trailing edge. As the case with all the previously discussed parameters, the higher the taper ratio, the higher the flutter speed and frequency. It is worth mentioning that the effect of the taper ratio on the flutter speed becomes minor if the inertial axis is shifted towards the mid-chord of the wing.
In the absence of the engine and the taper ratio, the non-dimensional flutter speed and frequency are plotted versus the inertial axis location and presented in Figure 40 for different values of viscoelastic damping. It can be observed that the influence of the viscous damping of the wing becomes of great importance when the inertial axis is located closer to the elastic axis. Here, it can be concluded that for better stability conditions, the mass center of the wing is preferred to be located nearer to the elastic center.

Figure 39: Effect of Inertial Axis Location on Flutter Condition of a Clean Elastic Wing for Different Taper Ratios
Figure 40: Effect of Inertial Axis Location on Flutter Condition of a Clean Uniform Wing for Different Values of Viscoelastic Damping
4.7 Density Ratio

Plots of the non-dimensional flutter speed and frequency versus the density ratio ($\mu$) for a clean elastic wing under unsteady aerodynamic loading for different taper ratios are shown in Figure 41. According to the definition of the density ratio, a value of $\mu = 0$ implies that the wing has no mass. In addition, low values of density ratio ($\mu < 2$) can exist if either the air density or the root chord-length is very high. Since these values are not realistic, the range of density ratio $2 \leq \mu \leq 25$ is considered. As shown in Figure 41, the density ratio can significantly affect the stability characteristics. As the density ratio increases from 2 to 3, the flutter speed drops dramatically. Values of density ratio greater than 3, enhance the dynamic stability of the wing (i.e., increase the flutter speed). Figure 41.b indicates that the flutter frequency rises as the density ratio increases from 2 to 5 and then drops for density ratios greater than 5. The same behavior is obtained for the three different taper ratios. It is also observed from the plots that the taper ratio has slight influence on the flutter speed at low density ratios. However, for high values of density ratio, the taper ratio becomes of great effect as the aeroelastic stability of wings with higher taper ratio is better. As for the flutter frequency, it is observed that tapered wings have higher flutter frequency for any value of density ratio as compared to uniform wings.

Overall, it can be noticed that for higher altitudes (lower air densities, higher density ratios) the flutter speed rises, providing a wider stability region for the wing. In fact, aircrafts are more expected to experience dynamic instabilities while flying at low altitudes. Moreover, as the aircraft burns more fuel while flying, the total mass of the wing per unit span ($m$) reduces and accordingly the density ratio will decrease. Therefore, the aircraft will be less dynamically stable.
In the absence of the engine and the taper ratio, the non-dimensional flutter speed and frequency are plotted versus the density ratio for different values of viscoelastic damping as shown in Figure 42. The results reveal that introducing a viscoelastic damping to the wing would raise the flutter speed and reduce the flutter frequency at any value of density ratio. Similar results are obtained by Haddadpour and Firouz-Abadi (2006) and Beheshtinia et al. (2017) for 2D typical wing section. It is also revealed from the results that for low density ratios, the viscoelastic damping...
has a significant effect on the flutter frequency. Nevertheless, the effect of the viscoelastic damping on the flutter frequency becomes minor for high values of density ratios.

Figure 42: Effect of Density Ratio on Flutter Condition of a Clean Uniform Wing for Different Values of Viscoelastic Damping
4.8 Frequency Ratio

Plots of the non-dimensional flutter speed and frequency versus the bending-to-torsion frequency ratio ($\sigma$) for a clean elastic wing under unsteady aerodynamic loading for different taper ratios are illustrated in Figure 43. Usually, the value of the bending-to-torsion frequency ratio is less than unity. This is mainly because of the fact that the uncoupled bending frequency of the structure is less than the torsional frequency. It is observed from Figure 43 that as the frequency ratio increases up to a certain value (e.g., $\sigma = 0.55$ for $c_t = 0$), the flutter speed is reduced. Higher values of frequency ratio would cause the flutter speed to increase significantly. For low frequency ratios ($\sigma < 0.3$), the flutter speed is lower for wings with small taper ratio. The effect of the taper ratio on the flutter speed becomes of great influence for frequency ratios greater than 0.4. It is noticed that a wing with a taper ratio of 0.4 may not experience flutter if the frequency ratio is greater than 0.7. In fact, this is an interesting finding where the flutter characteristics of tapered wings can be enhanced by increasing the frequency ratio. It is also indicated from the figure that the flutter frequency rises for higher values of frequency ratio. Higher taper ratios increase the flutter frequency for frequency ratios less than 0.6. It is worth mentioning that at about $\sigma = 0.6$, the flutter frequency is the same for any taper ratio.
The effect of the bending-to-torsion frequency ratio on the non-dimensional flutter speed and frequency for a clean uniform wing under unsteady aerodynamic loading is illustrated in Figure 44 for different values of viscoelastic damping. For low frequency ratios, the viscoelastic damping has minor effect on the flutter speed and frequency. Nevertheless, it can be observed that higher values of viscoelastic damping can significantly improve the dynamic stability of the wing for high frequency ratios.

Figure 43: Effect of Bending-to-Torsion Frequency Ratio on Flutter Condition of a Clean Elastic Wing for Different Taper Ratios
According to the obtained results, it is shown that flutter may not happen to wings with viscoelastic damping when the frequency ratio exceeds a certain value.

Figure 44: Effect of Bending-to-Torsion Frequency Ratio on Flutter Condition of a Clean Uniform Wing for Different Values of Viscoelastic Damping
4.9 Radius of Gyration

In the absence of the engine and the viscoelastic damping, variations of the non-dimensional flutter speed and frequency versus the dimensionless radius of gyration about the elastic axis \( r_a \) are obtained and presented in Figure 45 for different taper ratios. As noticed from the results, higher radius of gyration would considerably enhance the dynamic stability of the wing, which is expected indeed. According to the definition of the radius of gyration as given in Section 2.5, higher values of \( r_a \) implies that the wing structure has large mass moment of inertia (i.e., the wing is more resistant to rotation). Hence, a wing that resists the rotation motion (or torsion) would be more dynamically stable since the flutter occurs when the torsional vibration mode becomes unstable. The figure also reflects the fact that the taper ratio has an impact on the behavior of the flutter with respect to the radius of gyration.

In Figure 46, the non-dimensional flutter speed and frequency for a clean uniform wing under unsteady aerodynamic loading are plotted versus the radius of gyration for different values of viscoelastic damping. The results show that better stability is achieved when higher viscous damping is introduced to the wing. In addition, it is observed that the impact of the viscoelastic damping on the flutter speed is substantial for high radii of gyration.
Figure 45: Effect of Non-Dimensional Radius of Gyration on Flutter Condition of a Clean Elastic Wing for Different Taper Ratios
Figure 46: Effect of Non-Dimensional Radius of Gyration on Flutter Condition of a Clean Uniform Wing for Different Values of Viscoelastic Damping
Chapter 5: Conclusion

Aero-viscoelasticity represents the interaction of aerodynamics, viscoelasticity, and structural dynamics to model the behavior of the structures subjected to airflow. The formulation and computation of an aeroelastic problem requires a background in each of the constituent disciplines. Flutter is one of the significant phenomena of Aeroelasticity. It is important to analyze the flutter to avoid running into instability regions which may lead to catastrophic events.

The flutter of a tapered viscoelastic wing modeled as Euler-Bernoulli beam, subjected to a follower thrust force is analyzed. The quasi-steady and unsteady models are employed to simulate the aerodynamic forces. The Theodorsen’s model is assumed for the unsteady aerodynamic forces. The determinant method is utilized to solve the flutter problem since it requires less computational time and enables the inclusion of the effect of the viscoelastic damping.

For a clean uniform elastic wing (in the absence of the engine, taper ratio and viscoelastic damping of the material), the flutter speed of the Goland wing obtained in the current study is 136.45 m/s and the flutter frequency is 69.39 rad/sec, which are compared with 137.16 m/s and 70.7 rad/sec obtained by Goland (1945). The flutter speed of the HALE wing obtained in the present study is 33.46 m/s and the flutter frequency is 21.48 rad/sec, which are compared with 32.51 m/s and 22.37 rad/sec obtained by Patil et al. (2001). Therefore, the obtained results show excellent agreement with the original Goland and HALE wings.

The carried-out parametric investigation shows that the taper ratio, viscoelastic damping, engine thrust, engine mass, engine location, elastic axis location, inertial axis
location, wing density ratio, bending-to-torsion frequency ratio, bending-to-torsion rigidity ratio, and radius of gyration have considerable effects on the flutter characteristics of the aircraft wing. Therefore, all these parameters must be taken into consideration to get accurate flutter predictions.

5.1 Research Implications

The unsteady aerodynamic model provides more reliable results than the quasi-steady model which offers more conservative predictions. In most cases, the two models yield the same behavior. In some other cases, however, they give diverse results.

It is observed that increasing the taper ratio raises the flutter speed and flutter frequency, which leads to better stability characteristics. The current study shows that aircrafts flying at higher altitudes (higher density ratio) would be more dynamically stable as the flutter conditions are higher. Investigating the effect of the density ratio reveals that as the aircraft travels longer distances, the flutter speed reduces and, hence, the wing becomes less dynamically stable. It is also perceived that the viscoelastic damping property of the wing material can adequately reduce the structural vibration and enhances the stability of the wing. Based on the obtained results, increasing the value of bending viscoelastic damping ($\eta_E$) and/or the value of torsion viscoelastic damping ($\eta_G$) increases the flutter speed. Hence, improved dynamic stability is achieved.

According to the present analysis, the flutter speed and frequency are substantially influenced by the bending-to-torsion frequency ratio and the radius of gyration. Wing structures that have high frequency ratio ($\sigma > 0.5$) relish wider stability
regions. Nevertheless, wings with low frequency ratios are less dynamically stable (i.e., have lower flutter speed). In addition, the results show that as the radius of gyration about the elastic axis increases, both the flutter speed and frequency rise providing wider region of stability.

The results show that the location of the elastic axis may affect the dynamic stability of the wing significantly. Wings with an elastic axis located between 20% to 35% of the chord from the leading edge have almost the same flutter condition. However, wings that have an elastic axis located between 35% to 50% of the chord are much more stable. The inertial axis position plays an important role in the stability of the wing as well. As the results reveal, if the distance between the inertial axis and the elastic axis is large, the flutter speed drops. Therefore, it is recommended to have the elastic center closer to the wing mid-chord and to reduce the chord-wise distance between the inertial center and the elastic center. This will assure that higher flutter speed and better stability characteristics are achieved.

The obtained predictions indicate that the flutter speed and frequency are substantially influenced by the engine thrust and mass where higher engine thrusts and masses deteriorate the dynamic stability of the wing. It is also observed that the taper ratio, aspect ratio, and bending-to-torsion rigidity ratio rapidly affect the flutter boundary for high engine thrusts and masses.

The developed aeroelastic model show that the location of the engine affects the dynamic stability of the wing significantly. The influence of the engine location on the flutter characteristics is greater for tapered viscoelastic wings.
5.2 Key Findings

Based on the obtained results, the flutter characteristics of the wing can be enhanced by:

- Increasing the taper ratio.
- Using viscoelastic material.
- Increasing the torsional rigidity.
- Reducing the engine mass.
- Placing the engine away from the elastic axis towards the wing leading edge.
- Placing the engine away from the fuselage towards the wing tip.
- Placing the engine right below the wing (i.e., keeping the vertical distance between the engine’s center of gravity and the wing chord-line minimal).
- Reducing the static margin between the elastic center and the inertial center.

5.3 Future Work

A recommended future research would be to extend the parametric study to include more cases such as the effect of engine location on the flutter boundary for different values of engine thrust and mass. In addition, the developed aeroelastic model can be upgraded to cover more parameters such as the wing sweep and twist. Moreover, it is possible to investigate the effect of involving more external stores or engines and the location of each store/engine on the flutter characteristics of a viscoelastic tapered wing. Finally, this work can be validated by conducting numerical investigations using Finite Element Methods or by performing experimental testing as suggested in Appendix D.
References


List of Publications


Appendices

Appendix A: Derivation of the Lift Force and Twisting Moment Equations for Goland Wing

The general equations of motion for a clean elastic uniform wing are given as,

\[ m \ddot{h} + m_y \ddot{\theta} + (EI\dddot{h})'' = -L \]

\[ I_{EA} \ddot{\theta} + m_y \dot{h} - (GJ\theta)' = M \]

The unsteady aerodynamic lift and moment are (Theodorsen, 1935; Hodges & Pierce, 2011; Polliana et al., 2016):

\[ L = \pi \rho b^2 \left[ \dot{h} + U \dot{\alpha} - ab \dot{\alpha} \right] + 2\pi \rho UbC(k) \left[ \dot{h} + U\alpha + b \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \]

\[ M = \pi \rho b^2 \left[ ab \dot{h} - Ub \left( \frac{1}{2} - a \right) \dot{\alpha} - b^2 \left( \frac{1}{8} + a^2 \right) \ddot{\alpha} \right] + 2\pi \rho Ub^2 \left( a + \frac{1}{2} \right) C(k) \left[ \dot{h} + U\alpha + b \left( \frac{1}{2} - a \right) \dot{\alpha} \right] \]

Considering the differences in the annotation between the model analyzed in this thesis (Figure 47) and the model of Theodorsen’s notation (Figure 48), it is noted that:

\[ b \rightarrow \frac{c}{2} \quad \alpha \rightarrow \theta \quad 2\pi \rightarrow \frac{dc_k}{d\theta} \quad ab \rightarrow y_0 - \frac{c}{2} \quad a \rightarrow \frac{2y_0}{c} - 1 \]
Hence the lift force will be:

\[ L = \pi \rho \left( \frac{c}{2} \right)^2 \left[ \dot{h} + U \dot{\theta} - \left( y_0 - \frac{c}{2} \right) \ddot{\theta} \right] \]

\[ + \frac{dc}{d\theta} \rho U \left( \frac{c}{2} \right) C(k) \left[ \dot{h} + U \theta + \frac{c}{2} \left( \frac{1}{2} - \left( \frac{2y_0}{c} - 1 \right) \right) \dot{\theta} \right] \]
Rearranging the terms,

\[
L = \frac{1}{4} \pi \rho c^2 \left[ \dot{h} + U \dot{\theta} - \left( y_0 - \frac{c}{2} \right) \dot{\theta} \right] + \frac{1}{2} \rho U^2 c \frac{dc_L}{d\theta} C(k) \left[ \dot{h} \frac{h}{U} + \theta + \frac{c}{U} \left( \frac{1}{4} - \left( \frac{y_0}{c} \right) \right) \dot{\theta} \right]
\]

Finally,

\[
L = \frac{1}{4} \pi \rho c^2 \left[ \dot{h} + U \dot{\theta} - \left( y_0 - \frac{c}{2} \right) \dot{\theta} \right] + \frac{1}{2} \rho U^2 c \frac{dc_L}{d\theta} C(k) \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right]
\]

Similarly, the moment equation will be:

\[
M = \frac{1}{4} \pi \rho c^2 \left[ \left( y_0 - \frac{c}{2} \right) \dot{\theta} - U \left( \frac{3c}{4} - y_0 \right) \dot{\theta} - c^2 \left[ \frac{9}{32} + \frac{y_0}{c} \left( \frac{y_0}{c} - 1 \right) \right] \dot{\theta} \right] + \frac{1}{2} \rho U^2 c^2 \frac{dC_L}{d\theta} C(k) \left( \frac{y_0}{c} - \frac{1}{4} \right) \left[ \theta + \frac{\dot{h}}{U} + \frac{c}{U} \left( \frac{3}{4} - \frac{y_0}{c} \right) \dot{\theta} \right]
\]
Appendix B: Comparison between Exact and Approximate Theodorsen’s Function

The flutter speed, flutter frequency, and reduced frequency were obtained for the Goland Wing subjected to unsteady Theodorsen aerodynamic loading. The exact expression as well as the approximate expression of the Theodorsen’s function were adopted. Table 13 summarizes the results.

Table 13: Comparison between Exact and Approximate Theodorsen’s Function

<table>
<thead>
<tr>
<th></th>
<th>Reference Value (Goland, 1945)</th>
<th>Exact C(k)</th>
<th>Error (%)</th>
<th>Approx. C(k)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flutter Speed (m/s)</td>
<td>137.16</td>
<td>136.02</td>
<td>0.83</td>
<td>136.45</td>
<td>0.52</td>
</tr>
<tr>
<td>Flutter Frequency (rad/s)</td>
<td>70.7</td>
<td>70.06</td>
<td>0.91</td>
<td>69.39</td>
<td>1.85</td>
</tr>
<tr>
<td>Reduced Frequency (k)</td>
<td>0.471</td>
<td>0.471</td>
<td>0.0</td>
<td>0.465</td>
<td>1.27</td>
</tr>
</tbody>
</table>

As seen in the table, the exact Theodorsen’s function provide accurate flutter frequency but less accurate flutter speed. Overall, the inaccuracy associated with using the approximate representation of the Theodorsen’s function is considerably low.
Appendix C: Coefficients of the Equations of Motion

\[a_{11} = \int_0^1 \left( [1 - c_t \xi] f'' \right) f \, d\xi\]

\[a_{12} = \frac{\eta_E}{\sigma} \int_0^1 \left( [1 - c_t \xi] f'' \right) f \, d\xi\]

\[a_{13} = \frac{1}{\sigma^2} \int_0^1 [1 - c_t \xi] f^2 \, d\xi\]

\[a_{14} = \frac{M_e}{\sigma^2} \int_0^1 f^2 \delta_D(x_e - \xi) \, d\xi\]

\[a_{15} = \frac{M_e z_e^2}{\sigma^2} \int_0^1 f'' f \delta_D(x_e - \xi) \, d\xi\]

\[a_{16} = \frac{1}{2\mu\sigma^2} \frac{dC_L}{d\theta} \int_0^1 [1 - c_t \xi] f^2 \, d\xi\]

\[a_{17} = \frac{\pi}{4\mu \sigma^2} \int_0^1 [1 - c_t \xi]^2 f^2 \, d\xi\]
\[
a_{21} = \frac{y_a}{\frac{r_a}{2}} \int_{0}^{1} \left[1 - c_t \xi \right]^2 f \phi d\xi
\]

\[
a_{22} = \frac{\sqrt{\Omega}}{AR} \int_{0}^{1} P(x_e - \xi)H(x_e - \xi)f''\phi d\xi
\]

\[
a_{23} = \frac{M_ey_e}{r_a^2} \int_{0}^{1} f \phi \delta_D(x_e - \xi) d\xi
\]

\[
a_{24} = \frac{1}{2\mu r_a^2} \frac{dC_L}{d\theta} \int_{0}^{1} \left[1 - c_t \xi \right]^2 \left(y_0 - \frac{1}{4}\right) f \phi d\xi
\]

\[
a_{25} = \frac{\pi}{4\mu r_a^2} \int_{0}^{1} \left[1 - c_t \xi \right]^3 \left(y_0 - \frac{1}{2}\right) \phi f d\xi
\]
\[ b_{11} = \frac{y_a}{\sigma^2} \int_{0}^{1} [1 - c_t \xi]^2 f \phi \, d\xi \]

\[ b_{12} = \frac{AR}{\sqrt{\Omega}} \int_{0}^{1} P(x_e - \xi)H(x_e - \xi)\phi'' f d\xi \]

\[ b_{13} = \frac{2AR}{\sqrt{\Omega}} \int_{0}^{1} P H(x_e - \xi)\phi' f d\xi \]

\[ b_{14} = \frac{M_e y_e}{\sigma} \int_{0}^{1} \phi f \delta_D(x_e - \xi) d\xi \]

\[ b_{15} = \frac{AR}{\sqrt{\Omega}} \int_{0}^{1} P \phi f \delta_D(x_e - \xi) d\xi \]

\[ b_{16} = \frac{1}{2\mu \sigma^2} \frac{dC_L}{d\theta} \int_{0}^{1} [1 - c_t \xi] f \phi \, d\xi \]

\[ b_{17} = \frac{1}{2\mu \sigma^2} \frac{dC_L}{d\theta} \int_{0}^{1} [1 - c_t \xi]^2 \left( \frac{3}{4} - y_0 \right) f \phi \, d\xi \]

\[ b_{18} = \frac{\pi}{4\mu \sigma^2} \int_{0}^{1} [1 - c_t \xi]^2 \phi f \, d\xi \]

\[ b_{19} = \frac{\pi}{4\mu \sigma^2} \int_{0}^{1} [1 - c_t \xi]^3 \left( y_0 - \frac{1}{2} \right) \phi f \, d\xi \]
\begin{align*}
\textbf{b}_{21} &= \int_{0}^{1} \left( (1 - c_{t} \xi) \phi' \right)' \phi \, d\xi \\
\textbf{b}_{22} &= \eta_{c} \int_{0}^{1} \left( (1 - c_{t} \xi) \phi' \right)' \phi \, d\xi \\
\textbf{b}_{23} &= \int_{0}^{1} (1 - c_{t} \xi) \phi^2 \, d\xi \\
\textbf{b}_{24} &= \frac{AR^2 M_e}{r_a^2} \int_{0}^{l} \left( k_e^2 + z_e^2 + \frac{y_e^2}{AR^2} \right) \phi^2 \delta_D(x_e - \xi) \, d\xi \\
\textbf{b}_{25} &= \frac{P y_e \sqrt{\Omega}}{AR} \int_{0}^{l} \phi^2 \delta_D(x_e - \xi) \, d\xi \\
\textbf{b}_{26} &= \frac{1}{2 \mu r_a^2} \frac{dC_L}{d\theta} \int_{0}^{1} \left[ 1 - c_{t} \xi \right]^2 \left( y_0 - \frac{1}{4} \right) \phi^2 \, d\xi \\
\textbf{b}_{27} &= \frac{1}{2 \mu r_a^2} \frac{dC_L}{d\theta} \int_{0}^{1} \left( y_0 - \frac{1}{4} \right) \frac{dC_L}{d\theta} \left( \frac{3}{4} - y_0 \right) - \frac{\pi}{8} \left[ 1 - c_{t} \xi \right] \phi^2 \, d\xi \\
\textbf{b}_{28} &= \frac{\pi}{4 \mu r_a^2} \int_{0}^{1} \left[ 1 - c_{t} \xi \right] \left( \frac{3}{4} - y_0 \right) \phi^2 \, d\xi \\
\textbf{b}_{29} &= \frac{\pi}{4 \mu r_a^2} \int_{0}^{1} \left[ 1 - c_{t} \xi \right]^{4} \left( \frac{9}{32} + y_0^2 - y_0 \right) \phi^2 \, d\xi
\end{align*}
Appendix D: Potential Experimental Approach

In order to validate the developed aeroelastic model, experimental investigations can be conducted. A flat rectangular wing model can be used for Wind Tunnel testing. A possible test wing can be an Aluminum-6061 sheet of 2-mm thickness with a chord length of 10 cm and a half-span of 30 cm. The wing will be suitable to fit inside the wind tunnel available in the Aerospace Lab at the United Arab Emirates University.

To record the bending and torsional frequencies, two 6-axis accelerometers must be used. One accelerometer will be attached at the leading edge of the free end (wing tip) and the other one will be attached at the elastic center (which is mid-chord for a flat plate) of the free end. An Arduino code must be developed to extract the readings from the accelerometers. Then, the readings will be converted into frequencies in bending and torsion via a MATLAB script. It is important to have an angle of attack to stimulate the oscillation as the wing will have almost zero lift at zero angle of attack.

The wing model can be used to conduct the following experiments:

1) The effect of taper ratio: by preparing three test wings with three different taper ratios \( c_t = 0, c_t = 0.4 \& c_t = 0.8 \).

2) The effect of density ratio: by changing the material of the plate.

3) The effect of store/engine mass: by attaching a mass, such as a balancing Lead piece, to represent the store/engine and changing the mass of the attached piece.
4) The effect of store/engine span-wise location: by changing the location of the attached piece along the span.

5) The effect of store/engine chord-wise location: by changing the location of the attached piece along the chord.

The analytical solution of the flutter speed and frequency for the test wing can be obtained using the aeroelastic model developed in this thesis. After that, the theoretical results can be validated against the experimental records.

The following are 2 suggested test models with possible rigs that can fit in the wind tunnel at the United Arab Emirates University.

**Suggested Test Model 1:**

This test rig can be inserted through the circular window of the Wind Tunnel wall.

![Suggested Test Model 1](image)
Suggested Test Model 2:

This test rig can be fixed on the Wind Tunnel base.

Figure 50: Suggested Test Model 2
Appendix E: Best Paper Award

Effect of Engine Location on Flutter Speed and Frequency of a Tapered Viscoelastic Wing

authored by
Y.S. Matter, T.T. Darabseh and A-H.I. Mourad

has been awarded the best paper at
International Conference of Aerospace & Mechanical Engineering
held in Penang, Malaysia, November 21st – 22nd 2017

Associate Professor Ir. Dr. Parvathy Rajendran
Conference Chair
AEROMECH 2017