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Dynamics of Pancake Vortices in A Finite Stack Of Superconducting Copper Oxide Layers

Mariam Mustafa Ahmad Al Marzoqi

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DYNAMICS OF PANCAKE VORTICES IN A FINITE STACK OF SUPERCONDUCTING COPPER OXIDE LAYERS

A Thesis Submitted to
The Deanship of Graduate Studies
of The United Arab Emirates University

BY

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In Partial Fulfillment of The Degree of M.Sc in Materials Science and Engineering

2001
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Abstract

The high temperature superconductors are very promising materials in a wide range of applications, starting from the electricity power supply. They play a life saving function through imaging systems. Even major internet and communication advances depend on these materials.

Superconductivity in these materials occurs particularly in the copper-oxide planes. However, since these materials are type-II superconductors, magnetic fields can penetrate these materials in quantized amounts of flux called vortices without destroying completely superconductivity, but producing some resistance, due to vortex motion. In order to overcome the resistance problem, vortices must be pinned to prevent their motion and hence eliminate the resistance.

In this thesis, we will model the magnetic pancake vortices in a finite stack of superconducting layers. Pinning centers are uniformly distributed in each layer but the strength of these pinning centers is random from one layer to another to see their effect on the conductivity of the superconducting layers.

In chapter two, we apply equal but oppositely directed dc currents to the outermost layers, where two different cases are considered (1) free-pinning sample and (2) pinned sample. Decoupling current as function of the applied magnetic field as well as number superconducting layers is shown.

In chapter three, dc current is applied to one of the outermost layers. Velocities of outermost layers as well as displacement difference between the top layer and the layer below it are calculated. Moreover, time averaged voltages due to the motion of pancakes is computed for outermost layers. In addition, magnetic flux flow rate at the top layer and flux flow rate
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Abbreviations and symbols

\( \Lambda \)  Thin film screening length
\( \bar{j} \)  Current density
\( t \)  Time
\( \bar{E} \)  Electric field
\( \bar{h} \)  Microscopic magnetic field
\( m \)  Electron mass
\( n_e \)  Number of superconducting electrons per unit volume
\( e \)  Electron charge
\( \lambda_t \)  London penetration depth
\( \lambda \)  Penetration depth
\( \psi(r) \)  Order parameter wave function
\( \phi(r) \)  Phase of the wave function
\( \bar{r} \)  Position vector
\( T_c \)  Critical temperature
\( e^* \)  Charge of associated superconducting pairs
\( m^* \)  Mass of associated superconducting pairs
C.C.  Complex conjugate
\( c \)  Speed of light
\( \bar{a} \)  Vector potential
\( \hbar \)  Reduced Planck’s constant
\( K \)  Ginzburg-Landau parameter
\( \xi \)  Coherence length
K  Kelvin
$H$  Applied magnetic field

$H_c$  Critical applied magnetic field

$T$  Temperature

$J_c$  Critical current density

$H_{c1}$  Lower critical field

$H_{c2}$  Upper critical field

$F_p$  Pinning force

$s$  Interlayer spacing

$d$  Thickness of superconducting layers

$N$  Number of superconducting layers

$J_1(qR)$  First order Bessel function

$\phi_0$  Flux quantum

$\delta_{n,0}$  Kronecker delta

$\bar{\kappa}$  Sheet current density

$F$  Force between pancake vortices

$\bar{\delta}_j$  Displacement of pancake vortices

$\bar{g}$  Reciprocal lattice vector

$\bar{b}_i$  Reciprocal lattice basis vector

$\bar{a}_i$  Direct lattice basis vector

$A$  Area of unit cell

$a$  Lattice constant

$\eta$  Viscous drag coefficient

$B$  Average magnetic induction

$\bar{V}$  Time averaged voltage
$v_{r0}$ Common velocity of pancake vortex

$n$ Number of pancake vortices per unit area

$\phi$ Flux

$\dot{\phi}$ Flux rate

$I_0$ Current amplitude

$\Phi$ Applied current phase

$T_i$ Period of alternating current

$\nu$ Frequency

$P$ Power dissipation

$\overline{P}$ Average power dissipation
Chapter I: Introduction
1.1 Introduction

Superconductivity is a wonderful and challenging field of physics, because of its exceptional properties. Superconductors have a wide spectrum of applications.\textsuperscript{1,2} They have been used as electromagnets which generate large magnetic fields without losing energy. These superconducting magnets have applications in diagnostic medical equipment, such as MRI: Magnetic Resonance Imaging. Also, they are used in powerful particle accelerators, and studies of materials. Superconductors can have even more widespread practical applications if better superconducting compounds are discovered. For example,\textsuperscript{7,8} faster computers with larger storage capacities, magnetic levitation of high speed trains\textsuperscript{9}, and one of the most interesting of all is the efficiency of generation and transmission of electric power\textsuperscript{10}. As a result of this importance, J. George Bednorz & K. Alex Muller won a Nobel Prize in physics in 1987 after their discovery of superconducting materials at higher critical temperatures.

Superconductivity is the phenomenon that was first discovered by the Dutch physicist Heike Kamerlingh Onnes in 1911.\textsuperscript{3,12,13} It occurs in certain materials that demonstrate no resistance to the flow of an electric current. Although many physical properties of superconducting materials are not changed in the transition of the material from normal to superconductor state such as crystal structure and optical properties, they have the property of perfect conductivity, zero energy loss or zero resistance. Onnes found that the electrical resistivity of a mercury wire\textsuperscript{8} vanishes after cooling it to a temperature below 4 K. Very soon after this discovery, he discovered that a superconducting material could be returned to the normal state either by passing a sufficiently large current or by applying strong magnetic field to it. The highest temperature at which superconductivity occurs in a material is called critical temperature $T_c$. Below this transition temperature the resistivity of the material is zero. Moreover, Onnes\textsuperscript{14}...
found that the superconductor exhibited what he called persistent currents, electric current that continued to flow without an electric potential driving them. In one of Onnes experiments\(^1\), he started a current flowing through a loop of lead wire cooled to 4 K. After a year, the current was still flowing without significant current losses. In 1933 Walter Meissner and R. Ochsenfeld\(^3\) discovered that elemental superconductors are more than a perfect conductor of electricity. They exclude all magnetic flux and are perfect diamagnetic materials\(^2\). This causes currents to flow at the surface of the superconductor, which generate magnetic field inside the superconductor that just balances the field that would have otherwise penetrated the material. This effect is called the Meissner effect. It occurs only if the magnetic field is relatively small.

The importance of this experiment is the fact that although superconductors have zero electric resistance, they differ from perfect conductors\(^2\). But, how does the transition from normal state to superconducting state occur? Three famous theories of superconductivity are discussed next.

### 1.2 Theories of superconductivity

Many attempts had been made to explain the remarkable properties of superconductors. The early phenomenological theories expected that certain electrons are responsible for superconductivity and they were called as “super electrons”\(^2\). The most important theories are London, the BCS (Bardeen, Cooper, and Schrieffer) and Ginzburg-Landau theories\(^14\).

#### 1.2.1 London theory:

In 1935, shortly after the discovery of the Meissner effect, London brothers developed a phenomenological theory of superconductivity\(^2, 3, 8, 14-16\), which is referred to as London theory.
It was clear to them that this phenomenon, was an illustration of a quantum state of a macroscopic scale \(^{13, 14}\). This theory was capable of describing a large number of observations. The two basic equations of the London theory are given by:

\[
\frac{d}{dt}(\Lambda \vec{j}) = \vec{E}, \quad (1.1)
\]

\[
curl(\Lambda \vec{j}) = -\hbar, \quad (1.2)
\]

where the screening length \( \Lambda = \frac{m}{n_e e^2} \) and \( m, n_e \), and \( e \) are the mass, the number per unit volume, and charge of carriers of the super current, respectively. Eq. (1.1) is equivalent to say: the change of the current density with time is proportional to the electric field, \( \vec{E} \), and Eq. (1.2) describes the Meissner effect in a quantitative way \(^{2, 14}\). London theory led to important conclusions such as \(^2\), a decay of the external magnetic field in the thin surface layer, of order \( \lambda_t \), of the superconductor, as well as to the magnetic flux quantization, which was experimentally confirmed in 1961 \(^2\). In spite of the importance of the observations of London’s theory, it didn’t give any explanations about the origin of superconductivity in the microscopic scale.

1.2.2 The BCS theory:

In 1957, three American physicists, John Bardeen, Leon Cooper, and Robert Schrieffer \(^{14, 17}\) developed a simple model which is expressed in terms of advanced ideas of quantum
mechanics, but the main idea of the model suggests that the electrons in a superconductor condense into a quantum ground state and travel together collectively and coherently. This theory, which is called the BCS theory \(^4,^{16}\), says that two electrons in the superconductor can form a bound state, which is called the Cooper pair. These two electrons are of equal and opposite momenta and spin, they act as bosons. Hence, the superconducting particles are these pairs of electrons \(^2\). An important parameter associated with superconductivity is the coherence length. It can be defined as the smallest distance over which the electrons in a Cooper pair remain together. The BCS theory has given a good understanding of the phenomenon of superconductivity at low critical temperature.

### 1.2.3 Ginzburg-Landau theory:

Ginzburg-Landau theory is a semi phenomenological theory, which was constructed in 1950 but only justified only in 1959 on the basis of the BCS theory by Gorkov \(^1\). The advantage of Ginzburg-Landau theory over the BCS theory is that it can treat the case of a superconductor in a magnetic field in a much easier way. According to Ginzburg-Landau theory, a superconducting state can be described by a complex “order parameter” denoted by \(\psi(r)\) \(^2,^{14}\), where:

\[
\psi(r) = \left[ n_s(r) \right]^{1/2} \exp[i\phi(r)]
\]  

(1.3)

is expressed in terms of a phase \(\phi(r)\), and its magnitude is a measure of the degree of superconducting order at position \(r\) below \(T_c\). The description of a superconductor by \(\psi(r)\) is valid only for a superconductor whose properties change slowly on the scale of the dimensions
of the Cooper pair \(^2\). In the presence of an external applied electromagnetic field, the current equation is given by \(^{14}\):

\[
\vec{j} = -\frac{e'}{2m'} \left[ \psi^* (r) \left( \frac{\hbar}{i} \vec{\nabla} + \frac{2e'}{c} \vec{a} \right) \psi(r) + C.C. \right]
\]  

(1.4)

where \(e'\), \(m'\) are the charge and the mass associated with a superconducting pair, respectively. \(\vec{a}\) is the vector potential and \(C.C.\) is the complex conjugate. If \(n_s(r)\) varies slowly with respect to the phase \(\phi(r)\), then the current equation could be written as \(^{14}\):

\[
\vec{j} \approx \left( \frac{2e^2}{mc} \vec{A} + \frac{eh}{m} \nabla \phi(r) \right) n_s
\]

(1.5)

taking \textit{curl} of the Eq. (1.5), we obtain the result proposed by the London brothers\(^{14}\).

A famous parameter associated with Ginzburg-Landau theory is the ratio \(K\) of the magnetic field penetration depth \(\lambda(T)\) to the temperature dependent coherence length \(\xi(T)\), which is given by\(^{16, 18, 19}\):

\[
K = \frac{\lambda(T)}{\xi(T)}.
\]

(1.6)
According to parameter value $K$, the type of the superconductor is determined.

### 1.3 Josephson junction

Josephson\textsuperscript{14, 16} examined the quantum nature of superconductivity and proposed the existence of oscillations in the electric current flowing through two superconductors, separated by a thin insulating layer in a magnetic or electric field. The effect is known as the Josephson effect. The current flow is termed as the Josephson current, and the penetration of insulators by Cooper pairs is known as Josephson tunneling. Josephson expected and pointed out many possible associated applications, such as a detector of very high frequency radiation\textsuperscript{2}, but the requirement of maintaining the superconductor at very low temperatures, forms a severe hindrance to various applications. Developing higher temperature superconductors and better refrigeration techniques solve this problem\textsuperscript{4}.

### 1.4 Superconductors with higher critical temperature $T_c$

Until 1986 the highest critical temperature $T_c$ was 23.2 K in niobium-germanium compounds\textsuperscript{3}. Metallic and alloy materials are called low temperature superconductors, while ceramic based on copper oxides\textsuperscript{20, 21} are known to have high $T_c$. In 1986\textsuperscript{5, 6, 8}, George Bednorz and Alex Muller, did experiments on a particular class of metal oxide ceramics, which contains lanthanum, barium, copper, and oxygen, $La_{2-x}Ba_xCuO_4$, called perovskites. They found indications of superconductivity at 35 K, 12 K above the old record for a superconductor. In February 1987\textsuperscript{6} a perovskite ceramic material was found to superconduct at 90 K. This discovery was very significant because of possible use of liquid nitrogen as a coolant, which
cools 20 times more effective than liquid helium, an expensive and inefficient coolant. Because these materials superconduct at significantly higher temperatures they are referred to as high temperature superconductors. Since then scientists have experimented with many different forms of perovskites producing compounds that conduct over 130 K $^{5, 22}$. Large number of experiments, assumptions, or modifications of existing theories were performed on superconductors to investigate their properties, for better understanding of this phenomenon. Superconductors are classified according to their properties. For instance, depending on the transition temperature $T_c$, ceramic-based materials are considered to be high temperature superconductors, while metallic and alloy materials are described as low temperature superconductors. The most common classification of superconductors depends on their characteristic behavior in the presence of a magnetic field. According to this behavior, they are divided into two types: type-I and type-II superconductors.

1.5 Type-I superconductors

Superconductors of type-I are made, mostly, of a single material. They expel completely magnetic fields from penetrating into their interior$^{23, 24}$. They possess perfect diamagnetism. When an external magnetic field is applied, the transition temperature from the superconducting to the normal phase is sharp. Although this type of superconductors excludes the applied magnetic field from the center of the sample by establishing circulating currents on its surface that counteract the applied magnetic field, there is a certain penetration depth $\lambda$ inside the material$^{13}$, as shown in Figure 1.1.
Figure 1.1: Magnetic flux is excluded from the interior of the superconductor. $\lambda$ is the penetration depth inside the superconductor.

The penetration depth is a measure of the decay of a magnetic field in the interior of a superconductor. It is a function of temperature. At higher temperatures, the penetration depth gets larger:

$$\lambda(T) = \lambda(0) \left[ 1 - \left( \frac{T}{T_c} \right)^{4/12} \right]$$  \hspace{1cm} (1.7)

When $T_c$ of a superconductor is measured while applying a magnetic field $H$, the value of $T_c$ decreases with increasing $H$ and when this field exceeds a certain critical magnetic field $H_c$, the material returns to the normal state with finite resistance, as shown in Figure 1.2.
The maximum critical magnetic field, $H_c$, in any type-I (low temperature) superconductor is about 2000 Gauss (0.2 Tesla), which is quite low. An approximate expression that relates $H_c$ with temperature $T$ is:

$$H_c(T) \approx H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^4 \right]. \tag{1.8}$$

In spite of type-I superconductors' importance, they cannot be used as superconducting high-field electromagnets. However, there is another type of superconductors which is ideal for this application, they are called type-II superconductors.
1.6 Type-II superconductors

Superconductors of type-II are made of alloys or compounds of different materials, however vanadium, technetium and niobium are exceptions. They are formed from the transition elements and the actinide series. These materials are characterized by two critical magnetic fields, called $H_{c1}$ and $H_{c2}$, see Figure 1.3.

Below $H_{c1}$, the material acts as type-I. Between these two fields the superconductor is in a mixed state, it is also called vortex state or Shubnikov state. When the external magnetic field is increased, the transition from superconducting to normal state occurs after going through a wide

Figure 1.3: Relationship between temperature and magnetic field in type-II superconductors.
mixed state region. Back to the Ginzburg-Landau parameter $K$, in this type$^{16,18}$ $K > \frac{1}{\sqrt{2}}$, and for type-I superconductor $K < \frac{1}{\sqrt{2}}$. The transition from type-I to type-II occurs when $K = \frac{1}{\sqrt{2}}$, also this parameter could be written in terms of critical fields:

$$K = \frac{1}{\sqrt{2}} \frac{H_{c2}}{H_c}.$$  \((1.9)\)

This thesis will focus on the mixed state, where the material has “zero resistance” and partial flux penetration. Type-II superconductors can persist to several hundred thousand Gauss. When these superconductors are subjected to a strong magnetic field, they permit the field to penetrate through the sample in quantized amounts of flux, called vortices.

1.7 Magnetic vortices

Vortex structure differs according to the type of the superconductor. If the superconductor is isotropic (low-temperature superconductor), the vortex is called Abrikosov vortex, named after the Russian physicist who first predicted the state. Abrikosov vortex is a quantized unit of magnetic flux, which penetrates type-II superconductors. It has a tube shape. Otherwise, if the superconductor is anisotropic (layered superconductor), the vortex structure becomes more complicated. It$^{26-33}$ consists of an array of two-dimensional pancake vortices in CuO$_2$ planes weakly coupled by coreless Josephson vortices in between two superconducting planes, as shown in Figure 1.4.
If the material is not too anisotropic, it can be described by the continuous anisotropic Ginzburg-Landau or London theory. However, for materials with large anisotropy, such as strongly layered Bi- and Tl- compounds, the discrete Lawrence-Doniach model is applicable. The Lawrence-Doniach model describes these superconductors as an array of parallel thin superconducting planes weakly coupled by quantum mechanical coupling, the Josephson coupling. In the mixed state, large currents can lead to the dynamics of the vortices, so they produce resistance in the material and hence losses occurs. To overcome this problem one can "pin" these vortices to prevent their motion and hence eliminate the resistance due to vortex motion. Hard superconductors are of type-II with pinning centers. If the vortex state is well understood, superconductors can be enhanced a great deal. Vortex may go under phase transition under certain circumstances. Vortex lattice can freeze or melt; consequently current will increase or decrease. Introducing high magnetic fields to the superconductor vortex lattice may change to vortex glass phase, and applying higher temperatures vortex can melt and a vortex liquid phase results. Because of the complexity of the problem, simulations become important for understanding vortex phase diagram in the layered high temperature superconductor. In fact, three main forces are acting on an individual vortex:
(1) the Lorentz force: acts between the current and the vortex. (2) the lattice force: acts on a moving vortex and describing their interaction within and near the core region with the crystal lattice\textsuperscript{32, 46}. (3) the pinning force $F_p$: vortex pinning \textsuperscript{47, 48}. The pinning force permits to form a static vortex density, which leads to a bulk transport current density $\bar{j}$ given by:

$$\bar{j} = \left( \frac{c}{4\pi} \right) \nabla \times \bar{B}.$$  

(1.10)

Current density is flowing free of dissipation, and this indeed is highly important in applications. When the pinning force acting on a vortex is equal to the Lorentz force, the current density will be called critical or depinning current density $j_c$. There are many effective ways\textsuperscript{49-95} to pin these vortices. Li et al.\textsuperscript{57} state that "pinning centers are expected to be the most effective when the distance between pinning centers is comparable to the coherence length". Whereas Motwidlo et al.\textsuperscript{58} studied the effect of alloy and pin material on superconducting properties of NbTi and remarked that both the choice of the superconducting alloy and pin material affected the pinning force and the critical magnetic field. For example introducing defects by adding impurities\textsuperscript{49} to create artificial pinning arrays, or substituting of elements as done in Reference\textsuperscript{59} for yttrium in YBCO cause lattice mismatch; hence stress-field pinning occurs. Varanasi et al.\textsuperscript{60} investigated effects of rare-earth ion (Nd,La) substitutions to pure $YBa_2Cu_3O_7$ melt processed in air and found that this addition leads to marked improvements. Also Angst et al.\textsuperscript{61} had grown single crystals of $HgBa_2Ca_{n-1}Cu_nO_{2n+2+\delta}$ partly with substitution of $Ba$ by $Sr$ and $Hg$ by $Pb$ and $Re$ and investigated improvement of flux pinning properties by this chemical substitution. Kadyrov et al.\textsuperscript{62} achieved high critical current densities by the same method of introducing artificially designed pin structure of Nb in NbTi superconductor and obtained the maximum values when the Nb layer
thickness was designed to be approximately twice the coherence length in $\text{NbTi}$. Tripodi et al.\textsuperscript{63} worked with Sintered samples that have been hydrogenated using chemical absorption and concluded that the process improves its superconducting properties. Cooley et al.\textsuperscript{64} used artificial pinning center which geometry is determined by billet stacking was made by gun-barrel drilling to result a periodic flux pinning. They showed that highly uniform, periodic array of flux pinning centers can be obtained on a scale below 30 nm in $\text{Nb-Ti}$. Rizzo et al.\textsuperscript{65} used ferromagnetic artificial pinning centers in $\text{NbTi}$ superconductor and remarked that these pins enhancements are more than that of nonmagnetic pins for the given volume percent.

Another effective way of introducing defects is particle beam irradiation, which produce effective pinning centers in high temperature superconductor. Tkaczyk et al.\textsuperscript{66} studied textured $\text{TlBa}_2\text{Cu}_3\text{O}_7$ thick films with heavy ion irradiation, and observed enhanced critical current at high fields. Also irreversibility line was shifted to higher temperatures. Drost et al.\textsuperscript{67} used an oblique irradiation at various angles in order to study pinning by columnar defects. Zhao et al.\textsuperscript{68} used high-resolution transmission electron microscopy to study effects of different radiation damages. Ogikubo et al.\textsuperscript{69} high-energy heavy-ion irradiation followed by thermal annealing and showed experimental results on superconducting properties of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$. Thompson et al.\textsuperscript{70} investigated improved critical current density and extended irreversibility in single crystal $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8}$ as a result of linear defects caused by heavy ion irradiation. Metlushko et al.\textsuperscript{71} showed that by creating artificial pinning centers in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ single crystals and thin films either by Kr-irradiation or during the process of thin film growth, high critical current densities result. Also Siegal et al.\textsuperscript{72} enhanced critical current density significantly by $\text{H}^+$, $\text{Xe}^+$ or irradiation in epitaxial thin films of $\text{Ba}_2\text{YCu}_3\text{O}_{7.8}$ with little effect on critical temperature. Cutro et al.\textsuperscript{73} pinned $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ single crystals with heavy ion irradiation by $\text{H}^+$, $\text{He}^+$, $\text{Ne}^+$, $\text{Ar}^+$ that caused the critical current to persist at higher temperature and fields. Hu et al.\textsuperscript{74} investigated significant improvements of critical current measurement in $(\text{Bi})$. 
**PhH2Sr2Cu2O10/2Ag** tapes at 77 K following fast neutron irradiation that indicated to be an effective method to improve vortex pinning. Weaver et al.\textsuperscript{75} found that proton irradiation increases the vortex pinning energy in \( Tl2Ca2BaCu2O8 \) films. Field cooling\textsuperscript{78} the superconductor from above critical temperature, in the presence of a magnetic field results in a strongly pinned vortices. Miller et al.\textsuperscript{79} compared hot isostatically pressed \( Bi2Sr2CaCuOx \) films. One was pressed for 15 minutes and the other one for 120 minutes. The first sample had enhanced flux pinning because it contained a high density of dislocations and planar faults.

Although most of pinning centers include defect introduction, there is another type of pinning that is called “intrinsic pinning” and considered to be effective\textsuperscript{33}. Chakravarty et al.\textsuperscript{82} studied the effects of the intrinsic pinning in \( Bi2Sr2CaCu2O8 \) which exists in a direction perpendicular to the layers related to its strong structural anisotropy. Oussena et al.\textsuperscript{83} presented an experimental evidence of intrinsic pinning in hysteresis loops of untwined \( YBa2Cu3O7-x \) single crystals. Fuke et al.\textsuperscript{84} studied several kinds of \( YBCO \) thin films with different microstructures and showed that there is a relationship between the microstructure and pinning force, also boundaries between a-axis-oriented and c-axis-oriented grains were considered to be effective pinning centers. Consequently pinning of these vortices has many images and can be reached by several methods such as ordered oxygen-deficient phases\textsuperscript{86}, point and columnar defects\textsuperscript{87, 88}, pinning-induced stress\textsuperscript{89}, surface pinning\textsuperscript{90, 91}, random and periodic pinning\textsuperscript{92, 93}, twin boundary\textsuperscript{80, 81, 94}, lattice defects\textsuperscript{95}, anisotropic arrays of pinning centers as submicron holes “antidotes”, or magnetic dots produced by electron-beam lithography\textsuperscript{96} are among promising techniques. However, vortex migration in hard superconductor can always take place at finite temperature even when Lorentz force is smaller than pinning force, this is called thermally activated flux creep\textsuperscript{16, 97-102} and it plays a great role in the resistive measurements of these materials\textsuperscript{82, 103-105}. By reducing the temperature\textsuperscript{106, 107}, both the maximum pinning force and the maximum current density in a superconductor increase. So vortex dynamics may occur because of thermal
activation, high transport current, or magnetic field gradient, but by introducing effective pinning structures, the critical current will be enhanced, and the performance of superconducting thin films\textsuperscript{108}, such as SQUID\textsuperscript{36,109-112} sensors, can be improved drastically.

## 1.8 Pancake vortices in a finite stack of superconducting layers

We model the high temperature superconductor as an array of parallel thin films, which are Josephson decoupled. Our model applies to highly anisotropic material. Pe et al.\textsuperscript{113} applied transport currents to outermost layers in opposite direction and obtained an analytical expression for the coupling between vortices. They found that for fixed number of layers the decoupling current density increased with increasing number of layers. In this thesis, random pinning was presented to interior layers, such that its uniform in each layer but its strength is random from layer to layer, with the same other parameters in reference\textsuperscript{113} and the structure was greatly affected. However decoupling current wasn't enhanced. In addition, we showed that coupling between vortices is weak in highly anisotropic materials.

In reference\textsuperscript{114}, a dc current was applied to the top layer of a finite stack of superconducting layer. It involved zero and non-zero uniform pinning. To compare, we applied random pinning to the superconducting layers and found that pinning affects drastically the motion of vortices. Moreover, we suggest experimental ways such as measuring the difference in voltage and flux flow rate to determine the strength of coupling between vortices and detect the presence of pinning.
As a result of ac transport current, ac losses occur. Kerchner et al. \(^{115}\) noticed that as the current is raised above critical current, ac losses increase sharply. In this study we applied random pinning to superconducting layers with transport ac current to the top layer and calculated the ac losses for a finite stack of superconducting layers as function of frequency, which has been performed for the first time. The calculated ac losses are greatly reduced by the presence of pinning.

### 1.9 Thesis organization:

This thesis contains an introduction, three chapters, and a conclusion. In the second chapter the model is described in details. Direct current is applied to the outermost layers in opposite directions, and then the effects are studied on different number of superconducting layers. First, we study the case of pinning free system and then the case of introducing a random pinning force that differs between layers and is constant along each layer. In the third chapter, dc current is applied to one of the outermost layers, and then its effect is studied, also in the two cases with and without a random pinning force. Velocities of outermost layers, the difference between top layer and the layer just below it, magnetic flux difference, and time-averaged voltage are calculated. In the forth chapter alternating current is applied to one of the outermost layers, and its effect is studied in two cases: with and without random pinning. Also in this work velocities of top and bottom layers, the difference in position between top and the layer below, magnetic flux difference between top and bottom layers, and power dissipation are calculated. A conclusion that summarizes the results of the three chapters is presented at the end of the thesis. In addition, the algorithm of the program that calculates ac losses is shown in the appendix.
Chapter II: Equal transport dc currents applied to the outermost layers of a finite stack of superconducting layers in opposite directions with and without pinning
2.1 Introduction

The discovery of high temperature superconductors was accompanied by a significant number of experimental and theoretical methods, which were established to get a better understanding of these materials. In our model, the high temperature layered superconductor is taken as an array of parallel thin films. The vortices are stacks of two-dimensional pancake vortices joined by Josephson strings. There are three main assumptions in this model. First: finite number of superconducting layers $N$, which is the case for real samples. Second assumption is that these vortices have perfect hexagonal lattice. Third assumption is that the vortex structure in each layer doesn’t change under the influence of an electric field. They remain in the hexagonal lattice form through the experiment, for computational purpose. Also thermal fluctuations were not taken into account, so that the lattice won’t melt. However, pinning was investigated in the second part of the experiment. The effect of Josephson coupling between superconducting layers was not considered; so this model is applied for highly anisotropic material. Hence magnetic coupling is the only source of pancake vortices interaction.\textsuperscript{47, 97, 113}

Currents are applied to the outermost layers in opposite directions. When a current is applied to a certain superconducting layer and pancake vortex motion occurs in that layer, pancake vortices in other layers move also, although they are not subjected to a transport current.\textsuperscript{98, 116, 117}

Translation of the lattices at the outermost layers occurs because of the Lorentz force that resulted from these surface transporting currents. When the lattices at the top and bottom layers are displaced by an equal distance, and opposite directions and assuming that these lattices are fixed in their new positions, the lattices in the interior layers are displaced due to the interaction between pancake vortices in different layers. The main purpose is to simulate the effect of a current applied to outermost layers on the structure of the interior lattices, and how they will be displaced with respect to each other. This gives an insight about the nature and strength of the coupling between pancake vortices in different layers. Also to show the decoupling surface
current density, current at which the pancake vortices of outermost layers separate from the vortices of the interior layers for different number of superconducting layers, in three different cases. First, we study the case of a sample with no pinning centers. In the second case, the applied current circulates not only along the outermost layers, but could also run through a number of interior layers. In the third case, we study a sample with pinning centers uniformly distributed in each interior layer but the strength of these pinning centers is random from one layer to another. In each of these numerical experiments, we calculate the decoupling current as function of the applied magnetic field as well as the number of superconducting layers in the case of presence and absence of pinning centers.

2.2 Description of the model:

In order to study the pancake vortices in the case of finite number of superconducting layers, first let us take the simple case of a single pancake vortex as shown in Figure 2.1.

Let \( s \) be the interlayer spacing, \( t \) is the number of layers above the pancake and \( m \) is the number of layers below the pancake. Also we assume that the thickness \( d \) of the \( N \) superconducting layers is much less than penetration depth \( \lambda \).
2.2.1 Magnetic vector potential

Assuming that the pancake layer coincides with the plane \( z = 0 \). The layers then can be referenced by \( z = ns \) where \(-m \leq n \leq t\) (see Figure 2.1). In cylindrical coordinates, the vector potential \( a_\phi(\rho, z) \) is given by\(^{30,113} \)

\[
a_\phi(\rho, z) = \int dq A(q) J_1(q \rho) Z(q, z). \tag{2.1}
\]

Where \( J_1(q \rho) \) is the first order Bessel function of the first kind and \( Z(q, z) \) has the following form:

\[
Z(q, z) = \begin{cases} 
\alpha_{n,n+1} \exp(-qz) + \beta_{n,n+1} \exp(qz) & n < z/s < n+1, \quad -m \leq n \leq t-1 \\
\exp(-Q_n |n|s) & z/s = n, \quad -m \leq n \leq t \\
\gamma \exp(-qz) & z/s > t \\
\gamma' \exp(qz) & z/s < -m
\end{cases} \tag{2.2}
\]

By substituting \( z/s = t \) and \( z/s = -m \) in \( Z(q, z) \) given by Eq. (2.2) we obtain:

\[
\gamma = \exp(-Q_n ts)\exp(qts) \quad \text{and} \quad \gamma' = \exp(-Q_n ms)\exp(qms). \tag{2.3}
\]

respectively. Hence:

\[
Z(q, z > ts) = \exp(-Q_n ts)\exp(-q(z-ts)). \tag{2.4}
\]

and

\[
Z(q, z < -ms) = \exp(-Q_n ms)\exp(q(z + ms)). \tag{2.5}
\]

By substituting of \( z/s = n \) and \( z/s = n+1 \) in \( Z(q, z) \) given by Eq. (2.2) we obtain for

\(-m \leq n \leq t-1\), the following form:

\[
\begin{pmatrix}
\exp(-qns) & \exp(qns) \\
\exp(-q(n+1)s) & \exp(q(n+1)s)
\end{pmatrix}
\begin{pmatrix}
\alpha_{n,n+1} \\
\beta_{n,n+1}
\end{pmatrix}
= \begin{pmatrix}
\exp(-Q_n |n|s) \\
\exp(-Q_n |n+1|s)
\end{pmatrix} \tag{2.6}
\]
Solving for $\alpha_{n,n+1}$ & $\beta_{n,n+1}$, we get

$$\alpha_{n,n+1} = \frac{1}{2 \sinh (qs)} \left[ \exp(-Q_n|\eta|s) \exp(q(n+1)s) - \exp(-Q_{n+1}|\eta|s) \exp(q ns) \right] - m \leq n \leq t - 1$$

(2.6)

$$\beta_{n,n+1} = \frac{1}{2 \sinh (qs)} \left[ \exp(-Q_{n+1}|\eta|s) \exp(-q ns) - \exp(-Q_n|\eta|s) \exp(-q(n+1)s) \right] - m \leq n \leq t - 1$$

(2.7)

Substituting back into Eq.(2.2), $Z(q,z)$ can be rewritten as follows:

$$Z(q,z) = \begin{cases} \frac{1}{\sinh (qs)} \left[ \exp(-Q_n|\eta|s) \sinh(q(n+1)s - z) \right] + \exp(-Q_{n+1}|\eta|s) \sinh(q(z - ns)) & \text{for } n < z/s < n + 1, \quad -m \leq n \leq t - 1 \\ \exp(-Q_n|\eta|s) & \text{for } z/s = n, \quad -m \leq n \leq t \\ \exp(-Q_{n+1}|\eta|s) \exp(-q(z - ns)) & \text{for } z/s > t \\ \exp(-Q_{n+1}|\eta|s) \exp(q(z + ms)) & \text{for } z/s < -m \end{cases}$$

(2.8)

2.2.1.a The surface current density

The surface current density $K_\phi$, can be related to the vector potential $a_\phi$ as follows:

$$K_\phi(\rho,n) = \frac{-c}{2\pi \Lambda} \left[ a_\phi(\rho,z = ns) - \frac{\phi_s}{2\pi \rho} \delta_{n,0} \right], \text{ for } -m < n < t$$

(2.9)

Where $\Lambda$ is the two-dimensional thin film screening length denoted by $2\lambda^2/d^{77}$ and $\phi_s$ is the flux quantum defined as $hc/2e$, where $h$ is Planck’s constant, $c$ is the speed of light in vacuum, and $e$ is the electron charge. $\delta_{n,0}$ is the Kronecker delta. $K_\phi$ can also be expressed as the discontinuity in the radial component of the magnetic field across a superconducting layer.$^{30}$

Hence

$$K_\phi(\rho,n) = \frac{-c}{4\pi} \left[ \partial_z a_\phi(\rho,z) \bigg|_{z=ns} - \partial_z a_\phi(\rho,z) \bigg|_{z=ns} \right] \text{ for } -m < n < t$$

(2.10)
Equating Eq.(2.9) and Eq.(2.10) we obtain the equation for vector potential \( a_\phi (\rho, z) \) evaluated at 
\[ z = n_S \] 
given by:
\[
a_\phi (\rho, z = ns) = -\frac{2\pi \Lambda}{c} \left[ -\frac{c}{4\pi} \left( \partial_z a_\phi (\rho, z) \right)_{z=ns} - \partial_z a_\phi (\rho, z) \right]_{z=ns} + \frac{\phi_2}{2\pi \rho} \delta_{n,0}, \quad \text{for} \quad -m < n < t
\]
identically:
\[
a_\phi (\rho, z = ns) = \frac{\Lambda}{2} \left[ \partial_z a_\phi (\rho, z) \right]_{z=ns} + \frac{\phi_2}{2\pi \rho} \delta_{n,0}, \quad \text{for} \quad -m < n < t. \quad (2.11)
\]

2.2.1.b The layer containing the pancake vortex

Taking \( n = 0 \), it follows from Eqs.(2.1) and (2.11) that \( a_\phi (\rho, z = 0) \) is given by the following expression:
\[
a_\phi (\rho, z = 0) = \int_0^\infty dq A(q) J_1(q\rho). \quad (2.12)
\]
Taking the partial derivative of \( a_\phi (\rho, z) \) with respect to \( z \) at \( z = 0^\pm \), we will have:
\[
a_\phi (\rho, z = ns) \bigg|_{z=0^\pm} = \int_0^\infty dq A(q) J_1(q\rho) Z(q, z = ns) \bigg|_{z=0^\pm}. \quad (2.13)
\]
Substituting \( a_\phi (\rho, z = ns) \bigg|_{z=0^\pm} \) from Eq.(2.13) into Eq.(2.11) and combining it with Eq.(2.12) we obtain:
\[
\int_0^\infty dq A(q) J_1(q\rho) \left[ 1 - \frac{\Lambda}{2} \left( \partial_z Z(q, z) \right)_{z=0^\mp} - \partial_z Z(q, z) \right]_{z=0^\pm} = \frac{\phi_2}{2\pi \rho}. \quad (2.14)
\]
Using the following properties of Bessel functions:
\[
\int_0^\infty dq \rho J_1(q\rho) J_1(q'\rho) = \frac{1}{q} \delta(q - q'), \quad \text{and} \quad \int_0^\infty dq \rho J_1(q\rho) \frac{1}{\rho} = \frac{1}{q}. \quad (2.15)
\]
Eq.(2.14) can be rewritten as follows:

$$A(q) = \frac{\Phi}{2\pi} \left[ 1 - \frac{\Lambda}{2} \left( \partial_{z} Z(q, z) \bigg|_{z=0} - \partial_{z} Z(q, z) \bigg|_{z=0} \right) \right].$$

(2.16)

2.2.1.c Layers other than the pancake vortex layer

For $n \neq 0$: the vector potential has the following form:

$$a_{\varphi}(\rho, z = ns) = \frac{\Lambda}{2} \left[ \partial_{z} a_{\varphi}(\rho, z) \bigg|_{z=ns^+} - \partial_{z} a_{\varphi}(\rho, z) \bigg|_{z=ns^-} \right] \text{ for } -m \leq n \leq t, n \neq 0.$$  

(2.17)

On the other hand:

$$a_{\varphi}(\rho, z = ns) = \int_{0}^{\infty} dq' a_{\varphi}(\rho, z = ns') Z(q', z = ns) \text{ for } -m \leq n \leq t, n \neq 0.$$  

(2.18)

Taking the partial derivative of $a_{\varphi}(\rho, z)$ with respect to $z$ at $z = ns^+$ and $z = ns^-$, it follows from Eq.(2.18) that:

$$\int_{0}^{\infty} dq' A(q') J_{1}(q' \rho) \left[ Z(q', z = ns) - \frac{\Lambda}{2} \left( \partial_{z} Z(q', z) \bigg|_{z=ns^+} - \partial_{z} Z(q', z) \bigg|_{z=ns^-} \right) \right] = 0,$$

for $-m \leq n \leq t, n \neq 0$.  

(2.19)

Again using Bessel function property given by Eq.(2.15), one can verify that:

$$Z(q', z = ns) = \frac{\Lambda}{2} \left[ \partial_{z} Z(q', z) \bigg|_{z=ns^+} - \partial_{z} Z(q', z) \bigg|_{z=ns^-} \right], \text{ for } -m \leq n \leq t, n \neq 0.$$  

(2.20)

2.2.1.d Layers above the pancake vortex layer

Here we will consider $Z(q, z)$ given in Eq. (2.20) for: $n = t, t-1, t-2, \ldots, 1$.

For $z/s = t$: above the layer where the pancake vortex is located (see Figure 2.1).

Substituting $Z(q, z)$ given by Eq.(2.8) into Eq.(2.20), one can verify after simplification that:
\[ \exp(-Q_1s) \left[ \left( 1 + \frac{2}{\Lambda q} \right) \sinh(qs) + \cosh(qs) \right] = \exp(-Q_{t-1}(t-1)s). \] (2.21)

Let \( f(q) = \left( 1 + \frac{2}{\Lambda q} \right) \sinh(qs) + \cosh(qs). \)

Then Eq.(2.21) can be written as

\[ \exp(-Q_1s)f(q) = \exp(-Q_{t-1}(t-1)s). \] (2.22)

After substituting \( z/s = n > 0, \) such that \( 1 \leq n \leq t - 1 \) in \( Z(q, z) \) and combining Eqs.(2.8) and (2.20) we obtain the following form:

\[ \exp(-Q_{ns})g(q) - \exp(-Q_{n+1}(n+1)s) = \exp(-Q_{n-1}(n-1)s). \] (2.23)

where \( g(q) = 2 \left[ \frac{1}{\Lambda q} \sinh(qs) + \cosh(qs) \right]. \) (2.24)

Using Eqs.(2.22) and (2.23), the function in term \( \exp(-Q_{ns}) \) can be iteratively solved for \( n = t, t-1, t-2, \ldots, 1 \)

First, let us define the function \( h \) in terms of \( f(q) \) and \( g(q) \) as follows:

\[ h(q, n > 0) = \begin{cases} 
\frac{1}{f(q)} & n = 1 \\
\frac{1}{g(q) - h(q, n-1)} & n > 1 
\end{cases} \] (2.25)

Using Eqs.(2.23) and (2.25), \( \exp(-Q_{ns}) \) can be expressed at \( n = t, t-1, t-2, \ldots, 1 \) as follows

\[ \exp(-Q_1s) = h(q, t) \]

\[ \exp(-Q_2s) = h(q, t)h(q, t-1) \]

\[ \vdots \]

\[ \exp(-Q_{t-1}(t-1)s) = h(q, t)h(q, t-1) \ldots h(q, 3)h(q, 2) \]

\[ \exp(-Q_ts) = h(q, t)h(q, t-1) \ldots h(q, 3)h(q, 2)h(q, 1) \]
2.2.1. Layers below the pancake vortex layer

Applying Eq. (2.20) for layers $n = -m, -m + 1, -m + 2, \ldots, -1$, where $m > 0$, we get for $z/s = -m$ the following expression:

$$\exp(-Q_{-m} ms)f(q) = \exp(-Q_{-m+1}(m-1)s).$$  \hspace{1cm} (2.26)

For $z/s = n < 0$ such that $-m + 1 \leq n \leq -1$, after substitution of Eq. (2.8) into Eq. (2.20) we can write:

$$\exp(Q_{n} ns)g(q) = \exp(Q_{n-1}(n-1)s) = \exp(Q_{n+1}(n + 1)s).$$  \hspace{1cm} (2.27)

From the above equation, $\exp(-Q_{s} ns)$ can be solved, for $n = -m, -m + 1, -m + 2, \ldots, -1$, in the same manner:

$$\exp(-Q_{-1}s) = h(q,m)$$
$$\exp(-Q_{-2}s) = h(q,m)h(q,m-1)$$
$$\vdots$$
$$\exp(-Q_{-m+1}(m-1)s) = h(q,m)h(q,m-1) \ldots h(q,3)h(q,2)$$
$$\exp(-Q_{-m} ms) = h(q,m)h(q,m-1) \ldots h(q,3)h(q,2)h(q,1)$$

The form of $\exp(-Q_{s} n|s)$ can be rewritten in the following compact form:

$$\exp(-Q_{s} n|s) = \begin{cases} 
\prod_{p=0}^{n-1} h(q, m - p), & -m \leq n \leq -1 \\
1, & n = 0 \\
\prod_{p=0}^{n-1} h(q, t - p), & 1 \leq n \leq t.
\end{cases}$$  \hspace{1cm} (2.28)

Now, we can derive explicitly $A(q)$, from Eqs. (2.8), (2.16) and (2.28) as follows:

**Case one: Pancake layer at the top of the stack**

For $t = 0$, ($m = N - 1$), the term $A(q)$ will have the following expression:

$$A(q) = \frac{\phi_s \sinh \left( qs \right)}{\pi q} \left[ g(q) - h(q, N - 1) - \exp(-qs) \right].$$  \hspace{1cm} (2.29)
Case two: Pancake layer at the bottom of the stack

For \( m = 0, (t = N - 1) \)

We observe that \( A(q) \) given in Eq. (2.29) has the same form for \( t = 0, (m = N - 1) \) and \( m = 0, (t = N - 1) \).

Case three: Pancake layer lies between other layers

For \( t \neq 0 \) and \( m \neq 0 \):

Define:

\[
h(q, n = 0) = \exp(-qs).
\]

Then the expression of \( A(q) \) in Eq.(2.16) can be written as follows:

\[
A(q) = \frac{\phi_s}{2\pi} \left[ 1 - \frac{\Lambda}{2} \left( \frac{q}{\sinh(qs)} \left( \cosh(qs) + \exp(-Q_s) \right) - \frac{q}{\sinh(qs)} \left( \cosh(qs) - \exp(-Q_s) \right) \right) \right]^{-1}.
\]

Using Eqs.(2.24) and (2.28), we can obtain from Eq.(2.30) an expression of \( A(q) \) in terms of \( g(q) \) and \( h(q, n) \) as follows:

\[
A(q) = \frac{\phi_s \sinh(qs)}{\pi \Lambda q} (g(q) - h(q, t) - h(q, m))^{-1}.
\]

2.2.2 Coordinate transformation

We move the origin of the coordinate system to the bottom of the stack of layers to make the calculation simpler and take into account the influence of every vortex on the other vortices as shown in Figure 2.2.
The new form of $Z(q, t)$ and $\exp(-Q_n|n|s)$ becomes

$$Z_{old}(q, z) = Z(q, z, m) = \begin{cases} 
\frac{1}{\sinh(qs)} \left\{ \exp(-Q_{n,m}|n-m|s) \sinh\left(q(n+1)s - z\right) + 
\exp(-Q_{n,m}|n+1-m|s) \sinh\left(q(z-n)s\right) \right\}, & \text{for } n < z/s < n+1.0 \leq n \leq N-1 \\
\exp(-Q_{n,m}|n-m|s), & \text{for } z/s = n, 0 \leq n \leq N-1 \\
\exp(-Q_{n,m}(N-1)s) \exp(-q(z(N-1)s)), & \text{for } z/s > N-1 \\
\exp(-Q_{n,m}ms) \exp(q(z+ms)), & \text{for } z/s < 0 
\end{cases}$$  

and

$$\exp(-Q_n|n-m|s) = \begin{cases} 
\prod_{p=0}^{n-1} h(q, m-p), & 0 \leq n \leq m-1 \\
1, & n = m \\
\prod_{n=m}^{n-1} h(q, N-1-m-p), & m+1 \leq n \leq N-1 
\end{cases}$$  

(2.33)

where $n$ is the layer of the pancake vortex and $m$ is the layer at which $Z(q, z, m)$ is expressed.

Furthermore $A(q)$ can be written as

$$A(q, m) = \frac{\phi_s \sinh(qs)}{\pi} \left[ g(q) - h(q, N-1-m) - h(q, m) \right]^{-1}.$$  

(2.34)
2.2.3 Coupling force between two-dimensional pancake vortex lattices:

Assuming that at equilibrium all pancake vortices form vertical stacks along the z-direction. The force of pancake \( i \) due to pancake \( j \) is:

\[
\vec{F}(\rho_j, j, i) = \vec{K}(\rho_j, j, i) \times (z \frac{\phi_j}{c}),
\]

where \( \vec{K}(\rho_j, j, i) \) is the sheet current density produced by the pancake in layer \( j \) at the position of the pancake in layer \( i \). Notice that the net force on each pancake vortex is zero when all stacks of vortices are in perfect registry. From previous results, and by assuming that the magnetic field \( H \) is in the \( +z \) direction, Eq.(2.35) can be written as:

\[
\vec{F}(\rho_j, j, i) = \hat{\rho}(\varphi_j) \frac{\phi_j^2}{2\pi^2N^2} \int dq C(q, j, i),
\]

where

\[
C(q, j, i) = \frac{\sinh (qs) Z(q, i, j)}{g(q) - h(q, N - 1 - j) - h(q, j)}.
\]

and \( \hat{\rho}(\varphi_j) = \hat{i} \cos \varphi_j + \hat{j} \sin \varphi_j \) is the unit vector along \( \rho \).

Suppose that the lattice in layer \( j \) is translated by some displacement vector \( \vec{\delta}_j \), where \( |\vec{\delta}_j| < a \), the lattice constant. The net force on a pancake in layer \( i \) is:

\[
\vec{F}_c(\vec{\delta}_j, j, i) = \sum_l \vec{F}(l + \vec{\delta}_j, j, i).
\]

The summation is over all the lattice vectors \( \vec{l} \). \( \vec{F}_c \) can be represented by a summation over the reciprocal lattice vectors \( \vec{g} \) as:

\[
\vec{F}_c(\vec{\delta}_j, j, i) = \frac{1}{A} \sum_{\vec{g} \neq 0} \vec{G}(\vec{g}, j, i) \exp(i\vec{g} \cdot \vec{\delta}_j).
\]

The reciprocal lattice vector \( \vec{g} = n_1 \vec{b}_1 + n_2 \vec{b}_2 \), and \( n_1, n_2 \in (\ldots, -2, -1, 0, 1, 2, \ldots) \) for a hexagonal lattice (see Figure 2.3) can be calculated from direct lattice vectors \( \vec{a}_1, \vec{a}_2, \vec{a}_3 \) as follows:\(^{118}\):
\( \bar{a}_1 = a \hat{x}, \bar{a}_2 = \frac{a}{2} (\hat{x} + \sqrt{3} \hat{y}), \bar{a}_3 = \hat{z}, \) then we arrive at
\begin{align*}
\bar{b}_1 &= \frac{2\pi}{a} \left( \frac{x}{\sqrt{3} a} \right), \\
\bar{b}_2 &= \frac{4\pi}{\sqrt{3} a} \hat{y}.
\end{align*}

Figure 2.3: Hexagonal vortex lattice.

Using mathematical identities and properties of Bessel functions in Eq. (2.39) one can verify that:

\[ \tilde{G}(\bar{g}, j, i) = -\bar{g} \frac{i \phi^2}{\pi a^2} \frac{C(g, j, i)}{g^3}. \]  

(2.40)

Notice that that \( \tilde{G} \) satisfies the following identity:

\[ \tilde{G}(-\bar{g}, j, i) = -\tilde{G}(\bar{g}, j, i) \]

and hence Eq.(2.39) can be written as follows:

\[ \tilde{F}_c(\bar{g}, j, i) = \frac{\phi^2}{\pi a^2} \sum_{\bar{g} \neq 0} \frac{\bar{g}}{g^3} C(g, j, i) \sin(\bar{g} \cdot \bar{g}_j), \]

(2.41)

where \( g \) is the magnitude of \( \bar{g} \), and \( A = \frac{\sqrt{3}}{2} a^2 \) is the area of the unit cell for the lattice.
2.3 Equal transport currents applied to outer layers in opposite directions

Figure 2.4 represents the structure of one stack of $N$ pancakes aligned along the $z$-axis, where the bottom layer coincides with the plane $z = 0$. When a current is applied in opposite directions to the top and bottom layers, the bottom layer is displaced by $\delta_j$ whereas the top layer is displaced by $-\delta_j$. These layers are assumed to be fixed in their new positions, while the interior lattices adjust themselves to reach a new force-free configuration.

Suppose, for simplicity, that all pancake displacements occur along the $x$ direction (taken along the lattice vector $\vec{a}_1$), and that $x_i$ is the coordinate of the pancake in layer $i$; then the force balance equation for interior pancakes for $1 \leq i \leq N - 2$ is given by:
\[ \sum_{j \neq i} F_{cr}(x_j - x_i, j, i) - \eta \ddot{x} = m \dddot{x}, \]

where \( F_{cr} \) is the \( x \)-component of \( \vec{F}_r \), \( \eta \) is the viscous drag coefficient, and \( \dot{x}, \ddot{x} \) are the first and second derivatives of \( x \), with respect to time, respectively. \( m \) is the two-dimensional pancake vortex mass. Since \( m \) is negligible, last equation can be written as

\[ \sum_{j \neq i} F_{cr}(x_j - x_i, j, i) = \eta \ddot{x}. \quad (2.42) \]

From the symmetry of the displacement of the outermost layers, Eq. (2.42) will be solved only for half of the superconducting layers, i.e. \( \frac{N-2}{2} \) equations: an equation for each layer, and the displacement for the other half has the opposite sign. In case the current is applied to more than two outermost layers, less inner superconducting layers are taken into account. However, in the case of the existence of pinning forces that are uniform in each layer, but randomly distributed from layer to layer, the structure won’t remain symmetric any more, and all superconducting layers must be taken into account. The magnitude of the random pinning force is chosen within certain range, such that the net force can be calculated as follows:

\[ |\vec{F}| = |\vec{F}_r + \vec{F}_p| = \begin{cases} 0 & |\vec{F}_p| > |\vec{F}_r| \\ F_r + F_p & |\vec{F}_r| > |\vec{F}_p| \end{cases} \quad (2.43) \]

### 2.4 Results and discussion:

We take the interlayer distance \( s \approx 15 \mu m \) and \( \Lambda \approx 5.6 \times 10^5 s^{-1} \). The surface current densities and the decoupling surface current density are expressed in units of \( \frac{c \phi_0}{\pi \Lambda^2 A^2} \).
2.4.1 Case one: Current applied to top and bottom layers with no pinning (Figures 2.5-2.9)

Figure 2.5 shows the structure of one stack of pancakes in 31 superconducting layers. Different magnitudes of currents are applied to the top and bottom layers in opposite directions, and cause different displacements, $\delta_j = 0.1, 0.2, 0.3, 0.4$ of the lattice spacing, $a$. The lattice spacing is chosen to be $a = 103.02 \xi$, which is equivalent to an average magnetic induction of $0.1\,T$ (recall that: $B = \phi_s / A$). Notice that even when the top and bottom pancakes are very much displaced with respect to the interior pancakes, the latter remains almost in their initial position and manage to provide for a balancing force to the Lorentz force to the top and bottom layers. This shows how weak is the electromagnetic force between layers as opposed to the force due to the applied current. The portion of the interior layers is sketched in the inset of Figure 2.5, excluding top and bottom layers, to show clearly how the structure differs by applying different currents to the outermost layers. Comparing the displacement of different currents to the outermost layers with displacement of the interior, one clearly sees that interior lattices have relatively small displacements.

Figure 2.6 represents the variations of the surface current density versus the distance by which the top and bottom pancakes are displaced by. Different numbers of layers are introduced N= 3, 5, 11, 15, 21, 31, and 51 superconducting layers. The magnetic field perpendicular to the layers is taken to be $0.1\,T$. This figure shows that there is a maximum surface current density, which is called the decoupling surface current density, $K_d$. As long as $K_d$ is not reached, the interior pancakes (other than the top and bottom layers) are capable of generating forces on the bottom and top layers canceling the forces resulting from the applied currents and form a stable stack of pancake vortices. On the other hand, for currents greater than $K_d$, the forces due to interior pancakes cannot counteract the Lorentz force on the top and bottom layers due to the applied
Figure 2.5: Position of stacks of pancake vortices in 31 superconducting layers corresponding to different applied currents.
Figure 2.6: Variation of the surface current density as a function of the outermost pancake vortices displacements.
current. Therefore, the top and bottom pancake vortex lattices slip away from the corresponding stack of pancake vortices in the interior layers.

Figure 2.7 shows the decoupling surface current density $K_d$ versus the number of layers. The magnetic field perpendicular to the layers is $0.1 T$. The first indication of this figure is that as the number of layers increases, $K_d$ also increases. However, we expect that the decoupling current density is going to saturate when the number of layers is comparable to $\left(\frac{\Lambda}{s}\right)$, because the magnetic field produced by a pancake vortex has a length scale of $\Lambda$.

Figure 2.8 introduces the decoupling surface current density versus different lattice constants for 5, 11, and 31 superconducting layers. When the lattice constant is small, i.e. vortices are near each other, $K_d$ is high. By increasing the lattice constant, vortices have less effect on each other and $K_d$ becomes smaller. Similarly, Figure 2.9 shows $K_d$ for different values of applied magnetic fields for 5, 11, and 31 superconducting layers. At smaller fields, $K_d$ is smaller, because the interaction between vortices is weak. For higher applied fields, the interaction between vortices becomes stronger and $K_d$ has a larger magnitude. Also at higher fields saturation becomes quicker, as clearly demonstrated for 11 and 31 superconducting layers.

2.4.2 Case two: Current applied to more than one layer at the top and bottom (Figures 2.10-2.12)

Figure 2.10 is the same as Figure 2.6, but this time the current is applied to more than one layer at the top and bottom. In this graph the stack is made out of 31 superconducting layers and the current is applied at two layers both at the top and the bottom. After that the case of the current is
Figure 2.7: Dependence of the decoupling surface current density on the number of superconducting layers.
Figure 2.8: Decoupling surface current density as a function of lattice constant.
Figure 2.9: Decoupling current density corresponding to different applied magnetic fields.
Figure 2.10: 31 superconducting layers with an applied current to different number of outermost layers.
applied to three top and bottom layers is shown. The lattice constant is $a = 10.302s$ and the applied current causes a displacement of $\delta_j = 0.1a$. It is clear from this figure that when the applied current introduced just to two outermost layers, $K_d$ is higher than the case when the current is applied to four outermost layers. When the current injected to six outermost layers again a smaller $K_d$ results. Hence, applying current to more layers play a dominant role in decreasing $K_d$. From Figure 2.10 one can determine that a stack with current applied to six outermost layers can be broken easier than when the current is applied to a lesser number of outermost layers, because more pancake vortices are under the influence of the Lorentz force. Then it is easier to decouple these pancake vortices from the interior ones.

Figure 2.11 illustrates the structure of 31 superconducting layers with various applied current densities to six of the outermost layers. For the same state as in Figure 2.10. Also as in Figure 2.5 the interior pancakes remain almost in their initial positions despite of the high applied current. The inset portion in Figure 2.11 corresponds to a displacement $\delta_j = 0.1a$ is enlarged and shows how relatively small is the displacement of the interior pancakes as a result of weak electromagnetic coupling between layers. Figure 2.12 shows the structure of 31 superconducting layers where current is applied to two, four, and six of the outermost layers. The lattice constant is $a = 10.302s$. The applied current causes displacement $\delta_j = 0.1a$. From the inserted enlarged portion in Figure 2.12, one can see that there is a slight displacement in most of the interior pancakes, but with a small magnitude compared to $\delta_j$. The nearer lattices to outer layers are affected more, so that their motion is more noticeable. For a larger number of layers introduced to applied current, the motion of interior lattices is more pronounced. One can also investigate the effect of an applied current to the outer layers by varying other factors, such as the lattice constant, the applied magnetic field, etc.
Figure 2.11: Position of stacks of pancake vortices in 31 superconducting layers with different currents applied to outermost six layers, $\alpha = 10.302s$. 
Figure 2.12: Position of stacks of pancake vortices in 31 superconducting layers with current applied to different number of outermost layers, \( a = 10.302s \).
2.4.3 Case three: Current applied to top and bottom with random pinning distributed over the interior layers (Figures 2.13-2.18)

In fact, when the pinning force is applied, two different current densities must be taken into account: decoupling surface current density and depinning current densities. When the decoupling current density is larger than the depinning current density the interior pancake vortices are able to move. Otherwise, they remain in their initial positions. The current is applied to top and bottom layers. In Figure 2.13 a small pinning force is applied to the interior lattices with $B = 0.1 \, T$. Surface current density versus position of outermost layers is shown for $N = 3, 5, 7, 11, 15, 21, 31$ superconducting layers. Similar to Figure 2.6 there is a maximum current density $K_d$. Comparing Figure 2.13 with Figure 2.6, no change occurs in the surface current density profile.

In Figure 2.14 large pinning force is applied to the interior layers and the surface current density versus position of outermost layers is presented for $N = 5, 11, 21, 31, 51, 101$, where the applied field is $0.1 \, T$. Again the same values of surface current densities occur as in Figure 2.13 with small applied pinning and Figure 2.6 where no pinning is applied. This leads us to think that the effect of pinning is hidden by the symmetry of the applied current.

Taking into account different applied magnetic fields with small random pinning Figure 2.15 represents $K_d$ for $N = 3, 5, 7, 11$ superconducting layers. Similarly Figure 2.16 shows $K_d$ as a function of different applied magnetic fields but with large random pinning for $N = 5, 11, 21, 31, 51$ superconducting layers. As remarked for Figure 2.9, the saturation occurs very fast for high magnetic fields. The last four figures show clearly that pinning has nothing to do with the current density $K_d$.

Figure 2.17 illustrates how the structure looks like for eleven superconducting layers, when no pinning force is applied, compared to the structure when a small pinning and then a larger pinning force is applied for $B = 0.1 \, T$. The effect of the pinning is clear on the structure.
Figure 2.13: Surface current density versus positions of outermost pancake vortices with small pinning force applied to different superconducting layers.
Figure 2.14: Surface current density versus positions of outermost pancake vortices with large pinning force applied to different superconducting layers.
Figure 2.15: Decoupling surface current density versus applied magnetic field with small random pinning forces in different superconducting layers.
Figure 2.16: Decoupling surface current density versus applied magnetic field with large random pinning forces in different superconducting layers.
Figure 2.17: Positions of stacks of pancake vortices in 11 superconducting layers with no, small, and large pinning, (top and bottom layers are excluded).
When no pinning is applied, the structure is symmetric. However, after applying the pinning force, the structure is not symmetric any more. But when the applied pinning force is large, no displacement occurs for the interior pancakes ($1 \leq N < 10$). It is clear from Figure 2.17 that in the case of a small pinning force, the decoupling surface current density exceeds the depinning current density; and therefore vortices are able to move. On the other hand, when a larger pinning force is applied, the decoupling surface current density is smaller and the vortices remain in their initial positions. The comparison of the three cases of no pinning, small and large random pinning is shown in Figure 2.18 for eleven superconducting layers. Notice that there is no difference between the three cases. These results contrary to what one may expect, indicate that pinning force doesn’t enhance $K_d$. However, here we have a special case where equal current densities are applied to the outermost layers in opposite directions. The structure of pancake vortices in one stack is symmetric on the average. This symmetry hides the effect of pinning.
Figure 2.18: Decoupling surface current density versus magnetic field with no, small, and large random pinning applied to 11 superconducting layers.
2.5 Summary

We have discussed the effects of applying equal current densities to the top and bottom layers in opposite directions and we found that there is a maximum surface current below which the interior pancakes can produce a force to balance the forces resulting from the applied currents. However, above $K_d$ the top and bottom pancakes slip away from their corresponding stack of pancakes. Also, the displacements of top and bottom pancakes are large compared to the displacements of the interior pancakes. When the current is applied to more outer layers, the current density $K_d$ becomes even smaller. The last part dealt with samples containing random pinning in the interior layers. A striking result was observed: that no matter the value of the pinning force, $K_d$ is not affected at all. This is due to the symmetry of the applied current.
Chapter III: Transport dc current applied to one of the outermost layers of a finite stack of superconducting layers with and without pinning
3.1 Introduction

According to Vysotsky et al. \textsuperscript{104} "recent achievements in technology of high temperature superconductors paved the way for creation of real, full size high temperature superconducting magnets". However, a limitation of critical current density at high temperatures and high magnetic fields is still of great concern. To overcome this problem, pinning is introduced\textsuperscript{49, 59, 103, 119}. So static and dynamic properties of vortices in the mixed state in high temperature superconductors, with and free of pinning, are especially noteworthy\textsuperscript{120}. In this chapter, as in the first chapter, layered superconductors in the mixed state is taken into account, treating these vortices to be stacks of two-dimensional pancake vortices and assume as in chapter one, the number of superconducting layers, \(N\), to be finite. Also it is assumed that these vortices have perfect hexagonal lattice and that this structure remains unchanged throughout the experiment. Thermal fluctuations and effects of Josephson coupling are not taken into account.

Transport dc current is applied only to one of the two outermost layers. For the sake of comparison, these superconducting layers are studied with and without pinning, which is uniformly distributed in each layer but its strength varies randomly from one layer to another. An important goal is the understanding of random pinning effects on the superconductor properties. Hence a detailed study of magnetic coupling between two-dimensional pancake vortices is taken into account. The dynamics of pancake vortices is simulated in a finite stack of Josephson-decoupled layers in the presence of a magnetic field \(H\) directed perpendicular to the layers\textsuperscript{114}. Velocities as a function of time of the outermost layers are calculated, positions difference as a function of time between top and the layer below is shown as well. Also flux-flow voltage due to the motion of pancakes is computed by attaching voltage-measuring circuits to the top and bottom layers. In addition, the magnetic flux rate on the top layer and the flux rate difference between outermost layers also are calculated. All these quantities give an insight to the position and motion of the vortex lattices in different superconducting layers.
3.2 Description of the model:

Consider the case of finite stack of $N$ Josephson-decoupled superconducting thin films aligned along the $z$-axis with the bottom layer coinciding with the plane $z = 0$. If we apply a dc transport current $I_{\text{top}}$ to the top layer along the $x$-axis and neglect the effects of vortex pinning, pancake lattices in different layers will move in the $x$ direction\textsuperscript{114} as shown in Figure 3.1:

![Figure 3.1: Effect of direct current applied to the top layer and a magnetic field perpendicular to the layers of the superconductor.](Image)

In this case, using Eq. (2.42) we will have:

$$\eta \ddot{x}_i = \sum_{j \neq i} F_{\text{x}}(x_j - x_i, j, i) + \frac{\Phi_0}{c} K_{\text{x}}^{\text{top}} \delta_{i, N-1},$$

where $x_i(t)$ is the displacement of the pancake lattice in layer $i$ from the equilibrium position at time $t$. At $t = 0$, all layers are in perfect registry, i.e. $x_i(0) = 0$. $F_{\text{x}}$ is the $x$ component of the magnetic coupling force between a pancake in layer $j$ and another pancake in layer $i$. As in chapter two, $F_{\text{x}}$ can be written as follows:
\[ F_{x_j}(x_j - x_i, j, i) = \frac{\phi_0^2}{\pi \Lambda^2 A} \sum_{g 
eq 0, k} g_x C(g, j, i) \sin(g_x(x_j - x_i)). \] (3.2)

where \( g_x \) is the \( x \) component of \( g \). In the presence of a random pinning in all superconducting layers, the force in Eq (3.2) will be affected as mentioned previously in chapter two. From the solution of \( N \) force-balanced equations, the corresponding lattice velocities can be calculated.

### 3.3 Flux-flow voltage

Flux-flow voltage occurs as a result of the dynamics of the vortices. In order to calculate this voltage, two voltage-measuring circuits are attached to the outermost layers as shown in Figure 3.2, with a dc transport current density \( K_{v, \text{top}} \).

![Figure 3.2: Voltage measuring circuits attached to the outermost layers of the superconductor.](image)

We define \( V_{\text{top}} \), \( V_{\text{bot}} \) as the time averaged voltages per unit distance between contacts. When a dc current \( K_{v, \text{top}} \) is applied to the top layer, two different regimes are observed corresponding to the value of \( K_{v, \text{top}} \). When \( K_{v, \text{top}} \) is small, velocities of top and bottom layers coincides after the transient time and a steady state is reached; so \( V_{\text{top}} = V_{\text{bot}} \) and they have the following form:

\[ V_{\text{top}} = V_{\text{bot}} = \frac{\phi_0}{cA} v_{x0} \cdot \] (3.3)
where \( v_{rO} \) is the common velocity. However, when \( K_v^{\text{op}} \) is larger than a certain decoupling surface current density, velocities of top and bottom layers differ, and are periodic with time. So in this regime:

\[
\overline{V}_{\text{top}} = \frac{\varphi_0}{cA} \left[ x_{N-1}(t + T) - x_{N+1}(t) \right] \tag{3.4}
\]

\[
\overline{V}_{\text{bot}} = \frac{\varphi_0}{cA} \left[ x_0(t + T) - x_0(t) \right] \tag{3.5}
\]

where \( T \) is the common period.

### 3.4 Magnetic flux flow

Suppose that two SQUIDs are attached to the outermost layers of a stack of finite superconducting layers (see Figure 3.3).

![Figure 3.3: SQUID attached to the superconductor to measure flux flow due to the dc current applied to the top layer.](image)

The number of pancake vortices per unit area \( n \) is given by

\[
n = \frac{1}{\sqrt{3}/2 \ a^2} \tag{3.6}
\]
Using the relation

\[ B \frac{\sqrt{3}}{2} a^2 = \phi_*, \]  

(3.7)  

we obtain

\[ \frac{1}{\sqrt{3}} \frac{1}{2} a^2 = \frac{B}{\phi_*}. \]  

(3.8)  

Then the current of pancake vortices is given by:

\[ j = \frac{B}{\phi_*} \hat{x}. \]  

(3.9)  

Hence the flux through the SQUID is proportional to the current of vortices, and the magnetic flux \( \phi \) has the following expression:

\[ \phi = \text{Area of the SQUID} \times n \times \phi_* \]  

(3.10)  

Substituting in Eq.(3.10), we obtain the flux flow rate:

\[ \dot{\phi} = w \times \hat{x} \times B, \]  

(3.11)  

where \( \text{Area of the SQUID} = w \times x \), \( w \) is the width of the SQUID, and \( \hat{x} \) the velocity of the pancake vortex lattice. For small \( K_{v}^{top} \), there is no flux difference per unit time between the outermost layers, but once \( K_{v}^{top} \) reaches a certain decoupling surface current density, a flux difference between the top and bottom layers is expected.

### 3.5 Numerical results and discussion:

As in chapter two the interlayer spacing is taken to be \( s = 15 \text{\AA} \), \( \Lambda = 5.6 \times 10^{-4} \text{s} \) and the average magnetic induction \( B = 1 \times 10^{-1} \text{T} \). The applied surface current density \( K_{x}^{top} \) is expressed in units of \( \frac{c \phi_*}{\pi A^2 \Lambda^2} \). Velocities of top and bottom layers \( \hat{x}_{\text{top}} \) and \( \hat{x}_{\text{bot}} \) respectively, are presented in units.
The number of layers is taken to be \( N = 5 \) in all the figures illustrated in this chapter.

### 3.5.1 Velocities of outermost layers and displacement difference between the last two layers with and without pinning centers (Figures 3.4-3.10)

Velocities of top and bottom layers are shown as a function of time steps. Also the difference in position between the top layer and the layer below it \( (x_{\text{top}} - x_{\text{top-1}}) \) versus time is presented as well. Figure 3.4 shows \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) for \( K_y^{\text{top}} = 9 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2} \). From this figure one can see that both \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) are periodic. Figure 3.5 shows \( (x_{\text{top}} - x_{\text{top-1}}) \) in the presence of \( K_y^{\text{top}} = 9 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2} \). It is clear that the difference in position becomes larger as a function of time. In some regions, the difference becomes smaller and in other regions it increases again.

This occurs because the top pancake moves faster than the other pancakes, so the pancake below the top is so slow compared to it, but other pancakes slow down the top until eventually it decouples. After decoupling, the top pancake is negligibly affected by the pancakes below it, and as a result it moves faster and the difference becomes larger. Then the top pancake approaches another set of pancakes and again it slows down because the interaction between the top pancake and the rest of the pancakes below it becomes stronger. In Figure 3.6 \( K_y^{\text{top}} = 2 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2} \) as a surface current density, which is sufficiently small. Figure 3.6(a) represents \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \). In the transient time the top pancake decelerates, whereas the bottom pancake accelerates. After this transient time, \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) coincide. In other words, the top and bottom pancakes have a common velocity \( v_{x_0} \). Figure 3.6(b) shows \( (x_{\text{top}} - x_{\text{top-1}}) \).
Figure 3.4: DC current applied to the top layer with no pinning introduced.

\[ N = 5, \quad \varkappa^{\text{top}} = 9 \times 10^{-3} \frac{c\phi_s}{\pi A^2 \Lambda^2} \]
Figure 3.5: Direct current applied to the top layer with no pinning introduced.

\[ N = 5, \quad K^{\text{top}} = 9 \times 10^{-2} \frac{c \phi_{s}}{\pi A^{2} \Lambda^{2}}. \]
Figure 3.6: Dc current applied to the top layer with no pinning introduced.

$N=5, \quad K_{\text{top}}^{\text{eff}} = 2 \times 10^{-2} \frac{c \phi^2}{\pi A}$. 
In transient regime the difference increases. Then it becomes constant after a while, which indicates that no decoupling occurs because of this amount of $K_{y\text{top}}$. In Figure 3.7, \[ K_{y\text{top}} = 9 \times 10^{-2} \frac{c\phi_0}{\pi A^2 \Lambda^2} \] in presence of a random pinning force is applied to all layers. Figure 3.7(a) shows $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$. Comparing this figure with Figure 3.4 one can see that both $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ decrease due to the pinning. The shape of $\dot{x}_{\text{bot}}$ becomes different from the case with no applied pinning, which indicates the effect of pinning. Figure 3.7(b) shows $(\ddot{x}_{\text{top}} - \ddot{x}_{\text{top-1}})$. Also comparing Figure 3.7(b) to Figure 3.5, the difference in the latter figure is smaller; so the decoupling is delayed. In Figure 3.8 $K_{y\text{top}} = 2 \times 10^{-2} \frac{c\phi_0}{\pi A^2 \Lambda^2}$ in the presence of a pinning force is applied. In Figure 3.8(a), it is obvious that $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ become smaller when compared with Figure 3.6(a), where no pinning was introduced. In other words, these pancakes slow down greatly. For the bottom pancake, it needed some time to depin where $\dot{x}_{\text{bot}} = 0$ at first, then it started to move, whereas the top pancake started to move but with smaller initial velocity. In Figure 3.8(b) again the difference $(\ddot{x}_{\text{top}} - \ddot{x}_{\text{top-1}})$ in the case of pinning is smaller than in the case of no applied pinning force (see Figure 3.6(b)). In order to know the effect of different sets of pinning on the decoupling between the pancake lattices the next two figures (Figures 3.9, 3.10) are presented. In Figure 3.9(a) another set of pinning is introduced to the superconducting layers, with $K_{y\text{top}} = 9 \times 10^{-2} \frac{c\phi_0}{\pi A^2 \Lambda^2}$. Comparing this figure with Figure 3.7(a), where a different pinning set is applied, and with Figure 3.4 where no pinning is applied, it appears clearly that periodic cycles in pinned samples are less than the unpinned sample, which means that pinning slows down these pancakes. Also in the two figures with pinning (Figures 3.7(a) and 3.9(a)) there is a difference in profile.
Figure 3.7: Dc current applied to the top layer with random pinning.

\[ N = 5, \quad K_{\text{top}} = 9 \times 10^{-2} \times \frac{c \phi_*}{\pi A^2 L^2} \]
Figure 3.8: DC current applied to the top layer with random pinning.

\[ N = 5, \quad K^{\text{top}} = 2 \times 10^{-2} \frac{c \phi_*}{\pi A^2 N^2}. \]
Figure 3.9: DC current applied to the top layer with random pinning.

\[ N = 5, \quad K_{\text{top}} = 9 \times 10^2 \frac{c\phi_*}{\pi A^2 \Lambda^2}. \]
The pancakes in Figure 3.9(a) move even slower, although pinning has the same range, i.e. maximum amount of pinning is the same, namely \( 5 \times 10^{-3} \frac{c \phi_0}{\pi A^2 \Lambda^2} \). Figure 3.9(b) shows \( (x_{\text{top}} - x_{\text{top}-1}) \) for the same pinning set used in Figure 3.9(a), where \( K_{y,\text{top}} = 9 \times 10^{-2} \frac{c \phi_0}{\pi A^2 \Lambda^2} \).

Comparing this figure with Figure 3.7(b), where another set of pinning is applied, one can see that in Figure 3.9(b) the difference becomes smaller and the decoupling takes a longer time, so that random pinning prevents early occurrence of decoupling. Figure 3.10(a) presents \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) with \( K_{y,\text{top}} = 2 \times 10^{-2} \frac{c \phi_0}{\pi A^2 \Lambda^2} \) in presence of a random pinning set other than the one in the case of Figure 3.8. \( \dot{x}_{\text{top}} \) decreases until \( \dot{x}_{\text{top}} = 0 \), whereas \( \dot{x}_{\text{bot}} = 0 \) all the time, which means that the bottom pancake stays pinned. So in both cases of pinning of Figures 3.8(a) and 3.10(a), the velocities \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) are less than that in case of no pinning, as first mentioned in Figure 3.6(a). In Figure 3.10(b) as in Figure 3.8(b), where another pinning set is applied to the superconducting layers, the difference \( (x_{\text{top}} - x_{\text{top}-1}) \) is less than in the case of no pinning.

### 3.5.2 Top and bottom time-averaged voltages with and without pinning centers (Figures 3.11, 3.12)

Figure 3.11 represents top and bottom time-averaged voltages \( \overline{V}_{\text{top}} \) and \( \overline{V}_{\text{bot}} \) versus applied surface current densities, for two samples: one without pinning and the other sample is pinned. In this graph, there are two different regimes. In the first regime where \( K_{y,\text{top}} \) is small, \( \overline{V}_{\text{top}} \) and \( \overline{V}_{\text{bot}} \) coincide, and they increase linearly with \( K_{y,\text{top}} \). The second regime is characterized by an increasing of \( \overline{V}_{\text{top}} \) and a decreasing of \( \overline{V}_{\text{bot}} \).
Figure 3.10: Dc current applied to the top layer with random pinning.

\[ N = 5, \quad K_{\text{rep}}^{\text{rep}} = 2 \times 10^{-2} \frac{c\phi_0}{\pi A^2 \Lambda^2}. \]
Figure 3.11: $\bar{V}_{\text{top}}$ and $\bar{V}_{\text{bot}}$ in presence of a dc current, with and without pinning.
The second regime starts when decoupling surface current density is reached or exceeded. When no pinning is applied, $V_{up}$ and $V_{bot}$ in the first regime are greater than in the case of applied pinning. In the second regime the unpinned sample has larger $V_{up}$ than the pinned one, but for $V_{bot}$ there is a small difference, and the pinned sample tends faster to zero. Figure 3.12 is the same as Figure 3.11 except that it has more than one case of different random pinning sets lying in different ranges to show different possible profiles. For large pinned samples the decoupling is very much delayed due to the great influence of randomly distributed pinning.

3.5.3 Magnetic flux flow rate in the top layer and magnetic flux flow rate difference between outermost layers with and without pinning (Figures 3.13-3.16)

In the next figures the magnetic flux flow rate difference between top and bottom layers $(\phi_{up} - \phi_{bot})$ is presented versus time, as well as the magnetic flux flow rate $\phi_{up}$ in top layer.

Figure 3.13 shows $(\phi_{up} - \phi_{bot})$ and in the inner graph $\phi_{up}$ for $K_{v,up} = 4 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$ in the case of no pinning. $(\dot{\phi}_{up} - \dot{\phi}_{bot})$ decays in the transient regime where there is no difference between the flux flow rate in the outermost layers. This means that the flux at the top and bottom layers are equal. The inset graph shows $\dot{\phi}_{up}$ which start to decrease in the deceleration period, then becomes constant. In Figure 3.14, double the surface current density as in the case of Figure 3.13, $K_{v,up} = 8 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$, is applied without introducing pinning. As in Figure 3.13, $(\phi_{up} - \phi_{bot})$ and in the inner graph $\phi_{up}$ are shown.
Figure 3.12: $\bar{V}_{up}$ and $\bar{V}_{hed}$ in presence of a dc current, with and without pinning.
Figure 3.13: Flux difference $(\phi_{a,p} - \phi_{a,u})$ in presence of a dc current

\[
K_y^{up} = 4 \times 10^2 \frac{c \phi_s}{\pi t^2 \Lambda^2}, \quad N = 5, \text{ with no pinning.}
\]
Figure 3.14: \((\phi_{\text{up}} - \phi_{\text{down}})\) in presence of a dc current \(K_{\nu}^{\text{imp}} = 8 \times 10^{-2} \frac{c \phi_i}{\pi A^2 \Lambda^2}\), \(N = 5\), with no pinning.
as well as \( \phi_{\text{top}} \) have a periodic shape, since \( K_{\text{top}} \) is above the decoupling surface current density as investigated from previous figures, whereas \( K_{\text{top}} \) shown in Figure 3.13 is considered to be below the decoupling current density. The periodic flux flow rate is due to the periodic velocity and it occurs only in the case of an applied current greater than the decoupling current. Figure 3.15 treats the same case as in Figure 3.13, but with two different random pinning sets. The difference in flux flow rate between the top and bottom layers is zero as in Figure 3.13 after a certain transient time. Comparing Figures 3.13 and 3.15, it is clear that the magnetic flux flow rate in the two cases are less than the case of no pinning. Although the two random pinning sets differ in magnitude, they are consistent in being less than in the case of no pinning force. Figure 3.16 shows the same case as in Figure 3.14 but with applying two random pinning sets within the same range. The shape is periodic here also. However, the number of cycles decreases. Hence, as stated earlier, decoupling will be late than in case of no pinning (see Figure 3.14) where vortices move more easily. These figures indicate that random pinning applied to superconducting layers delays the decoupling between the pancake lattices.
Figure 3.15: \((\phi_{\text{exp}} - \phi_{\text{het}})\) in presence of a dc current \(K_{\text{het}} = 4 \times 10^{-2} \frac{c\phi}{\pi A^2 \Lambda^3}\), \(N = 5\), with two different random pinning sets.
Figure 3.16: \( (\phi_{\text{up}} - \phi_{\text{low}}) \) in presence of a dc current \( K_{\text{up}} = 8 \times 10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2} \), \( N = 5 \). with two different random pinning sets.
3.6 Summary

In this chapter, pancake vortices in a finite stack of superconducting layers are subjected to a range of dc surface current densities flowing on the top layer with average magnetic induction \( B = 0.1 \, T \) and in the presence of random pinning. Velocities of outermost layers \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{hot}} \) were computed. The difference in positions between the upper two layers \((x_{\text{top}} - x_{\text{top-1}})\) is calculated. From these calculations we found that decoupling between pancake lattices occurred only for \( k_{\text{top}} \) higher than a certain decoupling current. This same idea is verified by measuring time-averaged voltages \( \bar{V}_{\text{top}} \) and \( \bar{V}_{\text{hot}} \) versus applied transport current density. There are two regimes: in the first regime, where the current is below the decoupling value, \( \bar{V}_{\text{top}} = \bar{V}_{\text{hot}} \) and they increase linearly with the current. Then when the decoupling surface current is reached, \( \bar{V}_{\text{top}} \) increases whereas \( \bar{V}_{\text{hot}} \) decreases. Then \( \phi_{\text{top}} \) and \( (\phi_{\text{top}} - \phi_{\text{hot}}) \) were computed. In all cases, it was obvious that random pinning inhibited or at least slowed the vortex motion, and consequently decoupling was delayed.
Chapter IV: Ac losses for a finite stack of superconducting layers with and without pinning
4.1 Introduction

High temperature superconductors have great technological weight, mainly in the mixed state where superconductivity coexists with magnetic vortices penetration. The vortex motion due to transport currents leads to resistance and hence power losses in the superconductor. Pinning is considered to be effective in inhibiting the motion of these vortices. In the preceding two chapters, the applied direct current was constant in magnitude and direction. However, in some cases, superconductors carry alternating current. In applications that run with ac current such as superconducting motors, transformers and generators, the problem of ac losses becomes significant. Due to this importance, ac losses have received a lot of attention as shown from experimental and theoretical works. Kerchner et al. determined that ac losses increase sharply as the maximum current in each cycle is raised above critical current while below the critical current the loss is relatively low. Also ac magnetic fields play an essential role in the ac losses. Celebi et al. believe that ac losses result from both hysteretic bulk pinning and vortex motion. Nevertheless, it is known that pinning is introduced into the superconductor to raise its critical current, and hence ac losses can be reduced.

The main goal of the study here is to simulate the dynamics of pancake vortices in a finite stack of Josephson-decoupled layers in case of applying ac transport current to the top layer assuming a perpendicular magnetic induction. Taking into consideration two cases: (1) free of pinning and (2) random pinning forces that are distributed uniformly in each layer. Taking into account different current amplitudes and frequencies, velocities of top and bottom layers are calculated as a function of time. Positions difference as a function of time between the top layer and the layer below it is calculated as well. In addition, magnetic flux flow rate difference between the outermost layers as a function of time is computed. Finally, the average power dissipation versus frequency is investigated.
4.2 Description of the model

As in the previous chapters consider a finite stack of $N$ Josephson-decoupled superconducting layers. Also it is assumed that the pancake vortices have perfect hexagonal shape that won’t change through the experiment. However, thermal fluctuations and Josephson coupling effects are neglected. At equilibrium, with applied perpendicular magnetic field all pancake vortices are aligned along the $z$-axis with the bottom pancake coinciding with the plane $z = 0$, as shown in Figure 3.1. AC transport current $k_{ac}^{\text{top}}$ is applied to the top layer. $k_{ac}^{\text{top}}$ has the following form

$$k_{ac}^{\text{top}} = I_e \cos \Phi.$$  

(4.1)

With the current amplitude $I_e$, and

$$\Phi = \frac{2\pi}{T_k} \times f.$$  

(4.2)

where $T_k$ is the period of the current $k_{ac}^{\text{top}}$, such that the frequency $\nu = \frac{1}{T_k}$.

Neglecting pinning effect, the pancake vortices in different layers will move according to the following equation

$$\eta \dot{x}_i = \sum_{j \neq i} \vec{F}_{ij}(x_j - x_i) + \frac{\varphi_0}{c} k_{ac}^{\text{top}} \delta_{i,j-1}.$$  

(4.3)

If the pinning is introduced, the force equation (3.2) will be affected in the same manner discussed in the last chapter. From the solution of $N$ force-balanced equations, the corresponding lattice velocities can be computed. Also by attaching SQUIDs to the outermost layers, the difference of flux flow between top and bottom layers is calculated.
4.3 Ac losses

In any application, where the superconducting material must carry an ac transport current or subjected to an ac external magnetic field the ac losses is considered to be an important parameter. Achievement of low ac losses offers enormous promise and is of crucial importance for practical products such as power transmission cables, generators, motors and many other ac applications. Low power dissipation is considered to be an essential prerequisite in the thought of replacing normal copper wire with superconducting wires. The power dissipation $P$ can be written as

$$P = F_{cx} \cdot v,$$  \hspace{1cm} (4.4)

where $F_{cx}$ is the force on the pancake vortex due to all other vortices and the Lorentz force due to applied current. $v$ is the velocity of the pancake vortex. Using Eq. (2.42), we arrive at

$$P = \eta v^2.$$  \hspace{1cm} (4.5)

Therefore the power loss for all superconducting layers at any time is

$$P(t) = \eta \sum_{i=0}^{\infty} v_i^2(t),$$  \hspace{1cm} (4.6)

where the sum is over all the layers. The average power dissipation is given by

$$\bar{P} = \frac{1}{T_k} \int_0^{T_k} P(t) dt,$$  \hspace{1cm} (4.7)

which can be written in a finite-difference form as:

$$\bar{P} = \frac{1}{T_k} \sum_{i=1}^{k} P(t_i) \Delta t,$$  \hspace{1cm} (4.8)

where $k$ is the number of time steps in one period of the alternating current. Then

$$\bar{P} = \frac{\Delta t}{T_k} \sum_{i=1}^{k} P(t_i).$$  \hspace{1cm} (4.9)
4.4 Numerical results and discussion:

As in the previous chapters the interlayer spacing \( s \approx 15 \, \text{Å} \), \( \Lambda = 5.6 \times 10^4 \, \text{s} \) and the magnetic induction is \( B = 0.1 \, \text{T} \). The ac transport surface current density \( j_{\text{top}} \) applied to the top layer is expressed in units of \( \frac{e \phi_c}{\pi A^2 \Lambda^2} \). Velocities of top and bottom layers \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \), are presented in units of \( \frac{\phi_c^2}{\pi \eta \Lambda^2 A} \). The flux flow rate difference is expressed in \( \frac{\sqrt{(\phi_c B)^3}}{\eta \pi \Lambda^2} \), and the average ac power loss is presented in units of \( \frac{B^2 \phi_c^2}{\pi^2 \eta \Lambda^4} \). The number of layers is taken to be \( N = 5 \) throughout all numerical experiments in this chapter.

4.4.1 Velocities of the top and bottom layers and the position difference

Velocities of the top and bottom layers \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) and the displacement difference between the top layer and the layer below it \( (x_{\text{top}} - x_{\text{top-1}}) \) versus time are shown.

4.4.1.a Current amplitude \( I_s = 9 \times 10^{-2} \, \frac{e \phi_c}{\pi A^2 \Lambda^2} \), with no pinning applied (Figures 4.1-4.3)

In Figure 4.1 the frequency is \( \nu = 5 \times 10^{-5} \). Then the period of the applied current density \( j_{\text{top}} \) is \( T_k = 20000 \). Figure 4.1(a) shows \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) versus time. At \( t = 0 \), \( j_{\text{top}} = I_s \) and velocity of the top layer \( \dot{x}_{\text{top}} \) has positive values until \( \cos \frac{\pi}{2} \) is reached at \( t = 5000 \), then it takes negative values when \( \frac{\pi}{2} < \cos \phi < \frac{3\pi}{2} \) which is the interval between \( t = 5000 \) and \( t = 15000 \).
Figure 4.1: Ac current is applied to the top layer, $N = 5$, $I_s = 9 \times 10^{-5}$, $\frac{c\phi_s}{\pi d^2 \Lambda^2} = \nu = 5 \times 10^{-5}$. 
At \( t = 5000 \) and \( t = 15000 \), \( k_y^{\text{top}} = 0 \) and \( \dot{x}_{\text{top}} \) is only due to the forces from other pancakes. In the interval from \( t = 15000 \) and \( t = 20000 \), \( \dot{x}_{\text{top}} \) has positive magnitude. At \( t = 20000 \), \( k_y^{\text{top}} = l_x \) and after that the cycle will start all over again. \( \dot{x}_{\text{top}} \) has larger values than \( \dot{x}_{\text{bot}} \) since the current is applied to the top layer only and the motion of the bottom pancake is only due to forces of the other pancakes. The peaks in \( \dot{x}_{\text{top}} \) leads us to think of decoupling occurrence where at these time steps, \( k_y^{\text{top}} \) has large values that leads to decoupling. Figure 4.1(b) shows the position difference \( (x_{\text{top}} - x_{\text{top}-1}) \) when an ac transport current is applied. This difference is periodic comparing to the increasing or constant form of the position difference in case of an applied dc transport current to the top layer, as seen in the chapter three. In Figure 4.2, \( \nu = 1 \times 10^{-4} \), that means the period of \( k_y^{\text{top}} \) is \( T_k = 10000 \). Figure 4.2(a) shows \( x_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) versus time. As a result of the frequency difference from that in Figure 4.1(a), the periodic shape is different, and the decoupling occurs more frequently. Figure 4.2(b) shows \( (x_{\text{top}} - x_{\text{top}-1}) \). Again the periodic shape also occurs here. In Figure 4.3, \( \nu = 5 \times 10^{-3} \) which is much higher than the previous frequencies. In Figure 4.3(a), \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) are shown. The decoupling occurs very early and \( \dot{x}_{\text{bot}} \) has very small magnitude compared to \( \dot{x}_{\text{top}} \). Figure 4.3(b) shows \( (x_{\text{top}} - x_{\text{top}-1}) \). The shape is periodic and the difference is smooth which may be a result of the big difference in position, in other words, the layer below the top one moved just slightly with respect to the top layer.
Figure 4.2: Ac current is applied to the top layer, \( N = 5 \), \( I_0 = 9 \times 10^{-2} \frac{c \phi_s}{\pi a^2 \Lambda^2} \), \( \nu = 1 \times 10^{-4} \).
Figure 4.3: AC current is applied to the top layer, \( N = 5 \), \( I = 9 \times 10^{-2} \frac{c\phi}{\pi \lambda^3} \), \( \nu = 5 \times 10^{-3} \).
4.4.1.b Current amplitude $I_\ast = 1 \times 10^{-2} \frac{e \phi_c}{\pi A^2 \Lambda^2}$, with no pinning centers (Figures 4.4-4.6)

In Figure 4.4 $\nu = 5 \times 10^{-5}$. Figure 4.4(a) displays $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$. $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ have the same periodic shape and resembling values, which is consistent with the fact that this current amplitude doesn’t cause decoupling. Figure 4.4(b) illustrates $(x_{\text{top}} - x_{\text{top-1}})$. The shape is periodic and the difference is very little compared to Figure 4.1(b) where the frequency is the same but higher current amplitude was applied to the superconductor. In Figure 4.5 $\nu = 1 \times 10^{-4}$ which is double the frequency in Figure 4.4. Figure 4.5(a) shows $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ that have the same periodic shape with adjacent magnitudes but the number of cycles increased compared with Figure 4.4(a). In the inner diagram $(x_{\text{top}} - x_{\text{top-1}})$ is shown for the same situation. In Figure 4.6 $\nu = 5 \times 10^{-3}$ which is very high frequency comparing with that in Figures 4.4 and 4.5. Figure 4.6(a) introduces $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$. The profile of the $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ is similar to that in Figure 4.3(a) where $I_\ast = 9 \times 10^{-2} \frac{e \phi_c}{\pi A^2 \Lambda^2}$ and $\nu = 5 \times 10^{-3}$, but only with smaller magnitudes. This means that at high frequencies the shape depends mostly on the frequency, and not on the current amplitude. Figure 4.6(b) represents $(x_{\text{top}} - x_{\text{top-1}})$. Here also the figure is the same as in Figure 4.3(b) except that it has smaller magnitudes because of the smaller current amplitude.

4.4.1.c Current amplitude $I_\ast = 9 \times 10^{-2} \frac{e \phi_c}{\pi A^2 \Lambda^2}$, with random pinning centers (Figures 4.7-4.12)

In all the preceding figures no pinning was introduced to the superconducting layers. In the next figures, the effect of introducing random uniformly distributed pinning to the superconducting
Figure 4.4: Ac current is applied to the top layer, $N = 5$, $I_c = 1 \times 10^{-3} \frac{c \phi_0}{\pi A^2}$, $\nu = 5 \times 10^{-5}$. 

\[\dot{x}_{\text{top}} \text{ and } \dot{x}_{\text{bot}} (\frac{\phi_0^2}{\pi A^2})\]
Figure 4.5: Ac current is applied to the top layer, \( N = 5 \), \( I_0 = 1 \times 10^{-2} \frac{c_0}{\pi^2 \Lambda^2} \).
\( \nu = 1 \times 10^{-1} \).
Figure 4.6: AC current is applied to the top layer, $N = 5$, $I_0 = 1 \times 10^{-2}$ $\frac{c \phi}{\pi l^2 A^2}$, $\nu = 5 \times 10^{-4}$. 

Number of time steps
layers is illustrated. In Figure 4.7 $v = 5 \times 10^{-5}$. Figure 4.7(a) shows $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$. Comparing this figure with Figure 4.1(a), it is clear that $\dot{x}_{\text{top}}$ in Figure 4.7(a) has a smaller magnitude and $\dot{x}_{\text{bot}}$ is almost zero. In addition, the peaks occur in smaller values at a later time in $\dot{x}_{\text{top}}$ curve. Figure 4.7(b) represents $(x_{\text{top}} - x_{\text{top-1}})$. Comparing Figure 4.7(b) with Figure 4.1(b) with the same conditions except that no pinning was introduced in the case of Figure 4.1(b) the shape of the curve differs slightly. Figure 4.8 displays $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ with a different set of random pinning within the same range. In this figure $\dot{x}_{\text{bot}} = 0$ at all times, whereas $\dot{x}_{\text{top}}$ has only minor peaks. Comparing this with Figure 4.7 where another set of pinning was applied and with Figure 4.1(a) where no pinning was applied, one can see clearly that in Figure 4.8, vortex motion is inhibited strongly by this strong random pinning set. In the inner graph, the difference in position $(x_{\text{top}} - x_{\text{top-1}})$ is shown. Also the difference is very small compared to Figure 4.7(b) where another set of random pinning was applied and with Figure 4.1(b) where no pinning was applied.

In Figure 4.9, we set $v = 1 \times 10^{-4}$. Figure 4.9(a) represents $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$. $\dot{x}_{\text{bot}}$ values are close to zero and at certain time intervals it becomes zero, which means that the applied current $k_{\text{top}}$ and forces due to other pancakes are not able to depin the vortex. Figure 4.9(b) shows $(x_{\text{top}} - x_{\text{top-1}})$. The curve has different forms: first upward then constant for a while followed by a downward. Moreover, for the rest of the curve in the case of pinning the difference is negative. Figure 4.10 displays $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$ for $v = 1 \times 10^{-4}$, the same as in Figure 4.9 but with altered random pinning set. The shape here differs a lot from that in Figures 4.9(a) and 4.2(a) where no pinning is introduced. In the inset graph of Figure 4.10 $(x_{\text{top}} - x_{\text{top-1}})$ is shown. Its profile also differs from that in the case of Figure 4.9(b) and Figure 4.2(b) where no pinning is applied. The position difference in Figure 4.10 takes both positive and negative values by almost equivalent amounts.
Figure 4.7: Ac current is applied to the top layer with random pinning, $N = 5$.

$I_0 = 9 \times 10^{-2} \, \frac{c \phi_s}{\pi A^2 \Lambda^2}$. $\nu = 5 \times 10^{-5}$. 
Figure 4.8: AC current is applied to the top layer with random pinning, \( N = 5 \).

\[ I = 9 \times 10^{-2} \frac{c \phi_0}{\pi A^2 \Lambda^2}, \quad \nu = 5 \times 10^{-5}. \]
Figure 4.9: Ac current is applied to the top layer with random pinning. $N = 5$.

$I_s = 9 \times 10^{-2} \frac{c \phi_s}{\pi A^2 \Lambda}, \nu = 1 \times 10^{-4}$. 
Figure 4.10: Ac current is applied to the top layer with random pinning. $N = 5$.

$I_\text{c} = 9 \times 10^{-2} \frac{c \phi}{\pi A^2 \Lambda^2}$, $\nu = 1 \times 10^{-4}$. 
In Figure 4.11, \( v = 5 \times 10^{-3} \). In Figure 4.11(a) \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) versus time are shown. Comparing Figure 4.3(a) where no pinning is present with this figure, we find that here \( \dot{x}_{\text{bot}} = 0 \) and \( \dot{x}_{\text{top}} \) has smaller values and is zero in certain intervals which means that in these intervals the pinning force effect is greater than both the applied current and forces due to other pancakes. Figure 4.11(b) displays \( (x_{\text{top}} - x_{\text{top-1}}) \) versus time. Comparing this figure to Figure 4.3(b) where no pinning is applied, there is a small difference in shape and magnitude. In Figure 4.12, another set of random pinning force is applied to the superconducting layers with the other parameters taken to be the same as in Figure 4.11. Comparing Figures 4.11(a) and 4.12(a), we can say that the pinning is stronger in Figure 4.12(a) and its effect is clearer on \( \dot{x}_{\text{top}} \) which has smaller magnitudes and is zero for longer intervals. However, the bottom pancake doesn’t move at all. Figure 4.12(b) shows \( (x_{\text{top}} - x_{\text{top-1}}) \). Comparing this figure with Figure 4.11(b) where another pinning set is introduced and Figure 4.3 where no pinning is applied, it’s obvious that the difference \( (x_{\text{top}} - x_{\text{top-1}}) \) in Figure 4.12(b) is smaller due to stronger pinning.

### 4.4.1.d Current amplitude \( I_0 \), with random pinning centers (Figures 4.13-4.18)

In Figure 4.13 \( v = 5 \times 10^{-5} \). Figure 4.13(a) shows \( \dot{x}_{\text{top}} \) and \( \dot{x}_{\text{bot}} \) versus time. With this current amplitude and pinning set applied, a minor amount of motion occurs at the top layer. However, the bottom pancake doesn’t move at all. Figure 4.13(b) presents \( (x_{\text{top}} - x_{\text{top-1}}) \) versus time. It is noticeable also that \( (x_{\text{top}} - x_{\text{top-1}}) \) is smaller than in the case of no pinning (see Figure 4.4(b)). In Figure 4.14, another set of pinning is introduced to the superconducting layers with the same frequency as in Figure 4.13.
Figure 4.11: Ac current is applied to the top layer with random pinning. $N = 5$.

\[ I_0 = 9 \times 10^{-2} \frac{c \phi_c}{\pi A^2} \] \[ \nu = 5 \times 10^{-3} \]
Figure 4.12: AC current is applied to the top layer with random pinning. $N = 5$.

$L_c = 9 \times 10^{-2} \frac{c}{\pi A^2 L^2}$, $\nu = 5 \times 10^{-3}$. 
Figure 4.13: Ac current is applied to the top layer with random pinning, $N = 5$, $I_c = 1 \times 10^{-2} \frac{c_i}{\pi A^2 \Lambda^2}$, $\nu = 5 \times 10^{-5}$. 

$x_{top}$ and $x_{bot}$
Figure 4.14: Ac current is applied to the top layer with random pinning. \( N = 5 \).

\[ I_\alpha = 1 \times 10^{-2} \frac{c_\alpha \phi_\alpha}{\pi A^2 \Lambda^2}, \quad \nu = 5 \times 10^{-3}. \]
Again no motion occurs in the bottom layer and just a slight motion occurs at the top layer, compared with Figure 4.13(a) where another set of random pinning is introduced and Figure 4.4(a) where no pinning is applied. In the inset of Figure 4.14 \((x_{top} - x_{top-1})\) is illustrated and it is of smaller value from both the other pinning set in Figure 4.13(b) and in the case of no pinning in Figure 4.4(b). Figure 4.15 displays \(x_{top}\) and \(x_{bot}\) for \(\nu = 1 \times 10^{-4}\). Again the bottom pancake stay in its position without any motion whereas the top pancake moves and becomes constant in long periods compared with the case of no pinning in Figure 4.5. \((x_{top} - x_{top-1})\) is shown in the inner graph. The difference \((x_{top} - x_{top-1})\) is smaller than in case of no pinning as shown in the inner graph in Figure 4.5. Figure 4.16 shows \(x_{top}\) and \(x_{bot}\) for \(\nu = 1 \times 10^{-4}\) with a different pinning set. Comparing this figure with the last figure we see that in this figure, the magnitude of \(x_{top}\) is smaller than that in the last figure, but the intervals of \(x_{top} = 0\) are smaller, which is due to different random pinning effects. Also the position difference \((x_{top} - x_{top-1})\) is presented and it is larger than in the case of no pinning, as shown in Figure 4.5. In Figure 4.17 \(\nu = 5 \times 10^{-3}\). Figure 4.17(a) shows \(x_{top}\) and \(x_{bot}\). The bottom pancake is not able to depin, hence \(x_{bot} = 0\). The top pancake moves but with smaller magnitude in contrast with the case of no pinning in Figure 4.6(a). Figure 4.17(b) shows \((x_{top} - x_{top-1})\). Comparing the position difference with that in Figure 4.6(b), one can see that in the case of pinning the position difference is smaller which maybe due to slow motion of top pancake and inhibited motion of the layer below as a result of pinning. In Figure 4.18, \(\nu = 5 \times 10^{-3}\), but with another applied random pinning set to the superconducting layers. Figure 4.18(a) illustrates \(x_{top}\) and \(x_{bot}\). Again the bottom pancake doesn’t move. However, the top pancake moves with a smaller velocity than in the case of no pinning, as shown in Figure 4.6(a).
Figure 4.15: AC current is applied to the top layer with random pinning, $N = 5$.

\[ I_s = 1 \times 10^{-3} \frac{\epsilon \phi_s}{\pi \sigma A^2}, \quad \nu = 1 \times 10^{-3}. \]
Figure 4.16: Ac current is applied to the top layer with random pinning, $N = 5$.

$I_x = 1 \times 10^{-2} \frac{c \phi_x}{\pi A^2 \Lambda^2}$, $u = 1 \times 10^{-4}$. 
Figure 4.17: AC current is applied to the top layer with random pinning. $N = 5$.

$I_s = 1 \times 10^{-2} \frac{c \phi_s}{\pi A^2 \Lambda^2}$, $\nu = 5 \times 10^{-3}$. 
Figure 4.18: Ac current is applied to the top layer with random pinning. $N = 5$.

$I_x = 1 \times 10^{-2} \frac{c \phi_x}{\pi A^2 \lambda^2}$, $v = 5 \times 10^{-3}$. 
In Figure 4.18(b), the difference \(x_{n+1} - x_{n-1}\) is more than in the case of other pinning set shown in Figure 4.17(b) and is comparable to the one in the case of no pinning as shown in Figure 4.6(b). From the last figures, we can say that for small applied current amplitudes the velocity becomes very small when pinning is introduced except for high frequencies.

### 4.4.2 Flux flow rate difference between outermost layers

The flux flow rate difference between outermost layers \(\phi_{top} - \phi_{bot}\) is presented as a function of time in presence of zero and random pinning.

#### 4.4.2.a Current amplitude \(I_0 = 9 \times 10^{-2} \frac{c\phi_s}{\pi A^2 L^2}\) (Figure 4.19)

Figure 4.19 shows \(\phi_{top} - \phi_{bot}\) for different frequencies in three cases without pinning and with two different sets of pinning, such that set two lies within a larger range that set 1. In Figure 4.19(a), \(\nu = 5 \times 10^{-5}\). The flux difference in the case of no pinning has larger magnitudes than in case of both pinning sets. For the first pinning set \(\phi_{top} - \phi_{bot}\) is mostly zero except in some intervals, where it has just small values. In Figure 4.19(b) higher frequency is taken into account \(\nu = 1 \times 10^{-4}\). For the first pinning set, the flux difference \(\phi_{top} - \phi_{bot}\) is very small, and even vanishes most of the time. This means that either no motion occurs for these vortices or that they are moving with identical velocities. In Figure 4.19(c) \(\nu = 1 \times 10^{-4}\). Here again, the flux difference in case of pinned samples is less than in the case of unpinned samples. Due to this first pinning set the flux difference is even smaller than in both cases of the other pinning set, but through this higher frequency the flux difference in case of pinning is higher than in case of smaller frequencies.
Figure 4.19: Flux difference between outermost layers when ac current is applied to the top layer with and without pinning, \( N = 5, I = 9 \times 10^{-7} - \frac{c \phi_0}{\pi A^2} \).
In Figure 4.19(d), where $\nu = 5 \times 10^{-3}$, although $(\dot{\phi}_{\text{nop}} - \dot{\phi}_{\text{hot}})$ is smaller in the case of pinning, the difference in all the situations is not so large. It is clear that in the case of pinning the flux difference is smaller, although as a result of the high frequency pinning effect is weakened.

### 4.4.2.b Current amplitude $I_0 = 1 \times 10^{-2}$ (Figure 4.20)

Figure 4.20 shows flux flow rate difference $(\dot{\phi}_{\text{nop}} - \dot{\phi}_{\text{hot}})$ for different frequencies. In Figure 4.20(a) frequency $\nu = 5 \times 10^{-3}$, which is considered to be a small frequency. The flux difference $(\dot{\phi}_{\text{nop}} - \dot{\phi}_{\text{hot}})$ in all the situations of zero and random applied pinning is just about zero. From the enlarged portion we can see that the flux difference in the case of zero introduced pinning is periodic whereas in the case of applied two different pinning sets within the same range, $(\dot{\phi}_{\text{nop}} - \dot{\phi}_{\text{hot}})$ vanishes in certain intervals. In Figure 4.20(b), we set $\nu = 1 \times 10^{-4}$. Despite the fact that the $(\dot{\phi}_{\text{nop}} - \dot{\phi}_{\text{hot}})$ is higher than in case of the smaller frequency, in the last figure the flux difference is still small and in case of pinning it becomes zero. In Figure 4.20(c), we set $\nu = 5 \times 10^{-4}$. The flux difference grows higher than in the case of $\nu = 5 \times 10^{-5}$ (see Figure 4.20(a)) and $\nu = 1 \times 10^{-4}$ (see Figure 4.20(b)) due to higher frequencies. In the case of pinning, in certain portions, the flux difference vanishes due to the pinning force applied to the superconducting layers. It is clear that at higher frequencies the flux difference becomes higher than in the case of low frequencies. In Figure 4.20(d), we set $\nu = 5 \times 10^{-3}$. The flux difference becomes much larger and the pinning effect is reduced due to this high frequency. The effect of pinning is reduced as the frequency gets higher.
Figure 4.20: Flux difference between outermost layers when ac current is applied to the top layer with and without pinning, $N = 5$, $J_s = 1 \times 10^{-2}$ $\frac{c \phi_s}{\pi A^2 \Lambda^2}$. 
4.4.3 Average power dissipation (Figures 4.21, 22)

Average power dissipation $\bar{P}$ is illustrated as a function of frequency for different pinned and unpinned samples. Figure 4.21 shows $\bar{P}$ with $I_*=1\times10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$. For the pinning-free sample, at low frequencies, power dissipation remains constant. When the frequency increases, $\bar{P}$ increases rapidly. In the case of pinning, though $\bar{P}$ increases in case of higher frequencies, this growth tends to be slower compared with the case of the unpinned sample. Figure 4.22 illustrates $\bar{P}$ with $I_*=9\times10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$. In the unpinned sample, the shape of $\bar{P}$ is different than what was shown in Figure 4.21. At low frequencies the power dissipation is high then it drops down and after that as the frequency gets higher $\bar{P}$ has increasing values. For the moderate pinned sample, also at low frequencies such as $\nu=5\times10^{-5}$ and $\nu=1\times10^{-4}$ the power dissipation is high. Then it declines and after a while it will increase again as the frequency becomes higher. At all points $\bar{P}$ in case of pinning is less than in case of no pinning. Also with this current amplitude, power dissipation is much higher than in case of $I_*=1\times10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$ as shown in Figure 4.21. At low frequencies, $\bar{P}$ in the large pinned sample vanishes. Then as frequency increases, power dissipation grows larger as in case of $I_*=1\times10^{-2} \frac{c\phi_s}{\pi A^2 \Lambda^2}$ in Figures 4.21. As can be seen pinning reduces the average power dissipation. The optimized pinning set could be chosen to reduce further the power dissipation. Notice also that a dip in power dissipation occurs when the frequency of the ac current is of the order of the frequency of the motion of the top pancake layer passing over the layers below. When the two frequencies are of the same order, the motion of the vortices is more coherent with the Lorentz force due to the applied ac current and hence, less energy dissipation.
Figure 4.21: Ac losses, with ac current applied to the top layer with and without pinning. \( N = 5 \). \( I_{\text{c}} = 1 \times 10^{-2} \frac{c \phi_v}{\pi d^2 \Lambda^2} \).
Figure 4.22: Ac losses, with ac current applied to the top layer with and without pinning. \( N = 5 \), \( I_c = 9 \times 10^{-2} \frac{c \phi_c}{\pi A^2 \Lambda^2} \).
4.5 Summary

Ac current was applied to the top layer in a finite stack of superconducting thin films, with a magnetic induction \( B = 0.1 \, T \). Two situations were considered: no pinning and random uniformly distributed pinning applied to the superconductor. Velocities of top \( \dot{x}_{\text{top}} \) and bottom \( \dot{x}_{\text{bot}} \) pancakes were computed. The difference in pancake positions \( (x_{\text{top}} - x_{\text{top-1}}) \) were calculated. Flux flow rate difference between outermost layers was shown. In addition average power dissipation as a function of frequency was figured. All these calculations were done under high and low current amplitudes with different high and low frequencies. For the sake of comparison, different sets of pinning were introduced to the superconducting layers. The power dissipation is reduced as we increase pinning, even at high frequencies. Hence, random uniformly distributed pinning improved superconducting electrical properties by reducing losses due to vortex motion.
Conclusion

High temperature superconducting materials are considered to be promising materials because of their wide range of applications. For example, they play a life-conserving role through medical imaging systems. Even internet and telecommunication growth are expected to depend on these stacked superconducting electronic materials. There are various further applications such as particle accelerators and magnetically levitated trains. However, these materials suffer certain problems like low temperature needs. Also in their mixed state, which is useful practically, magnetic vortices appear and their motion due to transport current or magnetic applied field causes power loss. In order to inhibit this motion pinning is introduced.

In this study a finite stack of superconducting thin films, as in real samples, with a pinning distribution are investigated.

In the second chapter two equal dc current densities were applied to the outermost layers in opposite direction. The interior pancake vortices rearrange themselves until they reach force-balanced configuration. This was done for different applied perpendicular magnetic fields. The current for which the outermost pancake vortices separate from the interior pancake vortices, is called the decoupling current. In addition, current was applied to more than two outermost layers. In the latter situation decoupling current density reduced sharply. In case of the applied pinning, the decoupling current density didn’t improved as was expected. We showed that the coupling in highly anisotropic materials is weak. Moreover, we showed that although pinning affects the motion of vortices, it doesn’t affect the coupling between vortices and we have to differentiate between the decoupling and depinning force.

In the third chapter dc current was applied to the top layer in presence of a perpendicular magnetic field. The superconductor was studied under no pinning and random uniform
distributed pinning. Velocities of outermost layers $\dot{x}_{\text{top}}$ and $\dot{x}_{\text{bot}}$, as well as positions difference between top layer and the layer below $(x_{\text{top}} - x_{\text{top-1}})$ were calculated. These calculations showed that for certain applied current decoupling doesn't occur. In this situation the positions difference remains constant. While in case of decoupling, positions difference increases as a function of time. In addition, time-averaged voltages of top and bottom layers ($\overline{V}_{\text{top}}$ and $\overline{V}_{\text{bot}}$) were shown. Two different regimes appear: in the first regime both $\overline{V}_{\text{top}}$ and $\overline{V}_{\text{bot}}$ are equal. The second regime that starts as soon as certain decoupling current density is reached. In this regime, $\overline{V}_{\text{top}}$ increased while $\overline{V}_{\text{bot}}$ tends to zero. Magnetic flux flow rate at the top layer and flux rate difference between outermost layers were calculated, and showed that decoupling didn’t occur for low applied currents. In all considered situations the pinned samples had inhibited or slowed vortices motion which indeed enhance superconducting properties. We suggest experimental ways to determine the strength of coupling between the vortices in different layers, to predict if the sample contains pinning, and to be able to calculate the average pinning force through measuring the difference in voltage and flux flow rate between top and bottom layers.

In the forth chapter ac current was applied to the top superconducting layer with different current amplitudes and frequencies. Also perpendicular magnetic field was introduced to the superconductor. Once more, two cases were taken into account: zero pinning and random uniformly distributed pinning. Velocities of top and bottom layers were calculated as well as difference in positions between the higher two layers $(x_{\text{top}} - x_{\text{top-1}})$. In addition flux flow rate difference between outermost layers was computed. These calculations compare unpinned samples to different pinned samples, which indicate a decrease in flux flow due to different pinning sets. In addition, average power dissipation was calculated as function of the
frequencies of the ac current. At low frequencies the vortex motion is limited then ac losses were lower, but at high frequencies vortex motion increases and hence ac losses increase. These ac losses are reduced under different pinning sets. This leads us to say that this pinning mechanism improved superconducting properties.
Appendix

Algorithm to calculate ac losses in presence of random pinning by two different methods.

Read $N$, $a$, $\Lambda$, $I_s$, $T_k$.

While ($i \leq N - 1$)

Apply random pinning to superconducting layers.

Current = $I_s \times \cos\left(\frac{2\pi}{T_k} \times time\right)$.

If current > pinning force

Position of top layer = current ± pinning force.

Time =0, ac_losses=0, power_loss=0.

While ($time \leq 1.5T_k$)

Let $f[k]=0$ (force initializing for all superconducting layers)

While ($j \leq N - 1$)

While ($i \leq N - 1$)

If ($i \neq j$) then $f[j]=f[j]+$ force ($x_i - x_j$).

(where force is a separate subroutine that calculates coupling force between pancake vortices)

For all superconducting layers except the top

If pinning[$i$]=0 then

$x_i = x_i + f[i]$

$E = f[i]$

Else if $\mid f[i] \mid > \mid$ pinning[$i$] then

$x_i = x_i + f[i] \pm$ pinning[$i$]

$E = f[i] \pm$ pinning[$i$]

Power_loss = power_loss + ($E \times f[i]$)

For the top layer, follow the last two steps with the addition of current effect.

$Power_loss = power_loss + (E \times f[i] + current)$

Reserve the last two previous positions in temp2 and temp3

Top velocity: tops=$(x_{N-1} - temp3_{N-1})/2$

Bottom velocity: bots=$(x_0 - temp3_0)/2$

For all interior superconducting layers

$Ac_losses = ac_losses + (x_i - temp3_i)^2 / 4$

$Ac_losses = ac_losses + tops^2 + bots^2$
References


Ren, Z. F., and Wang, J. H. 1993. Superior flux pinning in in situ synthesized silver-sheathed superconducting tape of Tl$_{0.5}$Pb$_{0.5}$Sr$_{1.6}$Ba$_{0.4}$Ca$_{0.8}$Y$_{0.2}$Cu$_{2}$O$_{y}$. Appl. Phys. Lett., 62 (23), 3025-3027.


وجود و اعداد مراكز الحبس. وقد وجدنا انخفاضاً في سرعة الدوامات وفرق الجبه و معدل الفيض المغناطيسي في حالة ووجود مراكز الحبس.

ثالثاً: نؤثر بتغير منتشر على الطبقية العليا للموصل الفائق و نحسب سرعة الدوامات المغناطيسية في كل من الطبقتين العليا و السفلية و فرق المسافة بين الدوامات المغناطيسية في الطبقية العليا و التي تليها، و الفرق في معدل الفيض المغناطيسي بين الطبقية العليا و السفلية. كذلك فقدان الطاقة مقابل التردد. علماً بأن كل الحسابات تمت في حالتين وجود و أعداد مراكز الحبس. وقد وجدنا نقصاً في سرعة الدوامات و الفيض المغناطيسي و فقدان الطاقة في حالة وجود مراكز الحبس.
الملخص

تعتبر الموصلات الفائقة التي تعمل في درجات حرارة عالية مواداً مثيرة للاهتمام في مجالات عملية واسعة، بدءاً من نقل الطاقة الكهروضانية. كذلك فإن هذه المواد تلعب دوراً فعالاً في إتقان حياة الناس من خلال نظم التصوير. إن نقلة نوعية في الاتصالات والإنترنت سيحتاج إلى هذه المواد.

من المبیني عليه أن الموصلية الفائقة في هذه المواد، النوع الثاني من الموصلات الفائقة، تظهر في طبقات أكاسيد النحاس. و بما أنها من النوع الثاني من الموصلات الفائقة فإن المجال المغناطيسي يدخلها في صورة كميات محدودة تسمى دوامات دون أن تدمر تماماً صفة الموصلية الفائقة لها، لكنها تنتج بعض المقاومة. من أجل التغلب على هذه المشكلة، يجب أن تحتسب هذه الدوامات في مكانها فمنع حركتها وبالتالي يتم التخلص من المقاومة التي تنتج عن هذه الحركة.

في هذه الأطراف نحالى هذه الدوامات المغناطيسية في الموصلات الفائقة ذات السمك المحدود في وجود مراكز حبس ثابتة التوزيع في كل طبقة لكنها عشاء عين من طبقة لأخرى لترى تأثيرها على الموصلية في الطبقات الفائقة التوصيل.

أولاً: نتظر بتباين ثابتين متعادلين في اتجاهين متصادين على الطبقات الخارجية للموصلي في حالتين مختلفتين (1) دون مراكز حبس (2) في وجود مراكز حبس. و نحسب تيار التفكك بين دوامات الطبقة الخارجية وباقي الطبقات مقابل المجال المغناطيسي و طبقات الموصل الفائق. كذلك نتظر بتباين على أكثر من طبقة خارجية في كل طرف و نحسب تيار التفكك في كل حالة. و قد وجدنا أنه كلا ما زاد عدد الطبقات المتأثرة بتباين المتصادين كما انخفضت قيمة تيار التفكك. أيضاً لم يتسكع تيار التفكك في حالة وجود مراكز الحبس.

ثانياً: نتظر بتباين ثابت على الطبقة العلوية للموصلي الفائق و نحسب سرعة الدوامات المغناطيسية في كل من الطبقة العلوية و السفلية و فرق المسافة بين الدوامات المغناطيسية في الطبقة العلوية و التي تليها و فرق الجهد في الطبقتين العلوية و السفلية، و معدل الفيض المغناطيسى في الطبقة العلوية، و الفرق في معدل الفيض المغناطيسى بين الطبقة العلوية و السفلية في حالة
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