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Abstract

In this paper the development of a new rectangular flat shell element is proposed. This element is called SBRPK-SBRIE. This element is used in the numerical analysis of thin structures based on the strain approach with linear elastic behavior. Combining bending and membrane elements yields the proposed element.The strain-based rectangular finite element for the thin plate bending element denoted SBRPK, and the strain-based membrane element denoted SBRIE. Several numerical examples have been conducted to assess the accuracy and reliability of the developed element compared with the theoretical results and other finite elements. Obtained results show its good performance compared to other elements in the literature.

Keywords: Flat shell element, Membrane element, Plate bending element, Strain approach, Thin structures

1 Introduction

Today, finite elements are used in most areas of engineering, from calculations of electromagnetic radiation from antennas to the fluid-structure interaction between the sea and a sailboat. It was in the field of structural mechanics that the Finite Element Method (FEM) was discovered and, even today, it is in this field that it is the most widely used and where the amount of research work is the greatest. This method as known today was presented in 1956 by Turner, Clough, Martin and Topp [1] and was first applied to the analysis of aircraft structural problems. Simultaneously, a new approach to the elements has been developed at Cardiff University (U.K.), known as the strain approach. This approach has been applied by Sabir and Ashwell [2], to develop a new class of elements for plane elasticity problems in Cartesian coordinates, then Sabir (1983) to develop a new class of elements for general elasticity problems [3], [4]. Belarbi and Maalem have developed a new element with static condensation for plane elasticity [5]. In 2016, Hamadi et al developed a new finite element for plane elasticity problems [6]. For plate bending problems, Belarbi and Charif have developed a hexahedral finite element "SBH8" based on the strain approach [7]. Belounar and Guenfoud formulated a new element for bending thick plates [8]. In 2016, Abderrahmani et al developed a finite element for bending thin plates [9], [10]. For the modeling of the shells, Belarbi (2000) formulated a flat quadrilateral shell element named ACM-SBQ4, obtained by superimposing the standard membrane element SBQ4 with the plate bending element ACM [11] , After that, in 2007, A.I. Mousa and M.H. El Naggar developed a new spherical rectangular finite element based on the formulation of lowered shells [12]. Abed-Meraim et al. (2013) new quadratic solid–shell elements and their evaluation on linear benchmark problems [13]. Hamadi et al. developed a finite element model for thin-shell structure linear analysis. [14]. Yan et al [26] proposed the HDF-HSF quadrilateral flat shell element, which was created by combining a hybrid displacement function plate bending element with a hybrid stress function membrane element. After that, in 2018, Guenfoud et al developed a new triangular thin flat shell finite element with drilling rotation based on the strain approach[27]. In 2019 Labiodh et al developed a new triangular finite element called DKT18RF, is a shell type element without rotation degree of freedom [15]. Simultaneously, Sangtarash et al. proposed a novel four-node quadrilateral element for shell structure analysis [28]. Sangtarash et al (2020) also developed a new three-node triangular Mindlin–Reissner flat shell element [29].

Numerical analysis of shell structures in large-scale industrial problems is always a challenge and received on an ongoing based on strong interest. Over the past three decades, finite element method (FEM) has been used as a numerical tool powerful to simulate the behavior of shell structures.

Shell elements are widely used to model curved geometry structures, they are based on classical shell theory and are very efficient, but difficult to develop. There are four types of shell elements: planar elements, curved elements, solid and axisymmetric elements. These elements can be classified according to the thickness of the shell and curvature of the average sheet. Planar shell elements were developed by combining membrane elements and plate bending elements. Reasoning by the superposition of two elements (flexural and membrane) [16].

This paper presents the formulation of a new flat shell strain based rectangular shell element called SBRPK-SBRIE for the analysis of thin structure. The element is obtained by superposition of the strain-based membrane element (SBRIE) and the strain based rectangular plate bending element (SBRPK). We used numerical problems to evaluate the performance of the proposed flat shell element. The result were very encouraging.

2 Formulation

The proposed rectangular shell element is a combination of a (**SBRIE)** strain based membrane element and (**SBRPK)** strain based plate bending element. The stiffness matrix of the shell element **SBRPK-SBRIE** is a combination of those elements, as shown in Fig.1

The shell elements generally have six degrees of freedom (d.o.f) in each node:

u_i, v_i, w_i Are the translations d.o.f

θ_{xi}, θ_{yi} Are the rotations d.o.f

θ_{xi} Fictive rotation

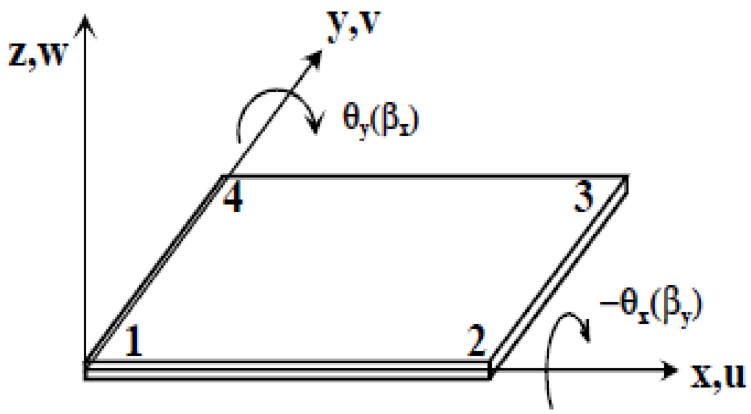

Plate bending element (SBRPK)

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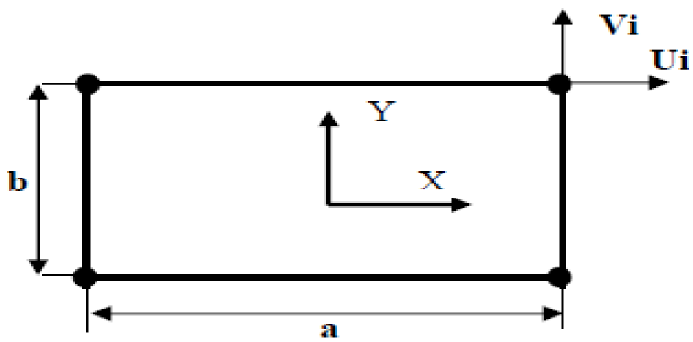

Membrane element (SBRIE)

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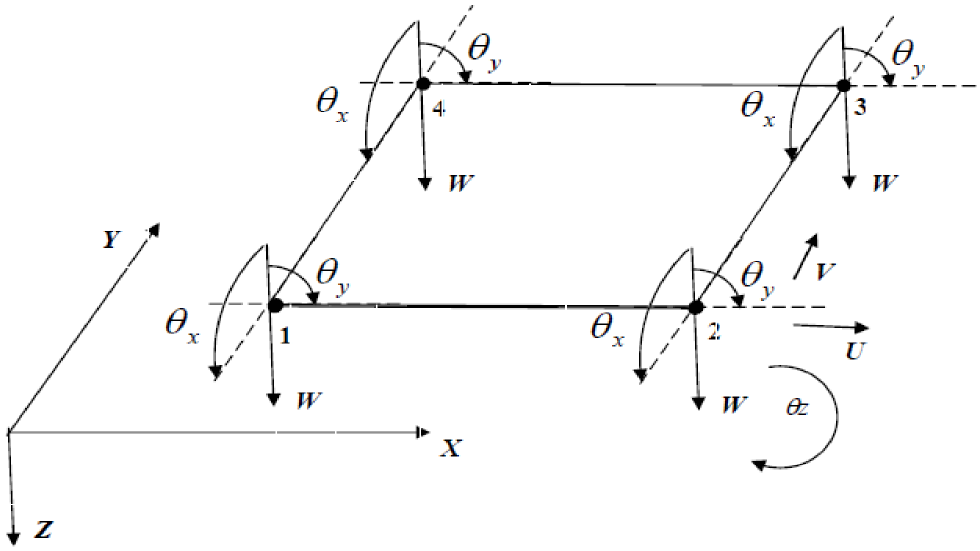

Flat shell element (SBRPK-SBRIE)

Figure 1: Construction of the shell element (SBRPK-SBRIE)

The bending stiffness matrix for each node is of order 3x3 and is symbolized by$[K_f]_{3x3}$.

The membrane stiffness matrix for each node is of order 2x2 and is represented by$[K_m]_{2x2}$.

The stiffness matrix for each node of the shell element is of order 6x6 can thus be represented by$[K_c]_{6x6}$.

$$[K_c]_{6x6} = \begin{bmatrix} [K_m]_{2x2} & [0]_{2x3} & 0 \\ [0]_{3x2} & [K_f]_{3x3} & 0 \\ 0 & 0 & K_r \end{bmatrix}$$

Or:

K_r : is the value of the fictitious stiffness added in each node.

2.1 Bending element (SBRPK)

Abderrahmani et al. [9] have developed a new rectangular finite element for thin plate bending based on the strain approach, with four nodes and 12 degrees of freedom, called SBRPK. As shown in Fig. 2. Analytical integration was used to evaluate the element stiffness matrix.

The displacement field:

$$w = a_1 - a_2x - a_3y - a_4\frac{x^2}{2} - a_5\frac{x^3}{6} - a_6\frac{x^2y}{2} - a_7\frac{x^3y}{6} - a_8\frac{y^2}{2} - a_9\frac{xy^2}{2} - a_{10}\frac{y^3}{6} - a_{11}\frac{xy^3}{6} - a_{12}\frac{xy}{2}$$

(1a)

$$\beta_x = a_2 + a_4x + a_5\frac{x^2}{2} + a_6xy + a_7\frac{x^2y}{2} + a_9\frac{y^2}{2} + a_{11}\frac{y^3}{6} + a_{12}\frac{y}{2}$$

(1b)

$$\beta_y = a_3 + a_6\frac{x^2}{2} + a_7\frac{x^3}{6} + a_8y + a_9xy + a_{10}\frac{y^2}{2} + a_{11}\frac{xy^2}{2} + a_{12}\frac{x}{2}$$

(1c)

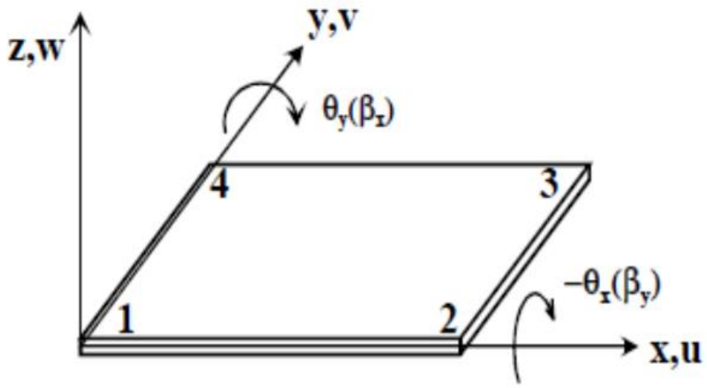

Figure 2: Geometry of the Rectangular plate element SBRPK

2.2 Membrane element (SBRIE)

The rectangular element SBRIE (Strain- Based Rectangular In-plane Element) [17] with two degrees of freedom (u_i, v_i) at each of the four corner nodes is schematically shown in Fig.3

The displacement field:

$$\begin{cases} U = a_1 - a_3 y + a_4 x + a_5 xy - a_7 \frac{y^2}{2} + a_8 \frac{y}{2} \\ V = a_2 + a_3 x - a_5 \frac{x^2}{2} + a_6 y + a_7 xy + a_8 \frac{x}{2} \end{cases}$$

(2)

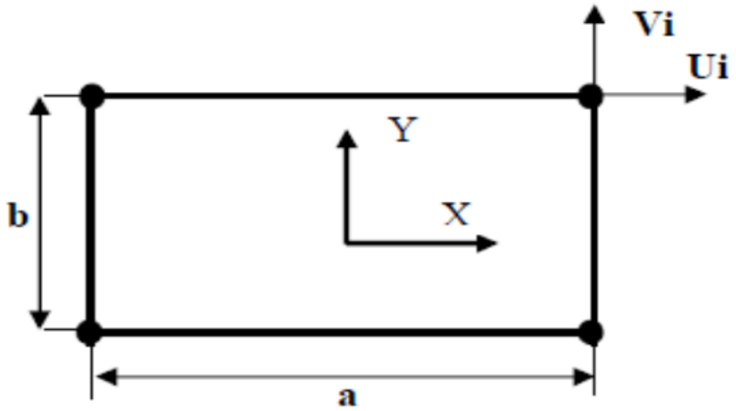

Figure 3: SBRIE element and coordinate system.

3 Numerical examples

The performance of this element **SBRPK-SBRIE** is estimated through a series of standard tests to show the use of the strain model.

The numerical results of several elements are used and compared with those obtained from this element and they are listed as follows:

SBH8: strain based hexahedron 8-node, is a three-dimensional element having eight nodes and 24 degrees of freedom, that is formulated for the analysis of thin and thick plates [7].

SBRP: strain based rectangular plate bending element, with 4 nodes and 12 degrees of freedom used for the analysis of thin and thick plates [8].

SBRPK: rectangular finite element for thin plate bending based on the strain approach with linear elastic behavior [9].

SBRIE1: the strain based rectangular in-plane element with an internal node [17].

SBRIER: The four nodes strain based rectangular in-plane element with an in-plane rotation with 12 DOFs [4].

ALLMAN: Allman proposed a triangular element with vertex rotation (3-DOFs).[18].

HEX20: quadratic solid–shell elements [13].

DKT18FR: shell finite element with no degree of freedom of rotation [15].

ACM-SBQ4: A flat quadrilateral shell element proposed by Belarbi [11].

ACM-RSBE5: A rectangular flat shell finite element proposed by Hamadi [14].

3.1 The patch tests for plate bending and plane membrane elements

Since the parent plate and membrane components can strictly pass all patch tests for plate and membrane elements, respectively, the resulting flat shell element SBRPK-SBRIE can also pass these tests.

3.1.1 Square plate bending problem

To assess the proposed element's correctness, a simply supported plate with a central point load is analysed (fig 4). The plate considered has aspect ratios L/h=100. This example has been studied by several authors ([9], [8], [7].and [19]).

The plate is made of isotropic material with a constant thickness of 0.2 in and length of 20. The material properties, we have: P=1, v=0.3 and E=10^6.

Table 1 shows the obtained results. These values are calculated using the use of different meshes. It can be observed that the convergence is monotone. The SBRIE-SBRPK element has a 1.33 percent inaccuracy when compared to the analytical value. This result shows the good performance of this element.

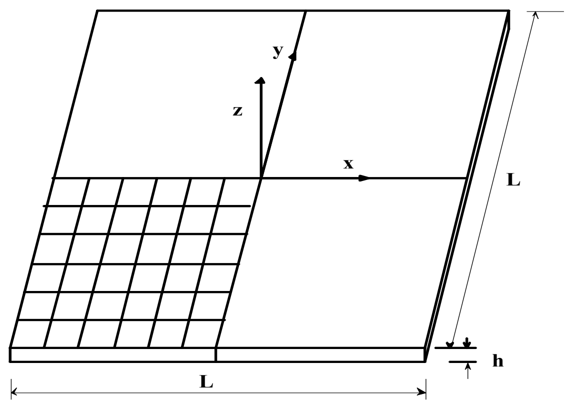

Figure.4: Geometry of the plate

Table 1: Simply support plate with central point load

Mesh	W_{max} (Normalized value)			
	SBH8	SBRP	SBRPK	SBRPK-SBRIE
2x2	0.220396	0.15762	0.80095	1.199
4x4	0.738189	0.72841	0.93695	1.0631
8x8	0.96270	0.96242	0.980224	1.0197
10x10	0.980655	0.98051	0.98670	1.0133
Analytical solution $\left[\frac{PL^2}{D}10^{-4}\right]$[19]	116.0			

3.1.2 Mac-Neal's elongated cantilever beam

The problems are presented to validate this element(SBRIE-SBRPK) . has been treated by Mac-Neal [20].The geometrical and materials characteristics of the structure are shown in Fig(5).

The results presented in Table 2 show that this element gives better results than all other elements (SBRIEIR, SBRIE1 and SBRIE).

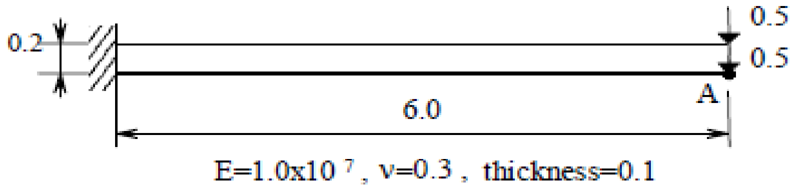

Figure 5: An elongated thin cantilever beam subjected to end shear

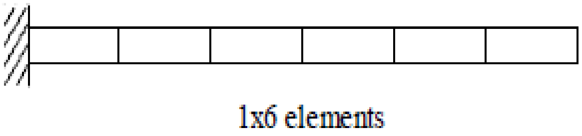

Figure 6: Example of mesh division

Table 2: Normalized deflection at point A, of a thin cantilever beam under shear

Normalized deflection				
Mesh	SBRIE	SBRIE1	SBRIEIR	SBRPK-SBRIE
1 × 6	0.9035	0.9035	0.9035	0.9861
1 × 12	0.9083	0.9083	0.9083	0.992
Analytical solution (0.1081)				

3.1.3 Short cantilever beam of Allman

The short cantilever beam is subjected to a uniform vertical load (fig 7).is a standard problem to test finite element accuracy. The parameters are shown in figure 7.

The results presented in Table 3 show that the SBRIE-SBRPK element gives the most accurate results than the other elements , the error is reduced to : 5.206 %

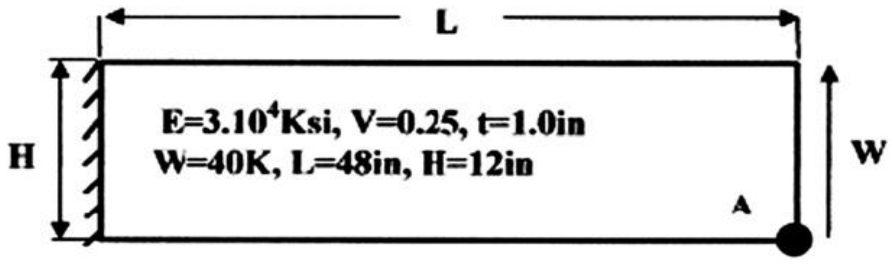

Figure 7: Cantilever beam under a tip load

Table 3: Vertical displacement at point A

Mesh	SBRIE	ALLMAN	SBRIEIR	SBRPK-SBRIE
4×1	0.3293	0.3027	0.3300	0.3368
Analytical solution [21] (0.3553).				

3.2 Clamped cylindrical shell with rigid diaphragm

The problem to be considered is that of a clamped cylindrical shell, as shown in Fig(8), which is supported by two rigid diaphragms and is subjected to two concentrated forces, is a standard problem for evaluating the performance of the thin shell element (R/h=100) in a bending-dominated problem with complex membrane and inextensible bending. Only 1/8 (ABCD) is examined due to the cylinder's symmetry. The symmetry conditions are: $W = \theta_Y = \theta_X = 0$ at AB, $V = \theta_X = \theta_Z = 0$ at BC and $U = \theta_Y = \theta_Z = 0$ at CD. The boundary condition: U= $W = \theta_Y = 0$ at AD. Geometry and properties are indicated in Fig 8.b

The results of this analysis are compared to the analytical solution based on the thin shell structures (R/h=100) given by Flugge [22] and Lindberg *et al* [23] below:

WC = -WC Eh/P = 164, 24 deflection under load P in point C only

The numerical results for this element SBRPK-SBRIE are compared with the reference solution, and numerical results reported in the literature of elements HEX20, DKT18FR, ACM-SBQ4 and ACM-RSBE5 (Table 4)

From the results obtained and shown in Table 4, the finest mesh used for the HEX20 element (20x4); the error obtained is 32.5%, the DKT18RF element (6x6); the error obtained is 8.6 %, the ACM-SBQ4 element (20x4); the error obtained is 4.4% and the ACM-RSBE5 element (20x4); the error obtained is 1.6% ,while that of this element (8x8) is 0.9, which represents an excellent performance of developed element.

The results obtained by element SBEPK-SBRIE show a monotonous convergence of displacement and agree well with the analytical solution. This result shows the good performance of this element.

Table 4: Clamped cylindrical shell, convergence of WC

Mesh	W_C (Normalized value)				
	HEX20	DKT18RF	ACM-SBQ4	ACM-RSBE5	SBRPK-SBRIE
4x4	0.140	0.763	0.618	0.649	1.183
6x6	0.328	0.914	0.821	0.842	1.11
8x8	0.523	-	0.904	0.955	1.009
20x4	0.675	-	0.956	0.984	-
Analytical solution	164.24				

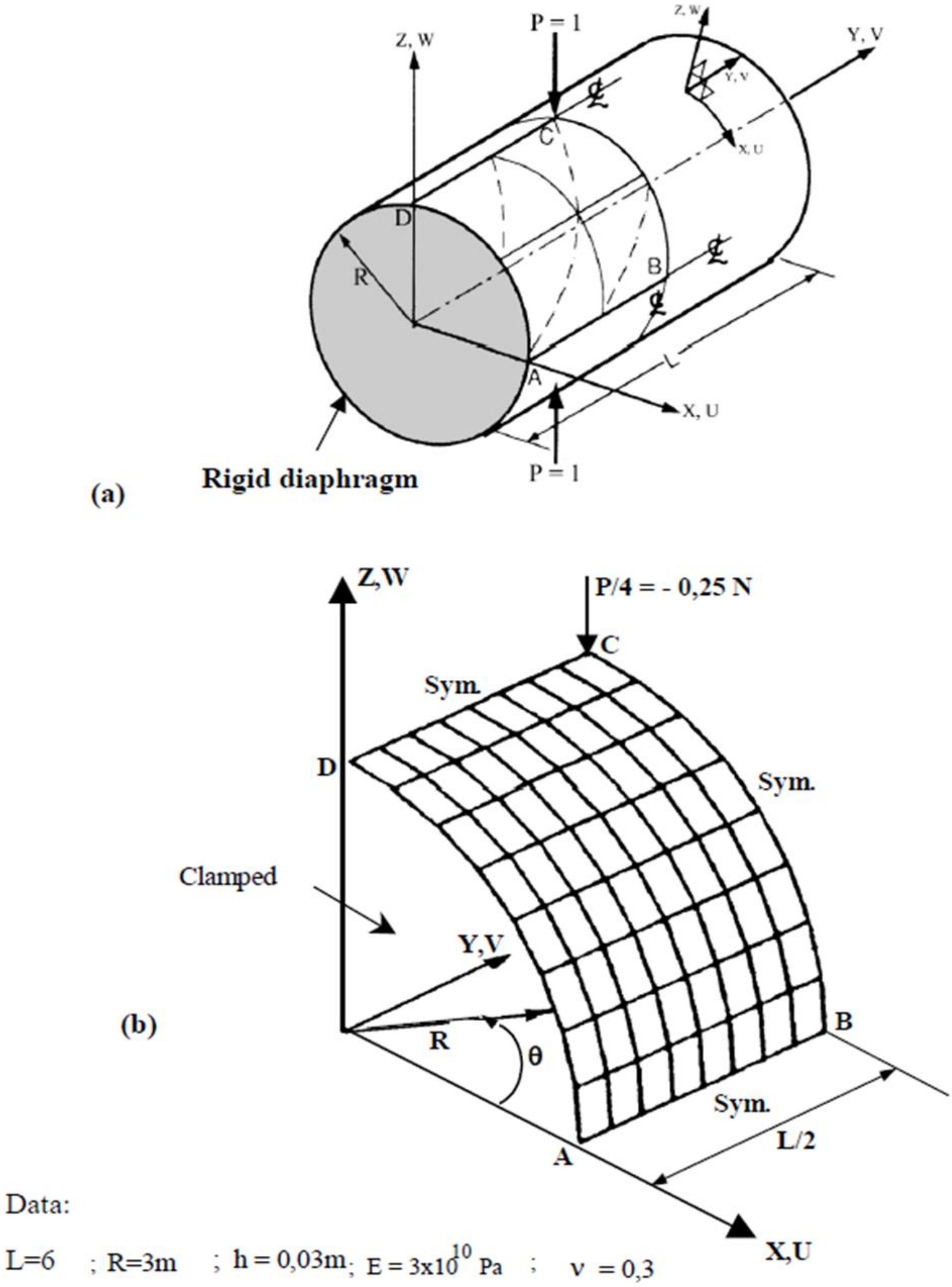

Figure 8: Clamped cylindrical shell

3.3 Pinched cylinder with free edges

In this example, a pinched cylinder with free edge is studied. The geometry and material properties are given in Fig (9). By reason of symmetries, Only (1/8) of the shell is studied by the element SBRPK-SBREI with different types of mesh.

The results of the SBRPK-SBRIE element (Table 5) are compared with the reference solution [20] with the other finite elements available in the literature. The results for deflections for the refined mesh (1x9) are in good agreement with an accurate reference solution.

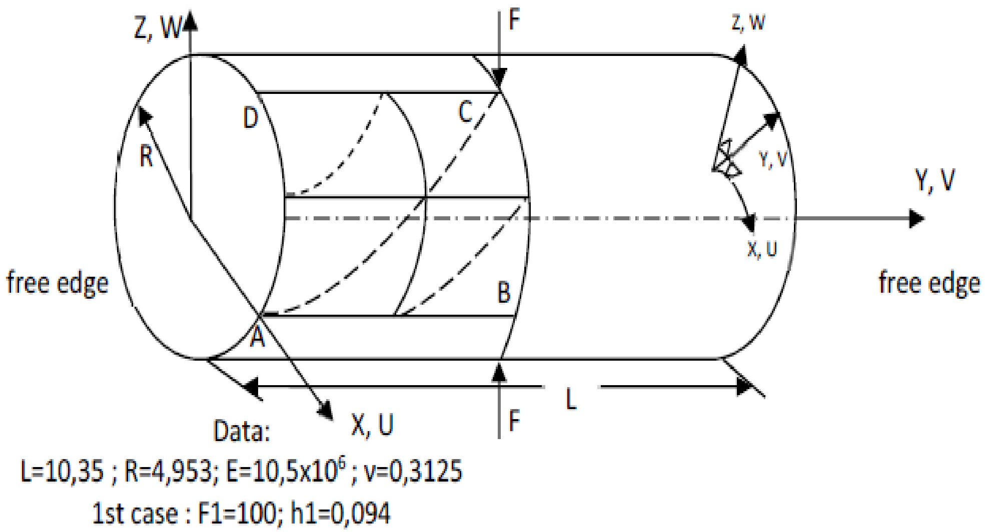

Figure 9: Pinched cylinder with free edges

Table 5: Pinched cylinder with free edges, convergence of WC

Mesh	W_C					
	[24]	[25]	DKT18RF	ACM-SBQ4	ACM-RSBE5	SBRPK-SBRIE
1x1	0.0025		2.75	0.0860	0.08763	1.57
1x3	0.1026	0.0297	0.7255	0.1041	0.1060	0.2387

1x5	-	0.0769	0.4077	0.1090	0.1116	0.1991
1x7	-	0.0987	0.2666	0.1102	0.1129	0.1286
1x9	-	0.1057	0.1852	0.1115	0.1134	0.1188
3x9	-	-	0.1153	-	-	-
Reference solution	0.1139					

4 Conclusions

The formulation of a flat rectangular thin shell element called SBRPK-SBRIE based upon the strain approach has been successfully developed and presented in this paper, is proposed by the combination of SBRIE membrane element and SBRPK plate bending element. Both are founded on a strain based formulation. The proposed flat shell element's performance is evaluated using a set of numerical benchmark problems. Numerical results obtained using the new element show its good performance in terms of accuracy and rate of convergence to the reference solutions for all tests. The shell elements resulting from this combination are robust, competitive and efficient. The performances of the SBRPK-SBRIE element have been demonstrated. This element can easily be coded up and integrated in to research/commercial soft-ware.

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