

August 2023

## Evaluation of Reliability Indices for Gas Turbines Based on the Johnson SB Distribution: Towards Practical Development

ELBAR Mohamed

*Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa 17000 DZ, Algeria, m.elbar@univ-djelfa.dz*

DJEDDI Ahmed Zohair

*Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa 17000 DZ, Algeria, az.djeddi@univ-djelfa.dz*

Ahmed Hafaifa

*1 Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa 17000 DZ, Algeria. 2 Department of Electrical and Electronics Engineering, Faculty of Engineering and Architecture, İstanbul Nisantasi University, 34398 Sarıyer, İstanbul, Turkey, a.hafaifa@univ-djelfa.dz*

CHARRAK Naas

*Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa 17000 DZ, Algeria, n.charrak@univ-djelfa.dz*

Abdelhamid Iratni

For more information and additional works at: <https://scholarworks.uaeu.ac.ae/ejer>

 Faculty of Science and Technology, University of Bordj Bou Arreridj, 34030 DZ, Algeria, iratni@univ-bba.dz  
Part of the [Risk Analysis Commons](#)

---

See next page for additional authors

### Recommended Citation

Mohamed, ELBAR; Ahmed Zohair, DJEDDI; Hafaifa, Ahmed; Naas, CHARRAK; Iratni, Abdelhamid; and colak, ilhami (2023) "Evaluation of Reliability Indices for Gas Turbines Based on the Johnson SB Distribution: Towards Practical Development," *Emirates Journal for Engineering Research*: Vol. 28: Iss. 2, Article 5.

Available at: <https://scholarworks.uaeu.ac.ae/ejer/vol28/iss2/5>

This Article is brought to you for free and open access by Scholarworks@UAEU. It has been accepted for inclusion in Emirates Journal for Engineering Research by an authorized editor of Scholarworks@UAEU. For more information, please contact [EJER@uaeu.ac.ae](mailto:EJER@uaeu.ac.ae).

---

## Evaluation of Reliability Indices for Gas Turbines Based on the Johnson SB Distribution: Towards Practical Development

### Authors

ELBAR Mohamed, DJEDDI Ahmed Zohair, Ahmed Hafaifa, CHARRAK Naas, Abdelhamid Iratni, and ilhami colak

## Evaluation of Reliability Indices for Gas Turbines Based on the Johnson SB Distribution: Towards Practical Development

Mohammed Elbar <sup>1</sup>, Ahmed Zohair Djeddi <sup>1,2</sup>, Ahmed Hafaifa <sup>1\*,2,3</sup>, Naas Charrak <sup>1</sup>, Abdelhamid Iratni <sup>4</sup>,  
Ilhami Colak <sup>3</sup>

<sup>1</sup> Applied Automation and Industrial Diagnostics Laboratory, Faculty of Science and Technology, University of Djelfa, 17000 DZ, Algeria

<sup>2</sup> Joint Research Team on Gas Turbines, University of Djelfa, 17000 DZ, Algeria  
Emails: [m.elbar@univ-djelfa.dz](mailto:m.elbar@univ-djelfa.dz); [a.z.djeddi@univ-djelfa.dz](mailto:a.z.djeddi@univ-djelfa.dz); [a.hafaifa@univ-djelfa.dz](mailto:a.hafaifa@univ-djelfa.dz); [n.charrak@univ-djelfa.dz](mailto:n.charrak@univ-djelfa.dz)

<sup>3</sup> Department of Electrical and Electronics Engineering, Faculty of Engineering and Architecture, İstanbul Nisantasi University, 34398 Saryer, İstanbul, Turkey  
Emails: [ahmed.hafaifa@nisantasi.edu.tr](mailto:ahmed.hafaifa@nisantasi.edu.tr); [ilhcol@gmail.com](mailto:ilhcol@gmail.com)

<sup>4</sup> Department of Electrical Engineering, Faculty of Science and Technology, University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj, 34030 DZ, El Anasser, Bordj Bou Arreridj, Algeria  
Email: [iratni@univ-bba.dz](mailto:iratni@univ-bba.dz)

(Received 27 Mars 2023 and Accepted on 25 July 2023)

### تقييم مؤشرات الموثوقية للتوربينات الغازية بناءً على توزيع جونسون SB : نحو تطوير عملي

#### ملخص

قدمت التطورات الحديثة في هندسة الكمبيوتر حلولاً فعالة لمعالجة وتحليل الأنظمة المعقدة والبيانات الضخمة. وبالتالي، فإن تعديل هذه البيانات وتوحيدها يلعب دوراً حاسماً في معالجة القضايا المتعلقة بمراقبة الأنظمة الصناعية. في هذه الدراسة، نقترح نهجاً موثقاً لتوربينات الغاز لتحديد وتوصيف تدهورها باستخدام البيانات التشغيلية. نقدم طريقة لضبط بيانات موثوقية التوربينات، والتي تحل التحديات المرتبطة بطبيعة بيانات التشغيل هذه. يتيح لنا ذلك تحديد وظيفة رياضية تقوم بنمذجة العلاقات بين معلمات موثوقية التوربينات وتقييم تأثير ممارسات الموثوقية من حيث التوافر. بالإضافة إلى ذلك، نحدد وظيفة البقاء على قيد الحياة ونستخدمها كنموذج توزيع مدى الحياة من خلال تقدير معلمات وظيفة Johnson SB. علاوة على ذلك، نحسب معدلات الفشل ومتوسط الوقت بين العمليات الجيدة لهذه الآلة الدوارة في ظل ظروف تشغيل مختلفة. تسمح لنا النتائج التي تم الحصول عليها بتقدير معلمات التوزيع التي تناسب بيانات موثوقية التوربين، والتي يتم التحقق من صحتها من خلال الاختبارات الإحصائية والرسومومية. نقوم بتقييم جودة الملاءمة باستخدام متوسط الخطأ التربيعي واختبارات الموثوقية مثل Kolmogorov-Smirnov.

#### Abstract

Recent advancements in computer engineering have provided effective solutions for processing and analyzing complex systems and big data. Consequently, the adjustment and standardization of this data play a crucial role in addressing issues related to the monitoring of industrial systems. In this study, we propose a reliability approach for gas turbines to identify and characterize their degradation using operational data. We introduce a method for adjusting turbine reliability data, which resolves the challenges associated with the nature of these operating data. This enables us to determine a mathematical function that models the relationships between turbine reliability parameters and evaluate the impact of reliability practices in terms of availability. Additionally, we determine the survival function and employ it as a lifespan distribution model by estimating the parameters of the Johnson SB function. Furthermore, we calculate the failure rates and mean time between good operations for this rotating machine under different operating conditions. The obtained results allow us to estimate the parameters of the distribution that best fit the turbine reliability data, which are validated through statistical and graphical tests. We assess the goodness-of-fit using mean square error and reliability tests such as Kolmogorov-Smirnov.

*Keywords: Reliability, Johnson SB distribution, gas turbine, survival function, parameter estimation.*

## 1. Introduction

The practice of reliability in modern industries plays a crucial role in meeting production requirements with the optimal quality and price ratio. Maintaining industrial equipment necessitates the implementation of operational safety strategies to ensure productivity and analyze operational behavior. To achieve this, a reliability approach is proposed in this work to evaluate the reliability indices of a gas turbine based on the Johnson SB distribution, utilizing the turbine's operating data with proper adjustment. The objective is to analyze and identify failures occurring during turbine operation and assess the significance of reliability parameters to make informed decisions regarding maintenance programming.

Several studies have been conducted in this area. Ahmed Zohair Djeddi et al. [2, 4] conducted an operational reliability study of a gas turbine using the three-parameter Weibull approach, and in [3, 5], they performed a comparative analysis by integrating the Weibull distribution to minimize turbine faults. Ahmed Hafaifa et al. [1], Rachid Belhadef et al. [33], and Nadji Hadroug [31] proposed reliability engineering techniques utilizing fuzzy logic. Ahmed Zohair Djeddi et al. [8] introduced approaches based on long-short-term memory networks and artificial neural networks. Other works focused on improving gas turbine reliability, such as Esakki Muthu [14], Egorov and Rabinovich [13], Choayb Djeddi et al. [11], Mouloud Guemana et al. [30], and Poozesh Mohammad et al. [32].

However, after conducting a comprehensive analysis of reliability data, the determination of reliability parameters requires the selection of a robust and practical distribution. This distribution should effectively model system failures and improve maintenance strategies by incorporating preventive and corrective practices. Various mathematical approaches, particularly stochastic distributions, have been employed to represent reliability distribution models. These include the exponential distribution, the Weibull distribution and its variations, as well as other commonly used reliability functions like the Gamma distribution, extreme value distribution, Birnbaum-Saunders distribution, and Log-normal distribution. Examples of such studies include the works of Ahmed Zohair Djeddi et al. [6-7], Ahsan Shazaib et al. [9], Jiang Renyan [20], Jinyuan Shi [21], Li Chun-Qing and Wei Yang [27], and Mingazov and Korobitsin [29].

The Johnson distribution, initially studied by Johnson Norman L. in 1949 [22], is a family of four-parameter probability distributions that serves as a transformation of the normal distribution. Since then, this transformation has been explored in

various studies. Florence George et al. [15] conducted parametric estimation using the Johnson system, and the selection of parameters for the Johnson transformation was studied by Slifker James and Samuel Shapiro [36]. Stanfield et al. [37] proposed modeling the inputs of a multivariate system using Johnson distributions. The Johnson SB distribution has found extensive application in representing empirical distributions of data characteristics. For instance, Flynn Michael [16] proposed data adjustment using the Johnson SB distribution for human exposure, Jordyn Koenig et al. [23] employed this distribution to model trunk kinematics, and Kamziah Abd Kudus et al. [24] estimated a nonlinear regression using the Johnson SB distribution. Jiang Chen et al. [19] conducted a systematic analysis of particle size effects based on the Johnson SB approach. Other applications include Hong Tang and Guowei Liang [17] and Rennolls Keith and Mingliang Wang [34].

In this context, the adjustment of gas turbine reliability data is performed using the Johnson SB distribution to model the relationships between reliability parameters and evaluate the impact of reliability practices on availability. This approach enables the survival analysis of the rotating machine and simplifies the processing of operating data, ensuring stable operation with maximum uptime and performance improvements using the Johnson SB distribution.

## 2. Johnson distributions for reliability assessment

In various fields, it is common to describe the life cycle of equipment using standard distributions that capture different phases: the period of youth, the period of useful life, and the period of wear or aging. In this work, we propose the utilization of the Johnson SB method to develop reliability indices for the studied gas turbine by adjusting its data. The Johnson SB method, introduced by Johnson in 1949 [22], involves a transformation that allows for improved fitting of statistical data by considering the four moments of a distribution (mean, standard deviation, asymmetry, and kurtosis). This probabilistic approach has been applied in numerous industrial applications, further enhancing its relevance.

Several studies have explored this method to advance its application in various contexts. For instance, Huynh Ngoc Phien and Wen-Bin Chow [18], Cugerone Katia and Carlo De Michele [12], Artur Lemonte and Germán Moreno-Arenas [10], Kottegoda [25], Leo Pio D'Adderio et al. [26], Mage David [28], Scolforo José Roberto Soares et al. [35], Takvor Soukissian [38], Teresa Fidalgo Fonseca et al. [39], and Zhang Lianjun et al. [40] have conducted studies to develop and refine this

probabilistic approach within different industrial domains.

In Johnson's concept, the transformed data should closely follow the line of a fitted normal distribution. This Johnson system contains four families of distributions which are generated by transformations of the form, given by equation (1), proposed in [22] and developed by [37-38, 40], is given by the following form :

$$z = \gamma + \eta k_i(x; \lambda, \varepsilon) \tag{1}$$

We take :

$$k_i(x; \lambda, \varepsilon) = f(u) \text{ with } u = \frac{x - \varepsilon}{\lambda} \tag{2}$$

where  $z$  is a standard normal random variable,  $x$  it is random variable,  $\eta$  it is a positive definite continuous form parameter,  $\gamma$  it is a continuous form parameter,  $\lambda$  is a positive definite scale parameter,  $\varepsilon$  is a localization parameter denoted also  $\xi$ ,  $f(\cdot)$  represents the four possible forms of the Johnson transformation called the transformation function and  $k_i(x; \lambda, \varepsilon)$  are chosen to cover a wide range of possible forms.

The Johnson distributions family can take four functions according  $f(\cdot)$ , depending on the variation of this function given by the formula (2).

The first form if  $f(u) = \ln(u)$  is the log-normal distribution of Johnson SL, given by [37]:

$$k_3(x; \lambda, \varepsilon) = \ln\left(\frac{x - \varepsilon}{\lambda}\right) \tag{3}$$

The second form of the Johnson SU distribution is an unbounded continuous probability distribution of a random variable, with  $f(u) = \ln(u + \sqrt{1 + u^2})$  is positive values, is given as follows [37]:

$$k_1(x; \lambda, \varepsilon) = \sinh^{-1}\left(\frac{x - \varepsilon}{\lambda}\right) \tag{4}$$

The third form is the Johnson distributions SB a bounded probability distribution, with  $f(u) = \ln\left(\frac{u}{1-u}\right)$ , is given by the following formula [37]:

$$k_2(x; \lambda, \varepsilon) = \ln\left(\frac{x - \varepsilon}{\lambda + \varepsilon - x}\right) \tag{5}$$

And the last form is the Johnson SN distribution which follows the normal probability law, given by [37]:

$$f(u) = u \tag{6}$$

Practically, reliability is a quantitative quantity and requires the knowledge of the distributions of the probability density function (pdf), in order to estimate it for each distribution. This pdf function for the case of the Johnson SB approach is given by the following formula:

$$f(x; \delta, \gamma, \xi, \lambda) = \frac{\delta}{\lambda x(1-x)\sqrt{2\pi}} e^{-\frac{1}{2}\left[\gamma + \delta \ln\left(\frac{x}{1-x}\right)\right]^2} \tag{7}$$

With  $\lambda > 0$ ,  $\delta > 0$ ,  $\xi < x < \xi + \lambda$ .

where  $x$  is the random variable,  $\gamma$  and  $\delta$  are the shape parameters,  $\lambda$  is the scale parameter,  $\xi$  is the location parameter.

The Johnson SB probability density function can have different shapes depending on the scale parameter value  $\gamma$ , as shown in Figure 1, and depending on the lifetime of the studied system.

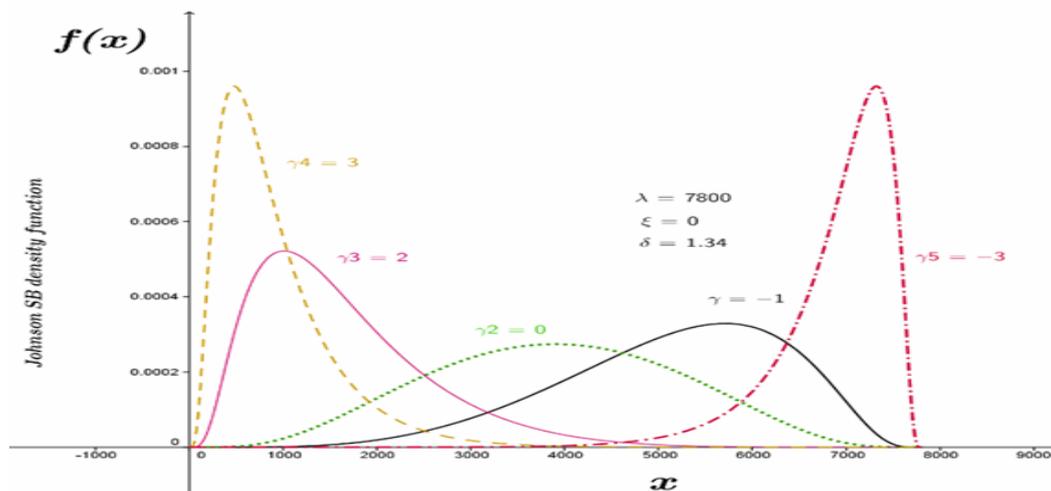
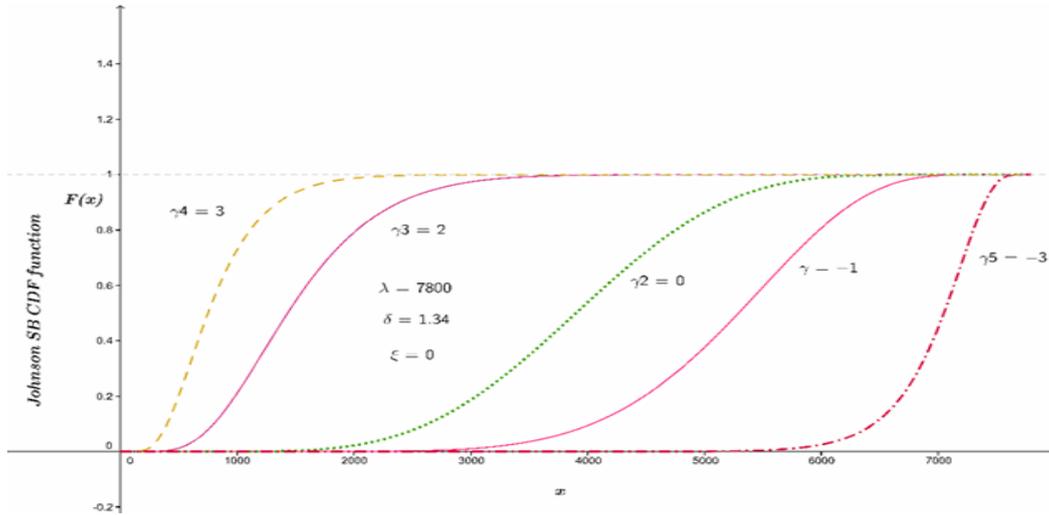


Figure 1: Variation of the probability density function (pdf) using the Johnson SB distribution

The cumulative effect of the probability density function is represented by the Johnson SB cumulative distribution function (CDF), as shown in Figure 2, given by the following formula:

$$F(x) = \Phi\left(\gamma + \delta \ln\left(\frac{x - \xi}{\xi + \lambda - x}\right)\right) \quad (8)$$

where  $\Phi$  is the cumulative distribution function of a standard normal distribution.

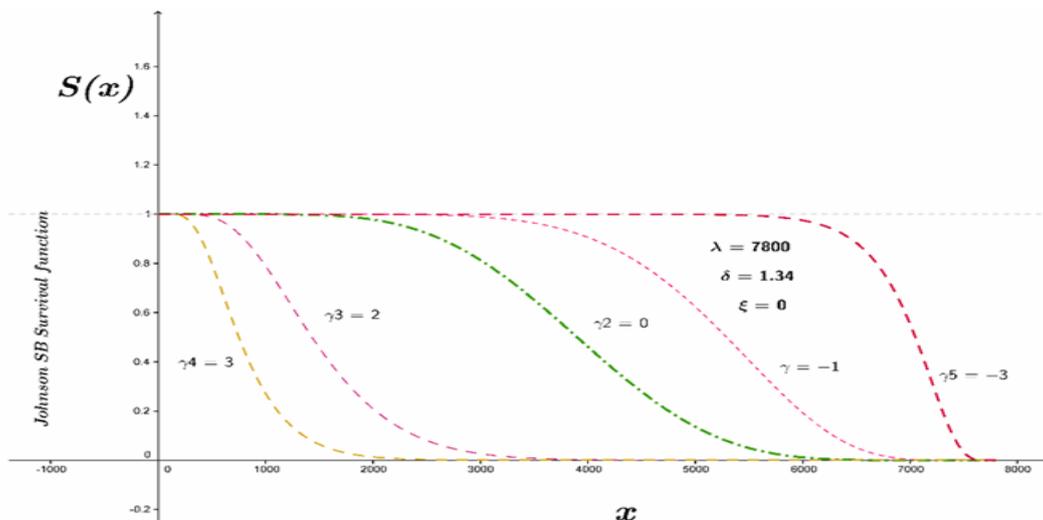


**Figure 2: Variation of cumulative distribution function (CDF) using Johnson SB distribution**

The variation of the survival function using the Johnson SB distribution is shown in Figure 3, this survival function is defined by:

$$S(x) = 1 - F(x) = 1 - \Phi\left(\gamma + \delta \ln\left(\frac{x - \xi}{\xi + \lambda - x}\right)\right) \quad (9)$$

This function represents the probability that the turbine does not have a failure, also called the reliability function. From this function we can deduce the system failure function, which is the distribution function and the probability density which is the derivative of this survival function.



**Figure 3: Variation of survival function using Johnson SB distribution**

The hazard function of the Johnson SB distribution model is expressed by equation (10) and is shown in Figure 4, it is obvious that the shape of this function depends only on the shape parameter. Therefore, this function represents the probability that at least one failure occurs during system operation. However, this risk probability

using the Johnson SB distribution is expressed by the following formula:

$$h(x) = \frac{f(x)}{S(x)} \tag{10}$$

with  $\xi < x < \xi + \lambda$ .

$$= \frac{\delta \cdot \lambda \cdot e^{-\frac{1}{2} \left[ \gamma + \delta \ln \left( \frac{x - \xi}{\xi + \lambda - x} \right) \right]^2}}{(x - \xi)(\lambda + \xi - x) \sqrt{2\pi} \left[ 1 - \Phi \left( \gamma + \delta \ln \left( \frac{x - \xi}{\xi + \lambda - x} \right) \right) \right]}$$

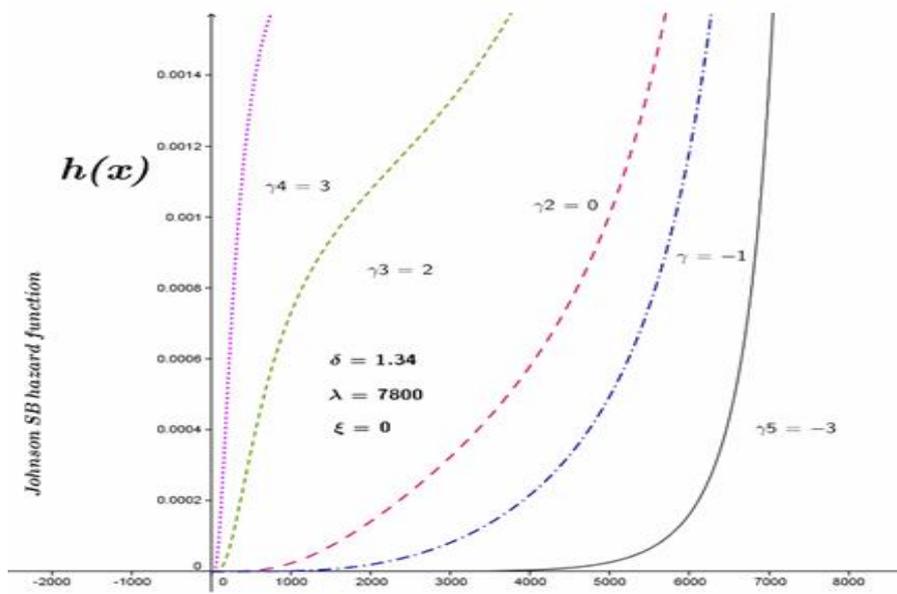


Figure 4: Variation of hazard function using Johnson SB distribution

It is obvious that, to obtain a better reading of the performance of the Johnson SB distribution, it is necessary to present the shape of the cumulative risk function, which is shown in Figure 5, this cumulative risk probability function is defined by the following relationship:

$$H(x) = -\ln(S(x))$$

$$= -\ln \left( 1 - \Phi \left( \gamma + \eta \ln \left( \frac{x - \varepsilon}{\lambda + \varepsilon - x} \right) \right) \right) \tag{11}$$

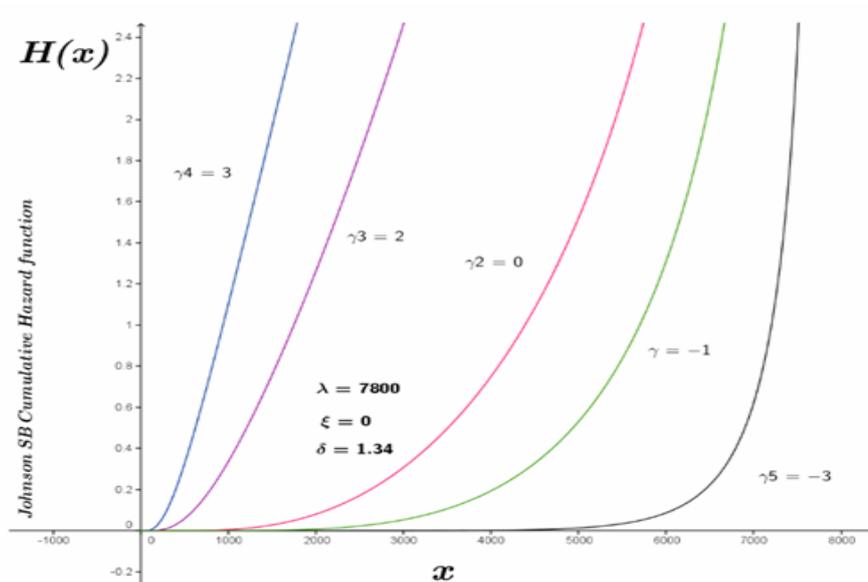


Figure 5: Cumulative hazard function variation using Johnson SB distribution

Therefore, the performance of the reliability estimation of a gas turbine engine based on the

Johnson SB distribution is investigated in the following sections. After selection and estimation

of the distribution parameters of Johnson SB applies the turbine reliability data examined with validation tests of the Kolmogorov-Smirnov type on the obtained reliability results.

### 3. Selection and estimation of reliability parameters based on the Johnson SB distribution

The procedure for selecting and estimating parameters by the Johnson SB distribution is done heuristically, that there should be a relationship between the distances in the tails and the distances in the central part of the distribution, which could be used to distinguish bounded from unbounded cases. This implied the following formalization [22]:

Firstly, assuming one of the transformations described by equation (1), we choose a fixed value of  $z > 0$  a standard normal random variable.

Thus, the four points of  $\pm z$  and  $\pm 3z$  give three intervals of equal length. The transformation (1) brings back four values of  $X$  which are no longer equidistant. It was assumed that for the bounded symmetric Johnson distribution, the distances between each outer and inner point would be less than the distance between the two inner points, and vice versa for the unbounded case.

Suppose that  $x_{-3z}$ ,  $x_{-z}$ ,  $x_z$  and  $x_{3z}$  are the values corresponding to  $-3z$ ,  $-z$ ,  $z$  and  $3z$  according to the transformation (1). We get :

$$\begin{cases} m = x_{3z} - x_z \\ n = x_{-z} - x_{-3z} \\ p = x_z - x_{-z} \end{cases} \quad (12)$$

For the choice of use of the Johnson SB distribution it is necessary that  $\frac{mn}{p} < 1$ , which is

verified, for the case of turbine data studied. Indeed, this specificity makes it possible, when applying this method, to distinguish between the four families of Johnson distributions and to know which distribution is the best for adjusting the reliability data.

For the selection procedure consists in the development of the following steps; The first step is the choice of an arbitrary value of  $z > 0$ , this choice is based on the number of data points to be used in the reliability study. Then in the second step, the percentile  $p_\xi$  corresponding to  $\xi = 3z$ ,  $\xi = z$ ,  $\xi = -z$ ,  $z$  and  $\xi = -3z$ , respectively is determined from a table of areas using the normal distribution. Then for each percentile  $\xi$  obtained from the data  $x^{(i)}$  corresponding to  $p_\xi$  is based on the following relationship:

$$\frac{\left(i - \frac{1}{2}\right)}{n} = x^{(i)} \quad (13)$$

Hence,  $x_\xi = x^{(i)}$  is the set of data points,  $x_\xi$  is the ordered observation  $r^{th}$  with  $i = np_\xi + \frac{l}{2}$  and  $i$  generally is not an integer, so should be interpolated.

And finally, the last step which is to calculate the sample values of  $m$ ,  $n$  and  $p$  from the values of the previous steps and use them to select the appropriate data family member.

### 4. Reliability evaluation of studied turbine

The production stability of gas compressor stations, using gas turbines, requires the implementation of robust monitoring approaches of their operation. For this purpose, the MS 5002C turbine reliability data set collected from the history of its operation, shown in Table 1, are used for the elaboration of these reliability parameters, these data contain the whole failures that appeared during the different periods of their operation. Hence, the proper functioning as well as the high performance of the MS 5002C gas turbine are the most sought-after means to ensure good production and transport of gas.

What is depends on the reliability factors, making it possible to program the maintenance actions to be carried out, to keep this turbine running. For this, based on the Johnson SB distribution and these properties, adjustments are made to determine and analyze the reliability indices of this turbine. In order to better know and understand the behavior and degradation phenomena observed during their operation.

Table 1. Reliability data for the studied gas turbine

Interval	Failure number	Class failures
0-590	2	0.05
590-1180	5	0.125
1180-1770	12	0.3
1770-2360	7	0.175
2360-2950	6	0.15
2950-3540	3	0.075
3540-4128	5	0.125

The obtained results concerning the main reliability performances of the studied turbine, based on Johnson SB are represented in Table 2, these parameters are obtained by calculations based on the real turbine data, using the estimation formulation of the distribution parameters of Johnson SB following:

$$z = \gamma + \eta \ln\left(\frac{x - \varepsilon}{\lambda + \varepsilon - x}\right) \quad (14)$$

With the continuous form parameter  $\eta$  is positive definite, given by:

$$\eta = \frac{z}{\cosh^{-1}\left(\frac{1}{2}\left[\left(1 + \frac{p}{m}\right)\left(1 + \frac{p}{n}\right)\right]^{\frac{1}{2}}\right)} ; \eta > 0 \quad (15)$$

And the shape parameter  $\gamma$  is determined by:

$$\gamma = \eta \sinh^{-1}\left[\frac{\left(\frac{p}{n} - \frac{p}{m}\right)\left(\left(1 + \frac{p}{m}\right)\left(1 + \frac{p}{n}\right) - 4\right)^{\frac{1}{2}}}{2\left(\frac{p}{m} \frac{p}{n} - 1\right)}\right] \quad (16)$$

Also, the scale parameter  $\lambda$  is positive definite, given by:

$$\lambda = \frac{p\left(\left(\left(1 + \frac{p}{m}\right)\left(1 + \frac{p}{n}\right) - 2\right)^2 - 4\right)^{\frac{1}{2}}}{\frac{p}{m} \frac{p}{n} - 1} ; \lambda > 0 \quad (17)$$

As well as, localization parameter  $\varepsilon$  also denoted  $\xi$ , is given by:

$$\varepsilon = \frac{x + x_{-t}}{2} - \frac{\lambda}{2} + \frac{p\left(\frac{p}{n} - \frac{p}{m}\right)}{2\left(\frac{p}{m} \frac{p}{n} - 1\right)} \quad (18)$$

By following Johnson SB's parametric reliability estimation procedure, we determine that with  $\frac{mn}{p^2} = 0.49 < 1$ , which confirms the correct choice of the reliability distribution. Hence, the examined turbine CDF cumulative distribution function is shown in Figure 6. Also, the quality of the adjustment of the examined turbine data is validated with the Kolmogorov-Smirnov test, with very satisfactory results obtained, given by Table 4.

Table 2. Obtained turbine reliability parameters based on Johnson SB

Johnson SB Distribution parameters	Validation	
	RMSE	MSE
$\gamma = 0.485340586$	0.0248307	0.0006166
$\eta = 0.750003713$		
$\varepsilon = 616.7630012$		
$\lambda = 4046.392651$		

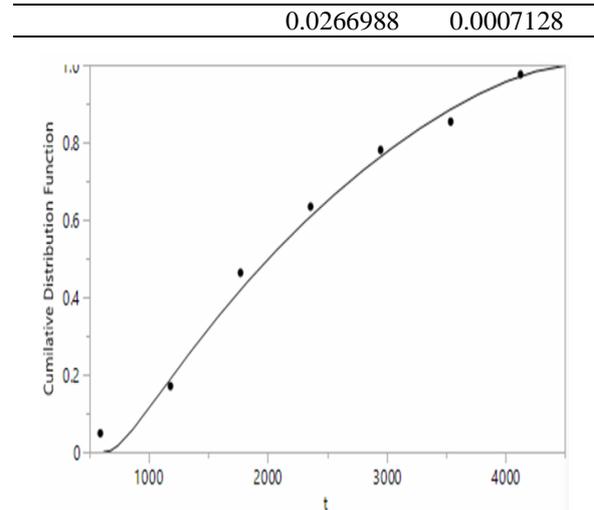


Figure 6: Cumulative distribution function

Unfortunately, the values obtained after determining these parameters were not the most accurate. Therefore, a fitting algorithm was used to estimate these reliability parameters of the studied turbine. Table 3 shows the best fitting values obtained for the case of the studied turbine. These results show that the estimated Johnson SB model is close to the real values of the various turbine reliability parameters, as indicated by the small values of the validation criteria (RMSE and MSE).

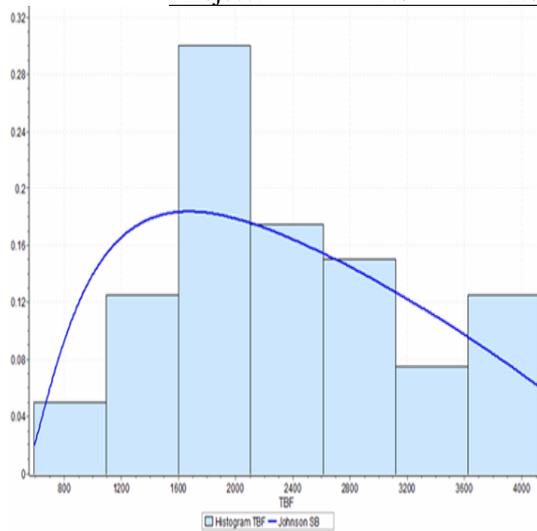
Table 3. Turbine reliability parameters obtained by Johnson SB after adjustment

Johnson SB Distribution parameters	Validation	
	RMSE	MSE
$\gamma = 1.668$	0.0248307	0.0006166
$\eta = 1.3668$		
$\varepsilon = -0.978$		
$\lambda = 0.0248307$		

From the results presented in Table 4, it can be concluded that the turbine reliability data matches the Johnson SB distribution, with the hypothesis  $H_0$  being accepted (NOT rejected) for all significance levels chosen with  $\alpha < 0.2$ . This can be confirmed by the data plot in Figure 7, which is close to the curve of the examined turbine estimated probability density function CDF.

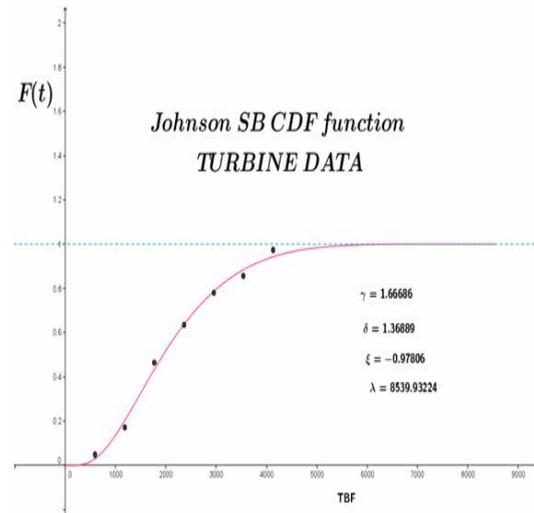
Table 4. Validation test of obtained Johnson SB parameters

<b>Kolmogorov-Smirnov test</b>					
Sample Size	7	-	-	-	-
Statistic	0.150232	-	-	-	-
P-Value	0.989231	-	-	-	-
Rank	1	-	-	-	-
$\alpha$	0.2	0.1	0.05	0.02	0.01
Critical Value	0.38148	0.43607	0.48342	0.53844	0.57581
Reject?	No	No	No	No	No

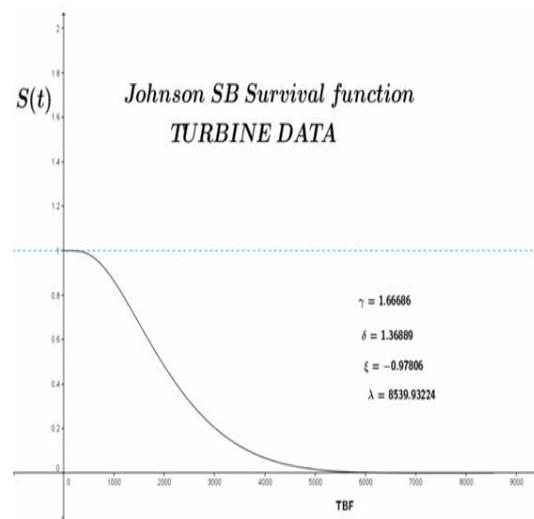


**Figure 7: Turbine probability density function based on Johnson SB distribution**

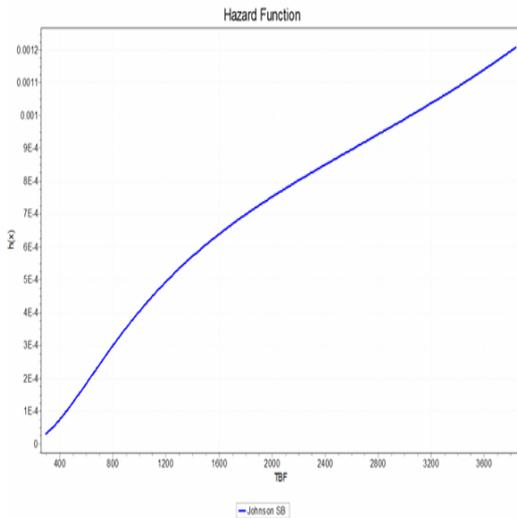
Figure 8 shows the turbine cumulative distribution function based on the Johnson SB distribution and Figure 9 shows the Survival function of this studied turbine. Furthermore, Figure 10 shows the turbine hazard function based on the Johnson SB distribution, from this cumulative hazard function we can see that there are only two distinct periods; The first is the stable failure period which is very short (0-800) and the second period is the wear-out failure period which starts early in this case study. Also, from the cumulative hazard function, shown in Figure 11, it can be said that the risk of failure is higher with the turbine operating time. Two intervals are observed approximately ( $< 1400$ ) and ( $> 1400$ ), it is as if there were two different accelerations, the first is greater than the second.



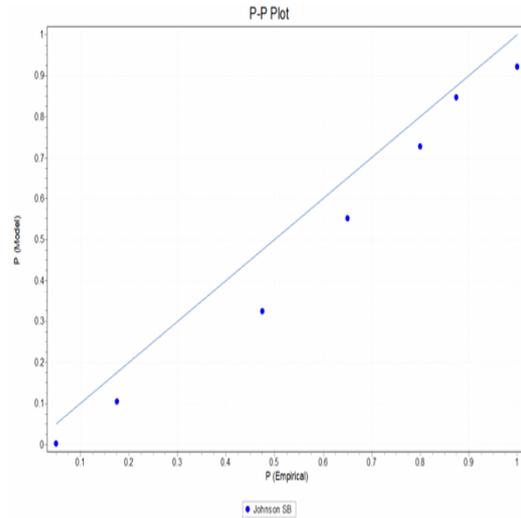
**Figure 8: Turbine cumulative distribution function based on the Johnson SB distribution**



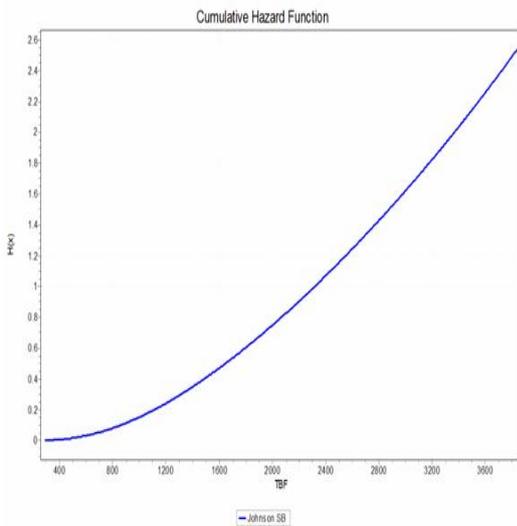
**Figure 9: Turbine survival function based on Johnson SB distribution**



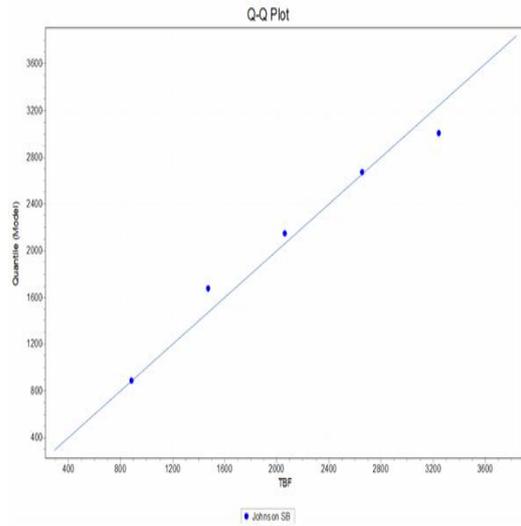
**Figure 10: Turbine hazard function based on Johnson SB distribution**



**Figure 12: PP Plot of turbine data based on Johnson SB distribution**

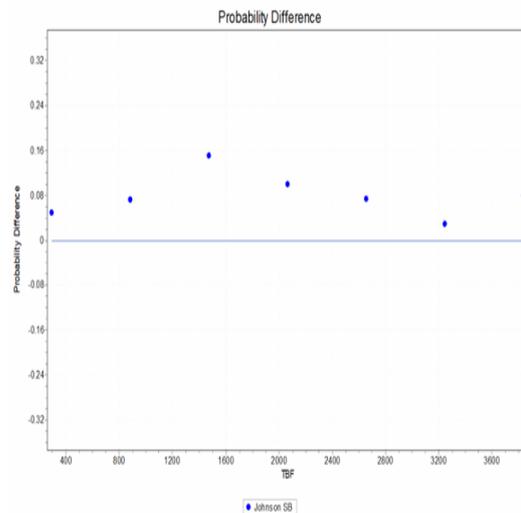


**Figure 11: Turbine cumulative hazard function based on Johnson SB distribution**



**Figure 13: QQ Plot of turbine data based on Johnson SB distribution**

Figures 12, 13, and 14 show the PP plot, QQ plot, and probability density function of the turbine data, respectively. These plots show that the Johnson SB distribution can be fitted to the turbine reliability data. The PP plot shows that the data forms a more or less linear shape, with no subset of points showing a marked deviation from the reference adjustment line. The QQ plot shows that the points are all almost on the reference line, which also indicates that the distribution can be fitted to the data. The evolution of the probability density function of the data further confirms this.



**Figure 14: Turbine probability density function based on Johnson SB distribution**

## 5. Conclusion

The present study presents a robust operational reliability evaluation methodology for gas turbines using the Johnson SU distribution, which offers notable advantages in reliability analysis for both functional and dysfunctional modeling of this rotating machinery. Through statistical determination of the gas turbine's reliability parameters using the Johnson SB distribution and subsequent numerical fitting, the estimation results were refined, enhancing the quality of parameter estimation.

The primary objective of this research is to provide comprehensive guidance for effectively monitoring gas turbine operation and ensuring a consistent and uninterrupted production process. Precise identification of availability factors and robust adjustments of reliability data are crucial for achieving this goal. To assess the quality of these adjustments, a series of graphical and statistical tests, including Kolmogorov-Smirnov, MSE, and RMSE, accompanied by graphical tools such as P-P Plot, QQ Plot, and Probability difference, were conducted. These tests validate the accuracy of the operational reliability parameter estimation for the gas turbine. By analyzing reliable data and incorporating operational experiences gained from the turbine's usage, this study sheds light on periods of optimal turbine operation and provides insights into potential malfunctions.

The results obtained from the assessment and estimation of the turbine's reliability parameters are both satisfactory and effective, offering invaluable information regarding optimal operation periods and instances of turbine malfunction. This study significantly contributes to the field by providing reliable and precise assessments of turbine reliability parameters, thereby enhancing our understanding of its operational characteristics. By implementing the proposed methodology, industry professionals can make informed decisions to optimize gas turbine performance, minimize downtime, and improve overall operational efficiency..

## Acknowledgments

The authors of this work express their sincerest thanks to the General Directorate of Scientific Research and Technological Development (DGRSDT), Algeria. For their support during the realization of this work within the joint research team on gas turbines with the Applied Automation and Industrial Diagnostics Laboratory, University of Djelfa, Algeria.

## References

[1]. Ahmed Hafaifa, Ferhat Laouad and Moulod Guemana, A New engineering method for fuzzy reliability analysis of surge control in

centrifugal compressor. *American Journal of Engineering and Applied Sciences*, 2009, vol. 02 no. 04, pp. 676-682.

- [2]. Ahmed Zohair Djeddi, Ahmed Hafaifa and Abudura Salam, Operational reliability analysis applied to a gas turbine based on three parameters Weibull distribution. *Mechanics*, 2015, vol. 21, no. 3, pp. 187-192.
- [3]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Abdelhamid Iratni, Abdellah Kouzou, Gas turbine reliability estimation to reduce the risk of failure occurrence with a comparative study between the two-parameter Weibull distribution and a new modified Weibull distribution. *Diagnostyka*, 2022, vol. 23, no. 1, p. 2022107.
- [4]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Abdelhamid Iratni, Reliability and availability study of a gas turbine based on usual approaches with a failure mode analysis. *Electrotehnica, Electronica, Automatica EEA*, 2021, vol. 69, no. 4, pp. 74-86.
- [5]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Abdellah Kouzou and Salam Abudura, Exploration of reliability algorithms using modified Weibull distribution: Application on gas turbine. *International Journal of System Assurance Engineering and Management*, 2017, vol. 8, pp. 1885-1894.
- [6]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Abudura Salam, Gas turbine reliability model based on tangent hyperbolic reliability function. *Journal of Theoretical and Applied Mechanics*, 2015, vol. 53, no. 3, pp. 723-730.
- [7]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Moulod Guemana and Abdellah Kouzou, Gas turbine reliability modelling based on a bath shaped rate failure function: modified Weibull distribution validation. *Life Cycle Reliability and Safety Engineering*, 2020, vol. 9, pp. 437-448.
- [8]. Ahmed Zohair Djeddi, Ahmed Hafaifa, Nadji Hadroug, Abdelhamid Iratni, Gas turbine availability improvement based on long short-term memory networks using deep learning of their failures data analysis. *Process Safety and Environmental Protection*, 2022, vol. 159, pp. 1-25.
- [9]. Ahsan Shazaib, Tamiru A. Lemma, Mebrahitom A. Gebremariam, Reliability analysis of gas turbine engine by means of bathtub-shaped failure rate distribution. *Process Safety Progress*, 2022, vol. 39, p. e12115.
- [10]. Artur J. Lemonte, Germán Moreno-Arenas, On residuals in generalized Johnson SB

- regressions. *Applied Mathematical Modelling*, 2019, vol. 67, pp. 62-73.
- [11]. Choayb Djeddi, Ahmed Hafaifa, Abdelhamid Iratni, Nadji Hadroug, XiaoQi Chen, Robust diagnosis with high protection to gas turbine failures identification based on a fuzzy neuro inference monitoring approach. *Journal of Manufacturing Systems*, 2021, vol. 59, pp.190-213.
- [12]. Cugerone Katia, Carlo De Michele, Johnson SB as general functional form for raindrop size distribution. *Water Resources Research*, 2015, vol. 51, no. 8, pp. 6276-6289.
- [13]. Egorov S.N., Rabinovich V.P., Increasing the operating reliability of steam-turbine rotors. *Strength of Materials*, 1982, vol. 14, pp. 1073-1076.
- [14]. Esakki Muthu S., Raghu V. Prakash, Mishra R. K., Sakthivel A., Thermo-Mechanical fatigue life assessment of a gas turbine rotor through reliability approach. *Journal of Failure Analysis and Prevention*, 2018, vol. 18, pp. 1361-1368.
- [15]. Florence George, Ramachandran K.M., Estimation of parameters of Johnson's System of distributions. *Journal of Modern Applied Statistical Methods*, 2011, vol. 10, no. 2, pp. 494-504.
- [16]. Flynn Michael R., Fitting human exposure data with the Johnson SB distribution. *Journal of Exposure Science & Environmental Epidemiology*, 2006, vol. 16, no.1, pp. 56-62.
- [17]. Hong Tang, Guowei Liang, Inversion of particle size distribution from spectral extinction data using the bimodal Johnson's SB function. *Powder Technology*, 2010, vol. 198, no. 3, pp. 330-336.
- [18]. Huynh Ngoc Phien, Wen-Bin Chow, Applicability of the Johnson SB-curve to annual stream-flow distributions. *Journal of Hydrology*, 1983, vol. 64, no. 1-4, pp. 15-24.
- [19]. Jiang Chen, Kaiwei Chu, Ruiping Zou, Aibing Yu, Andrew Vince, G.D. Barnett, P.J. Barnett, Systematic study of effect of particle size distribution in a dense medium cyclone by Johnson's SB function. *Minerals Engineering*, 2016, vol. 91, pp. 16-33.
- [20]. Jiang Renyan, Introduction to quality and reliability engineering. Springer, Series in Reliability Engineering, Science Press, Beijing and Springer-Verlag Berlin Heidelberg, 2015.
- [21]. Jinyuan Shi, Reliability design method for steam turbine blades. *Frontiers of Energy and Power Engineering in China*, 2008, vol. 2, pp. 363-368.
- [22]. Johnson Norman L., Bivariate distributions based on simple translation systems. *Biometrika*, 1949, vol. 36, no. 3-4, pp. 297-304.
- [23]. Jordyn Koenig, Hamid Norasi, Gary Mirka, Technical note: Using Johnson distributions to model trunk kinematics. *Theoretical Issues in Ergonomics Science*, 2021, vol. 22, no. 5, pp. 555-566.
- [24]. Kamziah Abd Kudus, Ahmad M.I., Jaffirin Lapongan, Nonlinear regression approach to estimating Johnson SB parameters for diameter data. *Canadian Journal of Forest Research*, 1999, vol. 29, no 3, p. 310-314.
- [25]. Kottegoda N.T., Fitting Johnson SB curve by the method of maximum likelihood to annual maximum daily rainfalls. *Water Resources Research*, 1987, vol. 23, no. 4, pp. 728-732.
- [26]. Leo Pio D'Adderio, Katia Cugerone, Federico Porcù, Carlo De Michele, Ali Tokay, Capabilities of the Johnson SB distribution in estimating rain variables. *Advances in Water Resources*, 2016, vol. 97, pp. 241-250.
- [27]. Li Chun-Qing, Wei Yang, Time-dependent reliability theory and its applications. Elsevier Science & Technology, United Kingdom, 2022.
- [28]. Mage David T., An explicit solution for SB, parameters using four percentile points. *Technometrics*, 1980, vol. 22, no. 2, pp. 247-251.
- [29]. Mingazov B.G., Korobitsin N.A., Use of probability indices for assessment of gas turbine power station reliability under commercial operation conditions. *Russian Aeronautics*, 2010, vol. 53, pp. 226-229.
- [30]. Mouloud Guemana, Ahmed Hafaifa and Mohamed Ben Rahmoune, Reliability study of gas turbines for improving their availability by ensuring optimal exploitation. *OIL GAS European Magazine*, June 2015, vol. 2, pp. 88-91.
- [31]. Nadji Hadroug, Ahmed Hafaifa, Abdelhamid Iratni, Mouloud Guemana, Reliability modeling using an adaptive neuro-fuzzy inference system: Gas turbine application. *Fuzzy Information and Engineering*, 2021. vol.13, no.2, pp. 154-183.
- [32]. Poozesh Mohammad, Tarighi Javad, Determination of timeliness cost using method of average workability probability based on reliability function of farm tractors. *Emirates Journal for Engineering Research*, 2020, vol. 25, no. 2, p. 2.
- [33]. Rachid Belhadeif, Ahmed Hafaifa, Nadji Hadroug, Mohamed Boumehraz, Reliability Evaluation Based on a Fuzzy Expert System: Centrifugal Pump Application, *Studies in*

- Informatics and Control, 2016, vol. 25, no. 2, pp. 181-188.
- [34]. Rennolls Keith, Mingliang Wang, A new parameterization of Johnson's SB distribution with application to fitting forest tree diameter data. *Canadian journal of forest research*, 2005, vol. 35, no. 3, pp. 575-579.
- [35]. Scolforo José Roberto Soares, Tabai Fernando César Vitti, De Macedo Renato Luís Grisi, Acerbi Jr F. W., De Assis A.L., SB Distribution's accuracy to represent the diameter distribution of *Pinus taeda*, through five fitting methods. *Forest ecology and management*, 2003, vol. 175, no 1-3, p. 489-496.
- [36]. Slifker James F., Samuel S. Shapiro, The Johnson system: Selection and parameter estimation. *Technometrics*, 1980, vol. 22, no. 2, pp. 239-246.
- [37]. Stanfield PM, Wilson JR, Mirka GA, Glasscock NF, Psihogios JP, Davis JR., Multivariate input modeling with Johnson distributions. In the Proceedings of the 28th conference on Winter simulation, 8 November 1996, pp. 1457-1464.
- [38]. Takvor Soukissian, Use of multi-parameter distributions for offshore wind speed modeling: The Johnson SB distribution. *Applied Energy*, 2013, vol. 111, pp. 982-1000.
- [39]. Teresa Fidalgo Fonseca, Carlos Pacheco Marques, Bernard R. Parresol, Describing maritime pine diameter distributions with Johnson's SB Distribution using a new all-parameter recovery approach. *Forest Science*, 2009, vol. 55, no. 4, pp. 367-373.
- [40]. Zhang Lianjun, Kevin C. Packard, Chuangmin Liu, A comparison of estimation methods for fitting Weibull and Johnson's SB distributions to mixed spruce fir stands in northeastern North America. *Canadian Journal of Forest Research*, 2003, vol. 33, no. 7, pp. 1340-1347.