Hierarchical control strategies for heaving wave Energy converters

Omsalama Mubarak Mohamed Saeed

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United Arab Emirates University
College of Engineering
Department of Electrical Engineering

HIERARCHICAL CONTROL STRATEGIES FOR HEAVING WAVE ENERGY CONVERTERS

Omsalama Mubarak Mohamed Saeed

This thesis is submitted in a partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

Under the Supervision of Dr. Addy Wahyudie

December 2015
Declaration of Original Work

I, Omsalama Mubarak Mohamed Saeed, the undersigned, a graduate student at the United Arab Emirates University (UAEU), and the author of this thesis entitled "Hierarchical Control Strategies for Heaving Wave Energy Converters", hereby, solemnly declare that this thesis is my own original research work that has been done and prepared by me under the supervision of Dr. Addy Wahyudie, in the College of Engineering at UAEU. This work has not previously been presented or published, or formed the basis for the award of any academic degree, diploma or a similar title at this or any other university. Any materials borrowed from other sources (whether published or unpublished) and relied upon or included in my thesis have been properly cited and acknowledged in accordance with appropriate academic conventions. I further declare that there is no potential conflict of interest with respect to the research, data collection, authorship, presentation and/or publication of this thesis.

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Abstract

This thesis is concerned with developing control strategies for a heaving sea wave energy converter. The objective of this study is to improve captured and converted powers in heaving wave energy converters using a robust hierarchical control strategy. This strategy consists of higher and lower hierarchical controllers. The higher controller provides a reference velocity signal, whereas the lower one follows the reference despite the uncertainties in the model. In this thesis, two novel methods for the higher controller are proposed. The objective of the first method is to design the intrinsic resistance that maximises the captured power while considering the constraint on the elevation of the buoy. For this purpose, a constrained optimisation quadratic problem is formulated as a function of the wave’s significant height and peak frequency. The objective of the second method is to maximise the captured power without exceeding the allowable level of the control force and a power take-off utilisation index. Similarly, three novel lower hierarchical controllers are proposed. The first method contains the PID (proportional-integral-derivative) augmentation with sliding mode control. This method has an interesting feature in which the dynamic model in the lower level is not needed. Hence, it can be categorised as a model-free controller. The stability and the speed of the convergence error is handled by the PID, while the robustness and tracking properties are conducted by the sliding mode control. The second method proposes a robust PID controller, which is designed using complex polynomial stabilisation. The last method proposes a novel lead-lag compensator, which is designed using $H_{\infty}$ theory with the objectives of maximising the robustness and tracking properties while minimising the control force of the power take-off device. The proposed methods are tested using nominal and perturbation cases in regular and irregular sea states. The resultant performance under different perturbation scenarios is compared with existing control techniques.

Keywords: Heaving wave energy converters, Hierarchical control strategy, Complex PID stabilisation, Robust control, Sliding mode control, Lead-lag compensator.
العنصر في محولات طاقة أمواج البحر: رأسية الحركة

ملخص

تتطلب هذه الأدوات بتطوير استراتيجيات هرمية للتحكم في محولات طاقة أمواج البحر رأسية الحركة. و تهدف إلى تحفيز التقدير على حرص الطاقة الميكانيكية الكاملة في الأمواج و تحويلها إلى طاقة كهربية. تعتمد الاستراتيجية على تصميم جهاز تحكم، أحدهما علوي و الآخر سفلي. يعني جهاز التحكم العلوي إصدار إشارة مرجعية أحدث في الاعتبار قيود و ضوابط معينة مثل الهد الأعلى لقوة التحكم و نسبة أعلى قيمة للطاقة المتحولة إلى متوسطها و أقصى ارتفاع يمكن أن يصل إليه الجزء الطافي من محول طاقة أمواج البحر. أما جهاز التحكم السفلي، فيقوم بضبط الإشارة الصادرة عن المحاج العلوي بشكل أني لا يتأثر بمتغيرات النظام أو الظروف الخارجية الشوكة. تم إعداد تقنيات لتصميم جهاز التحكم العلوي، وأولها تقوم على حل مسألة رياضية من الدرجة الثانية بدلالة الارتفاع المؤثر للموجة و ترددها. في حين تعتمد الطرقية الثانية كنقطة أقصى حصد يمكن للطاقة الميكانيكية الكاملة في الموجة بدون تجاوز القيمة المحددة لقوة التحكم و مؤثر جهاز حصد الطاقة و تحويلها. في المقابل، تم استخدام تقنيات لتصميم جهاز التحكم السفلي، و التي تتمثل في جهاز تحكم تناسلي تفاعلي بيضتي مدمج مع نظام تحكم إنزلاق. و في الطرقية الثانية تم استخدام جهاز تحكم تناسلي تفاعلي تفاعلي يتصف بالآلية. أما الطرقية الثالثة فاعتمدت تصميم جهاز تحكم على أساس طريقة التقدم و التأخير. تم التحقق من فاعلية التصميم باستخدام براز المحاكاة و تم تحليل النتائج و مقارنتها بنتائج استراتيجيات أخرى. و قد أثبتت التصميمات كفاءتها من ناحية تحقيق نسبة مرضية لحصد الطاقة و تحويلها في ظل القيود المحددة. كما تم اختيار فاعلية التصميم في وجود نسبة من المتغيرات و القوى الخارجية الشوكة.

مفهوم البحث الرئيسية: استراتيجية التحكم الهرمي، محول طاقة أمواج البحر رأسية الحركة، الطاقة المجددة.
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Dedication

To my beloved mother, husband, sons and daughters
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Chapter 1: Introduction

This chapter provides a summary describing the status of renewable energy and its contribution to global electricity production. Section 1.2 focuses on wave ocean energy, its potential, and how it is quantitatively estimated. Mechanisms of wave energy extractions, which are represented by Wave Energy Converters (WECs), are introduced in section 1.3. An overview of the available control techniques is given in section 1.4. Finally, the thesis structure is explained.

1.1 Renewable Energy

The world is experiencing a transition towards a sustainable energy future. Challenges and threats such as global warming, air pollution, and the expected depletion of non-renewable resources have prompted many countries to seek alternative renewable clean sources of energy. Furthermore, the modern technologies of renewable energy have matured and become more reliable. Political trends and international organisations that support this transition have emerged. For instance, the International Renewable Energy Agency (IRENA) advocates the sustainable utilisation and widespread adoption of all renewable energy types, to promote security, access, sustainable development, low-carbon economic prosperity, and growth. The adopted types include solar and wind energy, bioenergy, and ocean, geothermal, and hydroelectric energy [4].

Renewable power generation technologies have been available in the conventional market. Power plants based on these technologies accounted for 8% of global energy production in 2004. This share increased to 29% when large hydro plants were not included, and 40% with large hydro plants included [1]. The renewable energy share of global electricity production was 22.1% at the end of 2013 [5]. Figure 1.1 shows the share of each type at that time.

To a great extent, global approaches have driven the development of
renewable energy technologies by drawing in venture capital and creating markets that have realised economies of scale and bolstered innovation and progress. These trends have diminished expenses and stimulated managed development in the division. Since 2004, the number of nations supporting renewable vitality with direct financial backing has approximately tripled, from 48 to more than 140, and a steadily expanding number of developing nations are setting renewable vitality targets and ordering bolster strategies. Dependable and consistent strategy structures are expected to support the maintained sending of renewable energy. The industry needs consistent strategy systems with specific end goals to develop generation limits and design new methods. Countries with renewable energy policies in 2005 and 2013 are shown in figures 1.2a and 1.2b, respectively [1]. The United Arab Emirates appears in the 2013 map with a relatively moderate number of enacted policy types (3-5).

### 1.2 Potential of Wave Energy

Ocean waves are of considerable potential. The total amount of power contained in them is estimated to be 2 terawatts (TW), reaching the same magnitude
Figure 1.2: Countries with renewable energy policies, 2005 and 2013 [1]

As the world’s electricity consumption. According to conservative estimates, approximately 10-25% of this total power can be harnessed, suggesting that wave power could represent a considerable addition to total energy resources [6].

Wave energy density, denoted by $J$, is used to estimate the potential of sea wave energy in specific locations and is measured in $W/m$. Hence, an estimation of incident wave power can be obtained by multiplying $J$ by the width of the body
intercepting the wave. Theoretically, there are two sea-state types: monochromatic (a sea state with a single frequency) and polychromatic (a sea state with different harmonics). The latter sea state is the more realistic one. The wave energy density, \( J \), can be calculated for the monochromatic type in W/m using Equation 1.1 [7]: similarly, the polychromatic type can be calculated using Equation 1.2 [8].

\[
J = \frac{\rho g^2}{32\pi} TH,
\]

where \( \rho \) is the water density in kg/m\(^3\), \( g \) is the gravitational acceleration in m/s\(^2\), \( T \) is the wave period in s, and \( H \) is the wave height in m.

\[
J = \frac{\rho g^2}{2} \int_0^\infty \frac{S(\omega)}{\omega} d\omega.
\]

where \( S(\omega) \) is the wave spectral density. In ocean engineering literature, a variety of wave spectral models can be found. Pierson-Moskowitz spectra and JONSWAP spectra are examples of these models [9]. These models can be used to generate different spectra with different peak frequencies, \( \omega_p \), which in turn is used to generate polychromatic waves for simulation purposes.

Ocean waves represent a global resource of energy. Worldwide, their annual average power levels range from 5 kW/m to more than 60 kW/m. Low to moderate levels occur in sheltered and closed seas and in tropical regions, while higher levels are found in the Northern and Southern Hemispheres [2]. However, the most energetic locations are found in the Southern Hemisphere, because it experiences less seasonal wave variation [6]. The map in Figure 1.3 shows that the oceans are most energetic between the 50\(^\circ\) to 60\(^\circ\) latitudes. The highest power levels (greater than 60 kW/m) exist seaward along the coastlines west of Scotland (UK) and Ireland, Australia, southern Chile, New Zealand, northwest Canada, and Africa.

The data presented in Figure 1.3 was collected over a ten-year period. It is based on a 6-hourly time series of wave energy measurements at each point;
Figure 1.3: Ocean wave energy resources in kW/m from the ECMWF (European Center for Medium-Range Weather Forecasts) [2].

the model data was calibrated and validated against ten years of Topex satellite altimeter data for each point, as well as buoy data where available. The points are all offshore and do not mirror the waterfront wave atmosphere, which will typically be altogether different because of different shallow water impacts and seaside protections. Different software can be used to derive near-shore wave climates from open ocean data (e.g., the Worldwaves package, as well as various shallow-water models such as the SWAN model [2]).

Wave energy can be seen as solar energy's indirect form. However, it has a significantly greater power intensity of $2 - 3 \text{ kW/m}^2$; this is 4 to 6 times the power intensity of wind energy (i.e. $\sim 0.5 \text{ kW/m}^2$) and almost 10 times the power intensity of solar energy (i.e. $0.1 - 0.3 \text{ kW/m}^2$) [10]. In general, wind and solar power devices can generate power $20 - 30 \%$ of the time, while wave power devices are reported to generate power up to $90 \%$ of the time [3].

1.3 Wave Energy Converters (WECS)

Wave energy converters (WECS) capture the energy contained in ocean waves and use it to generate electricity. Numerous wave energy conversion concepts have
been developed all over the world; the number of patented wave energy conversion techniques in Europe, Japan, and North America exceeds 1,000. Although the concepts and designs of WECs vary significantly, they can be classified into the following main types.

- **Attenuator**
- **Point absorber**
- **Terminator**

Attenuators ride the waves. That is, they are placed parallel to the wave direction. Pelamis, developed by Pelamis Wave Power, is an example of this type. Figure 1.4a shows a Pelamis wave farm.

Point absorbers are floating structured devices that heave up and down. They can also rely on pressure differences, and hence be submerged under the water’s surface. The size of these devices is relatively small compared to the length of the incident wave. Therefore, the direction of the wave with respect to the device is not significant. There are a variety of point absorbers, including Ocean Power Technology’s Powerbuoy. Figure 1.4b shows a wave farm using Powerbuoys [3]. Another example of point absorbers is the Uppsala University single-body point absorber; it has a capacity of 40 kW and was installed in the Lysekil Project [11].

Terminator devices, on the other hand, are dependent on the direction of waves. When they are operated, the wave front must be parallel to the principle axis of the device; i.e. this axis must interrupt the wave by being placed perpendicular to its main direction. Salter’s Duck, shown in figure 1.4c, is an example of this type. It was developed at the University of Edinburgh [3].

Based on the mode of operation, WECs can be categorised into three main categories: 1) oscillating water columns that use trapped air pockets in a water column to drive a turbine, 2) oscillating body converters that are floating or submerged devices using wave motion (up/down, forwards/backwards, side to side)
to generate electricity, and 3) overtopping converters that use reservoirs to create a head and subsequently drive turbines [3,4]. A power take-off (PTO) system is used
to convert the wave energy into electricity. Table 1.1 shows these categories with additional classifications based on structures and locations, along with examples [4].

<table>
<thead>
<tr>
<th>WEC Type</th>
<th>PTO Technology</th>
<th>Structure</th>
<th>WEC Technology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillating water column</td>
<td>Air turbine</td>
<td>Fixed</td>
<td>Isolated:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>- Pico</td>
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<td></td>
<td>- UIMPET</td>
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<tr>
<td></td>
<td></td>
<td>Floating</td>
<td>Near shore (still at conceptual level)</td>
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<td></td>
<td></td>
<td></td>
<td>- Mighty Whale</td>
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<td></td>
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<td></td>
<td>- Ocean Energy</td>
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<td></td>
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<td>- Spexer</td>
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<td></td>
<td></td>
<td></td>
<td>- OceanFire</td>
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<tr>
<td>Oscillating bodies</td>
<td></td>
<td>Fixed at shoreline</td>
<td>Eco Wave Power</td>
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<tr>
<td></td>
<td>- Hydraulic motors</td>
<td></td>
<td>Essentially translation (heave):</td>
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<tr>
<td></td>
<td>- Hydraulic turbine</td>
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<td>- Aquabuoy</td>
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<td>- Electrical generator</td>
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<td>- IPS Buoy</td>
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<td>- FO3</td>
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<td>- Wavebob</td>
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<td></td>
<td></td>
<td></td>
<td>- PowerBuoy</td>
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<tr>
<td></td>
<td></td>
<td>Floating</td>
<td>Essentially rotating:</td>
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<td></td>
<td>- Pelamis</td>
<td></td>
<td>- Pelamis</td>
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<td></td>
<td>- PS Frog</td>
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<td>- SEAREV</td>
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<td></td>
<td></td>
<td>Submerged</td>
<td>Essentially translation:</td>
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<td></td>
<td>- AWS</td>
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<td>Reactor (bottom - hunged):</td>
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<td></td>
<td></td>
<td></td>
<td>- WaveRoller</td>
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<td></td>
<td>- Oyster</td>
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<td></td>
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<td>Fixed</td>
<td>Shoreline (with concentration):</td>
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<td></td>
<td></td>
<td></td>
<td>- TAPCHAN</td>
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<td></td>
<td>Shoreline (without concentration):</td>
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<td></td>
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<td>- SSG</td>
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<td></td>
<td>Low head hydraulic turbine</td>
<td>Floating (with concentration)</td>
<td>- Wave Dragon</td>
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<td></td>
<td>- WaveCat</td>
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</table>

Table 1.1: Wave energy technologies

There are different types of PTO systems, including rotary generators that employ energy transfer methods such as turbine transfer and hydraulic conversion. Direct electrical linear generators are also used. Figure 1.5 illustrates the variations in PTO systems [3].

1.4 Available Control Techniques

Numerous research efforts have been conducted to identify the best methods of extracting energy from waves. Previous research has focused on the concept and design of the primary interface; in addition, there is increasing interest in
developing control strategies to enhance efficiency and optimise the power train. In general, these control strategies are either passive or active.

Passive control strategies use electric circuits which are tuned to the dominant wave height and frequency of the site of interest. Available techniques utilise resistive loading and resonance circuits [12]. They are simple to implement and cost-effective. However, they have their drawbacks in terms of adaptivity. They are not able to respond to continuous changes in sea state, which results in a narrow absorption bandwidth.

On the other hand, active control strategies are more complex and expensive compared to passive techniques. However, they provide online control regimes that are able to change the WEC dynamics more rapidly. Accurate prediction algorithms are also required to implement a more rapid wave-to-wave control [13].

1.5 Objectives and Scope

1.5.1 Objectives

To take the advantage of simple design controllers with low computational cost that can be tuned offline, and at the same time achieve robustness and certain degree of
adaptivity, the following objectives are stated:

- Develop simple and low computational cost hierarchical control strategies to enhance the performance of the WEC in terms of captured energy, under constraints of maximum allowable control force and maximum allowable ratio of average converted power to its mean, or maximum allowable buoy's elevation.

- Improve the reference signal used in reference-based controllers with respect to different sea-states to achieve a certain degree of adaptability with an offline simple methods.

1.5.2 Scope of the Study

Heaving point absorbers WECs are composed of a mechanical part which represents the floating body interaction with the sea waves, and an electrical part that consists of the PTO and the power electronics setup required for grid connection. The scope of this study focuses on apply various hierarchical control strategies for a heaving wave energy converter based on the plant's mechanical model.

1.5.3 Limitations

Other aspects and parts are important for the complete wave to wire connection. However, they are out of scope of this study. They can be summarized as:

- Evaluating the control force based on the electrical parameters

- Controlling electrical components

- Back to back converters and connection to the grid
1.6 Research Problem and Thesis Structure

The main objective of this work is to develop robust control strategies to enhance the performance of a heaving WEC in terms of captured energy, under the constraints of a maximum allowable control force and maximum allowable ratio of maximum converted (electrical) power to its mean value, or maximum allowable elevation of the buoy. The proposed solution is a hierarchical control system (HCS) which consists of a high-level hierarchical controller (HHC) and a low-level hierarchical controller (LHC). The HHC provides the LHC with a reference signal based on design specifications and sea state, while the LHC provides robust tracking to the reference signal under the perturbations and/or disturbances of given parameters. Figure 1.6 shows the structure of the proposed HCS.

In conventional control methods of this particular system, reference velocity (signal) is generated by scaling the excitation force by a constant value, regardless of the sea state (i.e. the significant wave height, $H_s$, and the peak wave frequency, $\omega_p$). This procedure provides an inaccurate reference, as the velocity of the sea wave depends on its state. The HHC in the proposed method suggests two methods for generating the reference. The designed methods aim to find an optimal reference while considering design constraints. A trade-off is made between the reference’s optimality and the design constraints.

![System structure](image)

Figure 1.6: System structure
Figure 1.7: Heaving spherical WEC with a PMLG used as PTO mechanism

The adopted device is the Uppsala University heaving WEC or Uppsala point absorber. The device consists of a buoy, tether, and a permanent magnet linear generator (PMLG) placed on the seabed [14] as shown in Figure 1.7. The adopted mathematical models are the linear mechanical and electrical mathematical models developed in [15].

This thesis is organised into five chapters, including the introduction. Chapter 2 discusses the adopted model in [15] of the considered heaving WEC of Uppsala University [14]. The developed control strategies are presented in Chapter 3; simulation results and analyses are reported in Chapter 4. Finally, Chapter 5 focuses on the research findings and future works.
Chapter 2: Literature Review and Adopted System Models

2.1 Literature Review on Control Strategies

There is a lack of convergence on the best method of extracting energy from the waves and, although previous innovation has generally focused on the concept and design of the primary interface, questions arise concerning how best to optimize the power-train. Research and development studies in this area are rising, but are relatively immature compared to other renewable energy technologies. Available control strategies can be categorised to active and passive, model free, reference based, etc. Some are simple and cost effective, specifically passive ones, but have their drawbacks in term of adaptivity. On the other hand, active ones are more expensive and complex. Though, they have better performance in terms of adaptivity and robustness. Following subsections explores some of the available control regimes.

2.1.1 Latching Control

Latching control was first examined by Budal and Falnes in reference [16]. The objective behind latching control was to stall (i.e. latch) the motion of the device at the extremes of its movement (when velocity is zero), and to release it during good phase wave forces to maximise energy extraction. Latching control is discrete, highly non-linear, and by its nature sub-optimal. The challenge with this strategy was determining the optimum time to release the buoy from the latched phase [3]. This strategy also required involvement of mechanical devices and hence was much slower than electrical control techniques.
2.1.2 Reactive loading control

Reactive loading control was used to widen the efficiency range of a WEC on either side of the resonant frequency [17]. This theoretically optimal control strategy involved adjusting the dynamic parameters of the primary converter, such as the spring constant, inertia, and energy absorbing damping, to enable maximum energy absorption at all frequencies. Korde considered reactive control in [18], and found that velocity feedback could be used to adjust the damping coefficient provided by the PTO system to balance the radiation damping of the device to enable maximum permissible energy absorption. Optimal power absorption requires that the primary converter feels no reactive force (as at resonance) and that the energy absorption rate (damping) equals the rate at which kinetic energy is being radiated from the device. In practice, it was not possible for reactive control to be fully optimal due to velocities that could become extremely high. As such, constraints are necessary to safeguard against hazards of mechanical/electrical over driving [19].

2.1.3 Optimal Control

In [20] the optimal control law for a single nonlinear point absorber in irregular sea-states was derived, and proven to be a closed-loop controller with feedback from measured displacement, velocity and acceleration of the floater. However, a non-causal integral control component dependent on future velocities appeared in the optimal control law, rendering the optimal control law less useful for real time implementation. To circumvent this problem a causal closed-loop controller with the same feedback information was proposed, based on a slight modification of the optimal control law. The basic idea behind the control strategy was to enforce the stationary velocity response of the absorber into phase with the wave excitation force at any time. The controller was optimal under monochromatic wave excitation. It was demonstrated that the devised causal controller, in plane irregular sea-states, absorbed almost the same power as the optimal controller.
2.1.4 Stochastic Optimal Control

The method in [21] dealt with the stochastic optimal control of a wave energy point absorber with strong nonlinear buoyancy forces using the reactive force from the electric generator on the absorber as control force. The considered point absorber has only one degree of freedom, heave motion, which is used to extract energy. Constraints were enforced on the control force to prevent large structural stresses in the floater at specific hot spots with the risk of inducing fatigue damage, or because the demanded control force could not be supplied by the actuator system due to saturation. Further, constraints were enforced on the motion of the floater to prevent it from hitting the bottom of the sea or to make unacceptable jumps out of the water. The applied control law, which was of the feedback type with feedback from the displacement, velocity, and acceleration of the floater, contained two unprovided gain parameters, which were chosen so the mean (expected value) of the power outtake in the stationary state is optimized.

2.1.5 Ant Colony Optimization

The application of bio-inspired Ant Colony Optimization (ACO) meta-heuristic as decision support for real-time wave energy extraction was introduced in [22]. It was found that the performance of the proposed algorithm was appealing as it was able to provide optimal parameter values to the control model within short interval. This provided fast-tuning capability to the PTO while sparing sufficient time for the WEC to respond to the continuously changing sea states.

2.2 System Structure

The adopted device for this study is the Uppsala University heaving WEC or Uppsala point absorber [14]; the mechanical and electrical models developed in [15] are adopted for design and simulation purposes. The buoy and the hydraulic and mechanical forces acting on it represent the primary interface with ocean waves.
The theory and analysis that cover the buoy motion resulting from interaction with the waves are discussed in section 2.3.

The electrical part of the system is composed of the power take-off (PTO) mechanism, power converter, and the grid. The considered PTO converts the mechanical power directly to electrical power using a permanent magnet linear generator (PMLG). The mechanism is simple, as it does not involve a large number of conversion processes (compared to its hydraulic counterpart), as illustrated in Figure 1.5. However, the size of the generator must be large as the motion is slow [3]. Besides the PMLG's main function of converting mechanical power to electrical power, it is also used to regulate the motion of the buoy through the controller. In order to connect a WEC-based power farm to the grid, a back-to-back converter is used. It consists of an AC to DC converter on the machine side, known as a machine side converter (MSC). Another component is a DC to AC converter on the grid side, known as a grid side converter (GSC). Both are regulated by the controller to enhance performance. The back-to-back converter is out of the scope of this study. Figure 2.1 shows the system's structure with its mechanical and electrical parts emphasised.

![Figure 2.1: Structure of the heaving WEC system](image-url)
This study focuses on developing robust control techniques for the mechanical part of the system, to achieve optimum power extraction. The control force is generated by the PMLG, which is also used to convert the mechanical energy to electrical energy. Other control techniques must be applied on the electrical side to satisfy different requirements, one of which is to generate adequate control force using the current in the PMLG. However, the latter is again not within the scope of this study.

2.3 Acting Forces

Forces controlling the motion of the buoy include hydrodynamic forces that are induced by water-body interaction and by mechanical forces that are produced by the PTO mechanism and other mechanical components in the system. This section discusses the theory and formulae that control the hydrodynamic forces influencing the free motion of the point absorber WEC. The mechanical force generated by the PTO will also be discussed in the last part of this section. Altogether, these forces describe the controlled motion of the buoy and were used to develop the mechanical model described in section 2.5, using [15].

2.3.1 Linear Wave Theory

A simple and fairly accurate description of the propagation of waves on a fluid surface is given by the linear wave theory (LWT), introduced by George Airy in 1841. The provided description is subject to a few assumptions that are valid for many mediums (especially for water) [23]. For water waves, LWT is also used to analyse the forces and moments that any fixed or moving bodies/structures and water waves apply on each other. The flow of water particles is represented by a potential flow $\phi(t, x, y, z)$, which in turn represents a continuous scalar field, in which $t$ is time and $(x, y, z)$ are the Cartesian coordinates. The velocity vector of the
water particles $v$ can be expressed in terms of $\phi$ as [9]

$$v = \nabla \phi.$$  
(2.1)

where $\nabla$ is the gradient. Equation 2.1 is valid if and only if the potential is irrotational, which implies that

$$\nabla \times \nabla \phi = 0$$  
(2.2)

LWT is derived from the continuity equation, which describes the principle of conservation of mass [24] expressed by Equation 2.2 for a steady-state flow.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v),$$  
(2.3)

where $\rho$ is the water density. In the case of water at normal conditions, the fluid is assumed to be incompressible, and hence Equation 2.3 collapses to

$$\nabla \cdot v$$  
(2.4)

Substituting Equation 2.1 into Equation 2.4, we obtain the Laplace equation

$$\nabla^2 \phi = 0,$$  
(2.5)

where $\nabla^2$ is the Laplace operator. In order to solve the linear partial differential equation 2.5, we must define its boundary conditions. One dynamic and two kinematic boundary conditions are applied [24]. Figure 2.2 shows the parameters of a progressive wave travelling along the $x$ axis, $\eta(t,x)$ denotes the water free surface elevation along the $z$ axis. Hence, from this point onwards, all the corresponding variables are functions of $x$ and $z$ spatially and $t$ temporally. In the area near the free surface $\eta(t,x)$, it is obvious that the water particles in the potential flow oscillate
with the same velocity as the free surface. This is called the kinematic free surface boundary condition in the literature, and can be expressed as [24]

$$\frac{\partial \eta}{\partial t} = \frac{\partial \eta}{\partial z} |_{z=\phi(t,x)} \quad (2.6)$$

Figure 2.2: Wave Parameters

In order to formulate the boundary condition, we start with a Bernoulli equation 2.7. For a uniform pressure applied on an irrotational and incompressible flow, the Bernoulli equation expresses the pressure distribution on the free surface as [9]

$$\rho \frac{\partial \phi}{\partial t} + \frac{1}{2} \rho v^2 + \rho gz + p(t) = C(t), \quad (2.7)$$

where $g$ is the gravitational acceleration, $p(t)$ is the pressure at a certain point, and $C(t)$ is a time-dependent function. In the case of still water, $C(t)$ is set to zero. At the water surface, $p$ is selected to be the gauge pressure, which is relative to the ambient air pressure on the surface equal to zero. Note that Equation 2.7 is still nonlinear in $v$; however, for waves having a height $H$ that is smaller than their wave length $\lambda$, the second term (steady kinetic energy) of Equation 2.7 can be disregarded, leading to the following linear dynamic boundary condition [24]

$$\frac{\partial \phi}{\partial t} = -gz |_{z=\eta(t,x)} \quad (2.8)$$
The final boundary condition is applied on the seabed, which is assumed to be horizontal, fixed, and impermeable, such as [24]

\[ \frac{\partial \phi}{\partial z} \bigg|_{z=-h} = 0, \]  

(2.9)

where \( h \) is the water depth or the distance between the still water level \( (z = 0) \) and the seabed. The Laplace equation depicted in Equation 2.5 along with the boundary conditions shown in Equations 2.6, 2.8, and 2.9, are entirely linear and can be solved for \( \eta \) and \( \phi \) using the separation of variables principle [9]. Hence, for a progressive wave

\[ \eta(t,x) = \frac{H}{2} \cos(kx - \omega t), \]  

(2.10)

\( k \) is the wave number; it is equal to \( \frac{2\pi}{\lambda} \), while \( \omega \) is the wave frequency. The flow potential is represented as

\[ \phi(t,x,z) = \frac{Hg \cosh kh + z}{2\omega \cosh(kh)} \sin(kx - \omega t) \]  

(2.11)

The wave frequency can be found using the dispersion equation as follows

\[ \omega = \sqrt{gk \tanh(kh)} \]  

(2.12)

Note that \( k \) depends on \( \lambda \), which is unique for a specific \( h \) and \( ? \). Actually, there is no explicit solution for \( \lambda \) in terms of the known wave parameters. However, we can manipulate Equation 2.12 to get [9]

\[ \lambda = \frac{2\pi y}{\omega^2} \tanh \left( \frac{2\pi h}{\lambda} \right) \]  

(2.13)

In order to solve Equation 2.13 for \( \lambda \), we can plot each side of the equation, then the intersection of the two curves will be the corresponding value of \( \lambda \). An explicit solution for \( \lambda \) can be obtained only in deep water, where the term \( \tanh(kh) \)
approaches 1 as $\frac{h}{\lambda} \geq \frac{1}{2}$, and hence

$$\lambda = \frac{2\pi g}{\omega^2}$$

(2.14)

Despite the non-linearity of sea wave motion, LWT provides a linearised approximation of sea wave motion as it is subject to the following assumptions:

- Fluid is incompressible, irrotational, and inviscid

- Small wave heights are considered, i.e. $\frac{H}{\lambda} \ll 1$, which is the case in deep sea basins

2.3.2 Hydrodynamic Forces

As discussed in the previous section, the Laplace equation, along with its boundary conditions, is sufficient to find a solution for the flow potential if the progressive waves are not disturbed by a floating body. However, when such a body exists, we must solve more than one Laplace equation [7]. In this case, the flow potential is decomposed into three parts as follows:

$$\phi(t,x,z) = \phi_i(t,x,z) + \phi_d(t,x,z) + \phi_r(t,x,z),$$

(2.15)

where $\phi_i(t,x,z)$ is the incident undisturbed flow potential, $\phi_d(t,x,z)$ is the diffracted flow potential, and $\phi_r(t,x,z)$ is the radiated flow potential. The boundary conditions formulated in section 2.3 are sufficient to solve all of the terms in Equation 2.15. Therefore, for $\phi_r(t,x,z)$, an extra boundary condition is added for the oscillating body; that is

$$\frac{\partial \phi_r(t,x,z)}{\partial n} = v_n,$$

(2.16)

where $v_n$ represents the velocity of the floating submerged rigid body along the unit vector $\bar{n}$ [24]. Similarly, the boundary condition associated with the diffraction
component is added to solve for $\phi_d(t, x, z)$ [24]; that is

$$\frac{\partial \phi_d(t, x, z)}{\partial n} = -\frac{\partial \phi_i(t, x, z)}{\partial n}, \quad (2.17)$$

which shows that the magnitude of the normal gradient of the diffracted flow and the magnitude of the normal gradient of the incident flow are equal, with a $180^\circ$ phase shift.

**Excitation Force**

A motionless floating body in the open sea is exposed to sea waves. Such a body can be excited by the force applied by an incident wave. This force is called the wave excitation force, denoted by $f_{ex}(t)$. It is composed of the Froude-Krylov force (denoted by $f_{fk}(t)$), and the diffraction force (denoted by $f_d(t)$), and can be expressed as follows:

$$f_{ex}(t) = f_{fk}(t) + f_d(t),$$

$$f_{ex}(t) = \int \int \rho \frac{\partial \phi_i(t, x, z)}{\partial t} \hat{n} \, dS + \int \int \rho \frac{\partial \phi_d(t, x, z)}{\partial t} \hat{n} \, dS, \quad (2.18)$$

where $dS$ is the element surface area of the wet surface of the floating body. This can be seen as integrating the hydrodynamic pressure, which is represented by the first term of Equation 2.7 subject to the variation of the incident and diffracted flow potentials over the wet surface of the floating body. Because the size of the floating body for point absorbers is significantly smaller than the wavelength of the targeted energetic waves, the diffraction force $f_d(t)$ can be disregarded, making the Froude-Krylov force the dominant component of the excitation force, $f_{ex}(t) \approx f_{fk}(t)$ [7].
Radiation Force

When the floating body is excited, it induces waves radiating away from it. This hydrodynamic phenomena causes the surrounding water to apply a force on the submerged portion of the body; this force is called the radiation force \( f_r(t) \). It can also be obtained by solving a separate Laplace equation problem, using its boundary conditions. This force is proportional to the oscillating body velocity and acceleration in a causal relationship [25].

Because the analytical solution using partial differential equations for the radiation force is complicated, another method using specialised software and frequency domain is proposed and accomplished in [15]. This method will be explained in section 2.5.

Hydrostatic Buoyancy Force

Another force that is generated by the motion of the floating body is the hydrostatic (buoyancy) force \( f_b(t) \). It stems from the variation of the hydrostatic pressure expressed by the third term of Equation 2.7, as a result of the body movement. At equilibrium, the weight of the floating body is equal to the weight of the displaced water; therefore, any oscillation of the body will result in a mismatch between these two forces, as explained in [7]

\[
f_b(t) = mg - p g V_{disp},
\]

(2.19)

where \( m \) is the body mass and \( V_{disp} \) is the volume of the displaced water. At equilibrium, when the buoyancy force \( f_b(t) \) is zero, we can see the \( m = \rho V_{disp} \). Alternatively, we can say that the volume of the displaced water is equal to the volume of the wet surface of the floating body. For a freely oscillating body, \( f_b(t) \) is proportional to the body displacement from the equilibrium point and opposes
the direction of the movement, that is

\[ f_b(t) = -S_p e(t). \]  

(2.20)

where \( S_p \) is the buoyancy stiffness coefficient and is equal to \((\rho g A_w)\). For a spherical buoy, the device adopted in this study, the term \( A_w \) varies with the buoy’s vertical displacement and contains a non-linear term from five buoy excursions; the effect of this non-linear term can be disregarded, and hence [15]

\[ A_w \approx \pi r^2. \]  

(2.21)

where \( r \) is the radius of the oscillating body.

**Drag Force**

Morison’s equation describes the sea wave loads on a fixed or moving structure. For an oscillating body, the Morison’s equation is composed of inertia and drag terms, with two empirical coefficients for each term [26]. In [15], only the drag term is considered, which is a function of the signed square of the buoy velocity, as follows:

\[ f_d(t) = 0.5 \rho A_w C_D |v(t)| |v(t)|, \]  

(2.22)

where \( C_D \) is the drag coefficient, which is determined through experiments and depends on the flow conditions and Reynolds number [27].

**2.3.3 Power Take-off Force as a Control Force**

The main objective of developing controlling techniques for WECs is to maximise the captured energy from sea waves. For the point absorber, this occurs when the motion of the floating part is in resonance with the excitation force, i.e. having the same velocity [7]. Thus, in order to maximise the captured power, a controlled power take-off (PTO) mechanism is required. PTO could be mechanical, electrical,
or a combination of both [28]. In this work, an electrical direct drive PTO mechanism is used, namely, a permanent magnet linear generator (PMLG). As the name implies, direct drive enables the linear generator to be directly driven by the reciprocating motion of the WEC's buoy, as shown in Figure 2.3.

![Mechanism for extracting energy from sea waves using a heaving point absorber WEC](image)

Figure 2.3: Mechanism for extracting energy from sea waves using a heaving point absorber WEC

It is important to note that, besides using the PTO as an electricity generator that generates electricity from the mechanical power contained in sea waves, it is also used to control the motion of the WEC's buoy, or more precisely, apply the control force (or law), $f_c(t)$, on the oscillating WEC body (i.e. system actuator). This control force is generated by controlling the currents running in the PMLG [15].
2.4 Power Take-off

The forces applied on the oscillating buoy by water have been discussed in subsection 2.3.2. Evaluating methods of these forces will be discussed in section 2.5. However, a brief representation for the evaluation of mechanical forces, which we refer to as the PTO forces, is provided in this section. This includes the controlled electro-mechanical force applied by the direct drive actuator; in this case, the permanent magnet linear generator (PMLG), \( f_e(t) \), the restoring spring force, \( f_{rs}(t) \), and the mechanical friction force all influence the moving parts of the PMLG, \( f_f(t) \), and the end-stop force, \( f_{es}(t) \). All can be grouped as [15]

\[
 f_{pto}(t) = f_e(t) + f_{rs}(t) + f_f(t) + f_{es}(t). \tag{2.23}
\]

2.4.1 PTO Control Force

As mentioned earlier, the control force is enforced through the PMLG and the associated power converters. Similar to rotary machines, the reciprocating motion of the PMLG produces a time-varying synchronous flux due to the machine permanent magnets. This, in turn, induces voltage in the stator windings known as electromotive force (EMF), as stated in Faraday's law [29]. For the same segment thickness, permanent magnets can produce a magnetic flux 10 times greater than that produced by copper windings. Moreover, permanent magnet-based generators offer a more reliable solution, because less maintenance is required. Compared to induction machines of the same size, permanent magnet synchronous machines are smaller, owing to the lower copper volume in the stator. Moreover, high forces can be maintained at low speeds, which is better suited to slower systems such as heaving WECs [30]. The most widely used rare-earth permanent magnets are neodymium, iron, and boron (NdFeB) magnets [31]. When the machine is loaded, current starts to flow in the stator windings, resulting in a magnetic flux that opposes the main flux caused by the magnets. This effect is referred to as an armature
reaction. Because the magnetic field produced by the permanent magnets does not change with time, the armature reaction effect offers a means to control the motion of the machine [32]. Practically, the armature reaction magnetic field is altered by varying the current running in the stator circuitry, through varying the voltage at the generator terminals. The power converters are configured in a back-to-back scheme, namely, the machine-side converter (MSC) and the grid-side converter (GSC). The MSC is responsible of controlling the PMLG and hence the motion of the WEC’s buoy, while the GSC regulates the power fed to the grid and maintains the voltage of the dc link to ensure proper operation of the converter [33].

2.4.2 Restoring Force

Restoring force, \( f_{rs}(t) \), is another spring force besides the previously mentioned hydrodynamic buoyancy spring force. It acts on the sea-based PTO heaving WEC, and results from the spring units placed between the linear moving translator and the seabed [34]. The objective of placing these units is to bring down the buoy to equilibrium point post the wave crest, with the use of gravitational force. In addition, they help to maintain tether tension, in order to reduce nonlinearities due to a loose tether [14]. This force is directly proportional to the buoy displacement, \( z(t) \) as follows:

\[
 f_{rs}(t) = S_{rs}z(t),
\]  

(2.24)

where \( S_{rs} \) is the restoring spring coefficient.

2.4.3 Non-linear Forces

In addition to the previously mentioned forces, there are two additional non-linear forces acting on the PTO: friction force and end-stop force. These forces result in non-linear terms in the mechanical time domain model in section 2.5, which is assumed to be zero; hence, we obtain a linear WEC time-domain model.
Friction Force

Friction force in direct-drive WECs results from the movement of the translator with respect to the supporting assembly of the generator. Despite the use of bearings to facilitate the translator’s movement and a fixed air gap between the translator and the stator circuit, friction might increase with time owing to the rough environment of the sea [34].

Because of the high non-linearity associated with this type of force and its dependence on system specifications, it is very difficult to model. Therefore, it is usually modelled empirically, specifically for robotics applications which require high precision positioning [35]. In control systems, friction forces may produce undesirable consequences (steady state errors, limit cycles, etc.) in closed loops [36]. In the literature, available friction models can be classified as either dynamic or static models. A model of the latter, which are only dependent on the relative velocity of the moving bodies, is adopted in [15]. It consists of the classical Coulomb, viscous, and Stribeck effect components. Based on this model, the friction force can be expressed by:

\[ f_f(t) = \alpha_c \text{sign}(v(t)) + \alpha_v v(t) + (\alpha_s - \alpha_c) e^{-v(t) / \nu_s} \text{sign}(v(t)), \quad (2.25) \]

where \( \alpha_c \) is directly proportional to the normal force, \( F_n \). The proportionality constant is \( \mu_c \), which represents the Coulomb friction coefficient. The first term of Equation 2.25 represents the Coulomb force. In the second term, multiplying the viscous friction coefficient, \( \alpha_v \), by the body’s relative velocity results in the viscous friction force. The last term models stiction and Stribeck effect. Stiction is the friction force that must be overcome to move a body lying at rest, while the Stribeck effect models the behaviour of the friction force around the zero velocity to create an exponential decay, rather than an abrupt (discontinuous) drop [37].
End-stop Force

As with any machine, the PTO has its limitations. With respect to these limitations, and to prevent the PMLG translator from damaging the machine enclosure at large excursions, end-stop springs are used. In the Uppsala University WEC model, these springs are placed at the upper side of the PMLG enclosure. They become active when the translator maximum is reached during a very high wave crest [14]. On the other hand, the restoring springs prevent the extreme down motion of the translator when excited by harsh wave troughs [38]. The end-stop force, $f_{es}(t)$, can be expressed mathematically as:

$$f_{es}(t) = S_{es} [z(t) - \text{sign}(z(t))z_{limit}] H(|z(t)| - z_{limit}),$$  \hspace{1cm} (2.26)

where $H$ represents the Heaviside (unit step) function, $S_{es}$ is the end-stop spring coefficient, and $z_{limit}$ is the maximum displacement of the buoy that can be tolerated by the machine. In order to avoid abrupt movements of the translator at both ends, $S_{es}$ is chosen to be reasonably stiff.

During normal operation conditions, end-stop force is equal to zero. Thus, it is not considered in the linear time-domain model in section 2.5. On the other hand, the effect of $f_{es}(t)$ beyond the displacement limit, $z_{limit}$, is represented by enforcing bidirectional limitations on the translator displacement, and hence, the buoy displacement [15].

2.5 Mechanical Model

As mentioned earlier, the adopted mechanical model for this study was developed in [15]. It is a linear time-invariant (LTI) model in time domain and is explained later in this section. All control methodologies developed in this study were reference-based techniques that used the velocity of the striking wave as a reference. Thus, a method for evaluating the excitation force of the striking wave
and hence its velocity is discussed in this section. The same tool and methodology were used to evaluate the radiation force, in a manner that enabled it to fit into the LTI model.

Because the hydrodynamic forces acting on the WEC are a function of the sea waves' frequency and height, it is more convenient to model the heaving WEC in the frequency domain [25]. In general, a floating body in sea water has six degrees of motion freedom. However, only a single degree of motion freedom, heave mode, was considered in [15]. The reason behind this assumption is that, for a point absorber WEC of relatively small size with respect to the sea wave length, the heave mode is dominant, while other translational and rotational modes have a negligible effect on the accuracy of the WEC model [7].

Analytically solving the Laplace equations for the excitation and radiation force problems formulated in section 2.3 is a very complex task. Alternatively, a variety of numerical tools can be used. Some of these tools have been commercialized and others have been exclusively used in research centres. Among those software packages are WAMIT®, ANSYS® AQWA, and AquaDyn [6]. WAMIT® was the software deployed in [15] to model the excitation force, \( f_{ex}(t) \), and the radiation force, \( f_r(t) \).

### 2.5.1 WAMIT®

WAMIT® software was developed by a research group at MIT in 1987. This software, which is based on the boundary element method (BEM), is specifically used to solve the hydrodynamic problem of off-shore structures and vessels numerically. The basic version computes the velocity potential adjacent to the targeted wet surface of a rigid body, based on the linear wave theory discussed in subsection 2.3.1. Both the potential and the body are meshed into panels, upon which the problem is solved. This work used only one of the simple floating body geometries which were considered in [15]: the hemispherical buoy. In addition to
the buoy’s geometry, the sea depth and the considered wave frequency range were fed to WAMIT®. Based on the modelling criteria, the user has the freedom to select the required hydrodynamic variables [15].

2.5.2 Wave Excitation Force

The excitation force can be modelled, in terms of the undisturbed wave elevation \( \eta(t) \), in time domain by a bilateral convolution integral as follows:

\[
f_{ex}(t) = \int_{-\infty}^{\infty} k_{ex}(\tau - t)\eta(\tau)\,d\tau,
\]

(2.27)

where \( k_{ex}(t) \) is the excitation convolution kernel, which can be seen as the excitation response that occurs when the wave elevation at the centre of gravity of the floating body is a unit impulse function \( \delta(t) \). Thus, it is also known as the excitation impulse response function (IRF) [39]. The Fourier transform of \( f_{ex}(t) \) can be written as

\[
F_{ex}(j\omega) = \mathcal{F}(f_{ex}(t)) = \int_{-\infty}^{\infty} e^{j\omega t} f_{ex}(t)\,dt,
\]

(2.28)

where \( \mathcal{F}(\bullet) \) is the Fourier transform operator. Applying the Laplace operator, \( \mathcal{L}(\bullet) \), to \( f_{ex}(t) \), \( F_{ex}(j\omega) \) can also be expressed as

\[
F_{ex}(t) = \mathcal{L}(f_{ex}(t)) = F_{ex}(s)\big|_{s=j\omega},
\]

(2.29)

where \( S \) is the Laplace argument. Applying the Fourier transform to Equation 2.27, \( f_{ex}(t) \) can be expressed in frequency domain as

\[
F_{ex}(j\omega) = K_{ex}(j\omega)H(j\omega),
\]

(2.30)

where \( K_{ex}(j\omega) \) is the frequency domain wave excitation coefficient known as the excitation frequency response function (FRF), and \( K_{ex}(j\omega) \) is the Fourier transform of the wave elevation \( \eta(t) \) [40]. A numerical solution, using WAMIT®, was carried
out to obtain the magnitude and phase components of the excitation FRF $K_{ex}(j\omega)$ for a range of wave frequencies for the adopted semi-submerged spherical buoy.

However, the relation in Equation 2.27 is non-causal. That is, $f_{ex}(t)$ does not necessarily occur after the water surface is elevated at the buoy centre of gravity [7]. Therefore, for linearisation purposes, a causal approximation for $K_{ex}(j\omega)$ is required. This can be achieved by introducing a small time delay to the frequency values of $F_{ex}(j\omega)$ [39], thus we can say

$$\hat{K}_{ex}(j\omega) = K_{ex}(j\omega)e^{-j\omega\tau}.$$  \hspace{1cm} (2.31)

where $\hat{K}_{ex}(j\omega)$ is the causalised version of $K_{ex}(j\omega)$ and $\tau$ is the time delay. A frequency-based technique was adopted in [15] to approximate $\hat{K}_{ex}(j\omega)$. The technique utilises the frequency-dependent data generated by the hydrodynamic numerical tools, such as WAMIT®. These data are then fitted in transfer functions using frequency identification methods [39,40]. Then $\hat{K}_{ex}(j\omega)$ can be approximated in the Laplace domain, at $s = j\omega$, with a strictly proper transfer function as follows [15]:

$$\hat{K}_{ex}(j\omega) \approx \frac{N(s)}{D(s)} = \frac{b_{m}s^{m} + b_{m-1}s^{m-1} + \ldots + b_0}{s^n + a_{m-1}s^{m-1} + \ldots + a_0},$$ \hspace{1cm} (2.32)

where $N(s)$ is an $m^{th}$ order numerator polynomial and $D(s)$ is an $n^{th}$ order denominator polynomial. The geometry, and hence the hydrodynamics of the floating body, affect the order $n$ of the approximated excitation transfer function. Moreover, to ensure that $\hat{K}_{ex}(j\omega)$ decays asymptotically to zero as $\omega$ approaches infinity, the transfer function must be strictly proper, i.e. $n > m$. In addition, stability is imposed by stabilising the unstable poles of the identified transfer function [41].
Using MATLAB\textsuperscript{\textregistered} \textit{ircledR}, coefficients of were calculated as [10]

\[
\hat{K}_{ex} = \frac{\hat{F}_{ex}(s)}{H(s)} = \frac{-2145s^7 + 1.2 \times 10^4s^6 + \ldots}{s^8 + 0.9s^7 + 2.8s^6 + 1.5s^5 + 2.3s^4 + \ldots - 4.9 \times 10^4s^5 + 3.9 \times 10^4s^4 + \ldots} \frac{0.7s^3 + 0.7s^2 + \ldots}{4.2 \times 10^4s^3 + 1.3 \times 10^4s^2 - 5979s}{0.1s + 0.06} \tag{2.33}
\]

Thus, $\hat{F}_{ex}(j\omega)$ was approximated linearly to a transfer function in the frequency domain, which can be expressed later in the time-domain state-space representation.

### 2.5.3 Wave Radiation Force

Cummins [42] proposed an expression for the time-domain radiation force $f_r(t)$ for a vessel with zero forward speed as

\[
f_r(t) = -m_{\infty}a(t) - \int_0^t K_r(t - \tau)v(\tau)d\tau. \tag{2.34}
\]

Equation 2.34 is composed of two parts, a direct linear proportional relationship with the buoy's heave acceleration, $a(t)$. The proportionality coefficient, $m_{\infty}$, represents the body's added mass when the frequency approaches infinity. The other term models the dynamics of the fluid-memory, that is, the energy dissipated by the radiated waves and the inertial energy stored in the water adjacent to the buoy's wet surface. The radiation force, $f_r(t)$ exists only during the movement of the floating body, i.e. $v(t) \neq 0$. Therefore, the radiation impulse response function (IRF) is convolved with the buoy velocity, where $\tau$ is the integral time variable.

By using Ogilvie's relations, the radiation IRF $K_r(t)$ can be related to the radiation frequency domain components, namely, the radiation resistance
(damping), $R_r(\omega)$, and the radiation added mass, $M_r(\omega)$, as follows [43]:

$$R_r(\omega) = \int_0^\infty K_r(t) \cos(\omega t) dt.$$  \hspace{1cm} (2.35)

$$M_r(\omega) = m_\infty - \frac{1}{\omega} \int_0^\infty K_r(t) \sin(\omega t) dt.$$ \hspace{1cm} (2.36)

The radiation resistance $R_r(\omega)$ represents the energy dissipation term of the radiation force, while the radiation added mass $M_r(\omega)$ and the physical mass of the buoy $m$ can be seen as additional inertia terms. However, both of them vary with the wave frequency [41]. By applying an inverse Fourier transform on Equation 2.35, we can obtain $K_r(t)$ as follows:

$$K_r(t) = \frac{2}{\pi} \int_0^\infty R_r(\omega) \cos(\omega t) d\omega.$$ \hspace{1cm} (2.37)

The Fourier transform was performed on $K_r(t)$ to find the radiation FRF, $K_r(j\omega)$. Subsequently, Euler’s formula was utilised along with Equation 2.35 and Equation 2.36 to obtain

$$K_r(j\omega) = R_r(j\omega) + j\omega[M(\omega) - m_\infty].$$ \hspace{1cm} (2.38)

The frequency-domain representation of the radiation force $F_r(j\omega)$ was evaluated by taking the Fourier transform of Equation 2.34 and utilising Equation 2.38 as

$$F_r(j\omega) = K_r(j\omega)V(j\omega),$$

$$F_r(j\omega) = [R_r(j\omega) + j\omega M(\omega)]V(j\omega),$$ \hspace{1cm} (2.39)

where $V(j\omega)$ is the Fourier transform of the buoy velocity. Similar to $K_{\text{ex}}(j\omega)$, a frequency-domain identification technique was deployed to identify a stable
transfer function that estimates $K_r(j\omega)$. This technique resulted in [15]

$$K_r(s) \approx \frac{1.099 \times 10^6 s(s + 0.242)}{(s + 5.248)(s + 0.378)(s + 1.016 + j0.761)(s + 1.016 - j0.761)}$$  \hspace{1cm} (2.40)

### 2.5.4 Velocity of the Excitation Wave

Cummins proposed a time-domain equation that models the motion of a free-floating body on the water, while the body is exposed to the hydrodynamic forces applied by the water; this equation is based on Newton’s second law [42]. For a buoy oscillating freely in heave motion, Cummins’ equation of motion can be written as:

$$f_{ex}(t) - f_r(t) - f_b(t) = ma(t). \hspace{1cm} (2.41)$$

By applying a Fourier transform and substituting Equation 2.20 and Equation 2.39 into Equation 2.41, we obtain

$$F_{ex}(j\omega) - K_r(j\omega)V(j\omega) - S_bZ(j\omega) = mA(j\omega), \hspace{1cm} (2.42)$$

where $Z(j\omega)$ and $A(j\omega)$ are the Fourier transform of the heave displacement and acceleration, respectively. By substituting Equation 2.38 into Equation 2.42 and rewriting the latter in terms of $Z(j\omega)$, we obtain

$$[j\omega R_r(\omega) + S_b - \omega^2(m + M_r(\omega))]Z(j\omega) = F_{ex}(j\omega). \hspace{1cm} (2.43)$$

Utilising Equation 2.30, we can obtain the transfer function that relates the incoming wave elevation to the buoy’s heave displacement as

$$\frac{Z(j\omega)}{H(j\omega)} = \frac{K_{ex}(j\omega)}{j\omega R_r(\omega) + S_b - \omega^2(m + M_r(\omega))}. \hspace{1cm} (2.44)$$

Knowing that $V(j\omega) = j\omega Z(j\omega)$, Equation 2.44 can be rewritten to reflect the
transfer function that relates the wave elevation to the buoy heave velocity, such as

\[
\frac{V(j\omega)}{H(j\omega)} = \frac{j\omega K_{ex}(j\omega)}{j\omega R_r(\omega) + S_b - \omega^2 (m + M_r(\omega))}.
\]

(2.45)

Electrical Analogue

Analysing a problem using its electrical analogue is more convenient in some cases. For a freely-floating buoy, the hydrodynamic forces influencing the buoy can be seen as a single-loop electrical circuit, in which the excitation force is the potential source, and all other hydrodynamic forces are represented by an intrinsic impedance \(Z_{int}(j\omega)\). To reflect this analogue, Equation 2.45 was rewritten as [15]

\[
\frac{V(j\omega)}{F_{ex}(j\omega)} = \frac{1}{Z_{int}(j\omega)} = \frac{1}{R_r(\omega) + j[\omega(m + M_r(\omega)) - \frac{S_b}{\omega}]}. \tag{2.46}
\]

which means

\[
Z_{int}(j\omega) = R_{int}(\omega) + jX_{int}(\omega) = R_r(\omega) + j[\omega(m + M_r(\omega)) - \frac{S_b}{\omega}]. \tag{2.47}
\]

where \(R_{int}(\omega)\) and \(X_{int}(\omega)\) are the intrinsic resistance and reactance, respectively. Although the real part of the intrinsic impedance is represented only by the radiation resistance, \(R_r(\omega)\), any other resistive hydrodynamic forces can be included, such as the resistance of the linearised version of the viscous force [15].

In an attempt to evaluate the intrinsic impedance, \(Z_{int}(j\omega)\), the system can be handled as a mass-spring-damper system; hence, it can be modelled by a linear time-invariant second-order model of this form [15]

\[
\frac{1}{Z_{int}(j\omega)} = \frac{1}{-\omega^2 + 2\zeta\omega_0\omega + \omega_0^2}, \tag{2.48}
\]

where \(\zeta\) is the system damping ratio, which describes how rapidly the system oscillations are suppressed before coming to rest, and \(\omega_0\) is the natural frequency
of the system at which the phase of the intrinsic impedance changes from negative values to positive values and the system reactance becomes zero (i.e. the behaviour of the system changes from capacitive to inductive) [15, 44]. Utilising Equation 2.48 to Equation 2.48, we can derive the following expressions for $\omega_0$ and zeta

$$\omega_0 = \sqrt{\frac{S_b}{m + M_r(\omega)}}.$$  
(2.49)

$$\zeta = \frac{R_r(\omega)}{2 \sqrt{S_b(m + M_r(\omega))}}.$$  
(2.50)

2.5.5 State-Space Representation

A state-space model for the heaving spherical WEC was developed in [15], based on the hydrodynamic and mechanical forces that govern the buoy and its interaction with sea waves; these forces are discussed in this chapter. In a state-space model, a system can be modelled using a set of nonlinear differential equations of the following general form

$$\dot{x} = f(t, x, u),$$  
(2.51)

where $f \in \mathbb{R}^{n \times 1}$ is an $n^{th}$ order vector of nonlinear functions, $x \in \mathbb{R}^{n \times 1}$ is the $n^{th}$ order state vector, and $u \in \mathbb{R}^{m \times 1}$ is the $n^{th}$ order input vector [45].

For our heaving WEC system, the system states are the buoy’s heave (vertical) position and its velocity, $z(t)$ and $v(t)$ respectively. The first state can be measured by the mean of sensors, while the latter can be computed online using the position measurements. These are the main state variables that will be involved in design. There are four additional fictitious states. These four states, as discussed later in this chapter, resulted from the method used to model the radiation force, $f_r(t)$, and to impose it onto the state model. Thus, the system’s proposed model is composed of six state-space variables, and hence it is a sixth-order system. Because position and velocity represent the main state variables, all the forces that dictate the interaction between the buoy and the surrounding water were expressed as a function of these
two variables in the previous section, except for the wave excitation force, \( f_{ex}(t) \), which is independent on the buoy dynamics. Using Newton's second law, all forces acting on the buoy and the PMLG translator can be formulated as

\[
f_{ex}(t) - f_{r}(t) - f_{h}(t) - f_{rs}(t) - f_{d}(t) - f_{es}(t) - f_{f}(t) + f_c(t) = ma(t) . \tag{2.52}
\]

Earlier in this chapter, a frequency-domain linear representation for modelling the radiation force was derived as described in Equation 2.39. An equivalent time-domain representation is required, so that it can be integrated into one holistic time-domain model of the system. Several works have been conducted on how to handle the radiation convolution term in the time domain [40, 46]. In [15], the radiation convolution term was approximated by a fourth-order linear state-space model, as follows:

\[
\dot{q} = A_r q(t) + B_r v(t) \tag{2.53}
\]

\[
\int_0^t k_r(t - \tau)v(\tau)d\tau \approx C_r q(t) \tag{2.54}
\]

where \( q(t) \in \mathbb{R}^{4 \times 1} \) is the radiation state vector, the buoy's velocity \( v(t) \in \mathbb{R}^{1 \times 1} \) is the input, and the radiation force \( f_r(t) \in \mathbb{R}^{1 \times 1} \) is the model output. The state matrices of the model are \( A_r \), the radiation state matrix, \( B_r \), the radiation input matrix, and \( C_r \), the radiation output matrix.

Hence, for the overall WEC's state-space model, the state vector is \( x(t) = [z(t), v(t)q(t)]^T \), where \( x(t) \in \mathbb{R}^{6 \times 1} \) that is, \( x_1(t) = z(t) \), \( x_2 = v(t) \), and \( x_3(t) = q(t) \). The wave excitation force, \( f_{ex}(t) \), and the input control force, \( f_c(t) \), represent the inputs of the system. The model output \( y(t) \) is the buoy's heave velocity \( v(t) \). Accordingly,
the overall time-domain model can be summarised as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m+m_{\text{m}}} \left[ f_{\text{ex}}(t) - C_r x_3 - (S_h + S_{\text{fe}}) x_1 - 0.5 A_\omega C_D |x_2| x_2 \\
&\quad - S_{\text{es}} |x_1 - \text{sign}(x_1) z_{\text{limit}}| \mathbb{H}(|x_1| - z_{\text{limit}}) \\
&\quad - \alpha_c \text{sign}(x_2) - \alpha_c x_2 - (\alpha_s - \alpha_c) e^{-\left(\frac{\nu_2}{\nu_1}\right)} \text{sign}(x_2) + f_c(t) \right] \\
\dot{x}_3 &= A_r x_3 + B_r x_2 \\
y &= x_2 .
\end{align*}
\] (2.55)

Writing the model in state-space matrices form, we obtain

\[
\dot{x} = Ax + B(w(t) + u(t)) + \Theta
\] (2.56)

where

\[
A = \begin{bmatrix}
0 & 1 & \mathbf{0}_{1 \times 4} \\
-\frac{(S_h + S_{\text{fe}})}{m+m_{\text{m}}} & -u_c & -C_k \\
\mathbf{0}_{4 \times 1} & B_k & A_k
\end{bmatrix} \in \mathbb{R}^{6 \times 6},
B = \begin{bmatrix}
0 \\
\mathbf{1}_{m+m_{\text{m}}} \\
\mathbf{0}_{4 \times 1}
\end{bmatrix} \in \mathbb{R}^{6 \times 1}
\]

\[
\Theta = \begin{bmatrix}
-\frac{S_{\text{es}} |x_1 - \text{sign}(x_1) z_{\text{limit}}| \mathbb{H}(|x_1| - z_{\text{limit}})}{m+m_{\text{m}}} \\
-0.5 A_\omega C_D |x_2| x_2 - \alpha_c x_2 - (\alpha_s - \alpha_c) e^{-\left(\frac{\nu_2}{\nu_1}\right)} \text{sign}(x_2) \\
\mathbf{0}_{4 \times 1}
\end{bmatrix} \in \mathbb{R}^{6 \times 1}
\]

\[
C = \begin{bmatrix}
0 & 1 & \mathbf{0}_{4 \times 1}
\end{bmatrix} \in \mathbb{R}^{1 \times 6}
\]

Note that \(u(t)\) and \(w(t)\) represent the input control force, \(f_c(t)\), and the wave excitation force, \(f_{\text{ex}}(t)\), respectively. All the nonlinear terms were grouped in the matrix \(\Theta\). Assuming operation in a linear region, i.e. \(\Theta = 0\), the model formulated in Equation 2.56 can be reduced to

\[
\dot{x} = Ax + B(w(t) + u(t))
\] (2.57)

\[
y = Cx ,
\] (2.58)
where

\[
A = \begin{bmatrix}
0 & 1 & \mathbf{0}_{4 \times 6} \\
-\frac{(s_k + s_{ss})}{m + m_{oc}} & -\frac{s_k}{m + m_{oc}} & -\frac{G_k}{m + m_{oc}} \\
\mathbf{0}_{4 \times 1} & B_k & A_k
\end{bmatrix} \in \mathbb{R}^{6 \times 6}, \quad B = \begin{bmatrix}
0 \\
\frac{1}{m + m_{oc}}
\end{bmatrix} \in \mathbb{R}^{6 \times 1}.
\]

\[
C = \begin{bmatrix}
0 & 1 & \mathbf{0}_{1 \times 3}
\end{bmatrix} \in \mathbb{R}^{1 \times 6}.
\]

The developed state-space model above will be used to design linear controllers in chapter 3.

2.6 Electrical Model

The electrical model consists of the PMLG and the power converter module. This section summarises the adopted model developed in [15]. Figure 2.4 shows the d-q equivalent circuits of the PMLG that represent the synchronous frame direct and quadrature components. The Park transformation is used to transform the three phase voltages and current quantities to the synchronous frame components [47]. The d-q components of the stator voltage, \( v_s(t) \), at the terminal are formulated as the following equations

\[
v_{sd}(t) = R_s i_{sd}(t) - \omega_e(t) \lambda_{sq}(t) + \frac{d}{dt}(L_{sd} i_{sd}(t) + \lambda_{PM}),
\]

\[
v_{sq}(t) = R_s i_{sq}(t) - \omega_e(t) \lambda_{sd} + \frac{d}{dt}(L_{sq} i_{sq}(t)),
\]

\[
\lambda_{sd}(t) = L_{sd} i_{sd}(t) + \lambda_{PM},
\]

\[
\lambda_{sq}(t) = L_{sq} i_{sq}(t),
\]

![Figure 2.4: Equivalent circuit of the PMLG](image-url)
where \( i_s(t), R_s, \) and \( L_s \) are the stator current, the machine synchronous resistance, and inductor, respectively. In this study, the surface mounted PMLG is used. In this topology, the stator inductance quantities in the \( d \)-axis and the \( q \)-axis are almost identical, or \( L_{sd} \approx L_{sq} \) [33]. The variables \( \lambda_{PM} \) and \( \lambda_s \) are the permanent magnet flux and the stator flux linkage, respectively. The variable \( \omega_e(t) \) is the electrical angular frequency that is given by the following equation

\[
\omega_e(t) = \frac{2\pi \dot{z}(t)}{p_w}
\]

where \( p_w \) is the pole width of the PMLG. The converted (electrical) power, \( P_e \), is given by

\[
P_e(t) = \frac{3}{2} p \lambda_{PM} \omega_e(t) i_{sq}(t)
\]

(2.59)

where \( p \) is the number of the magnetic pole pairs. The stator active power, \( P_s(t) \), and reactive power, \( Q_s(t) \), are formulated as

\[
P_s(t) = \frac{3}{2} (v_{sd}(t)i_{sd}(t) + v_{sq}(t)i_{sq}(t))
\]

\[
Q_s(t) = \frac{3}{2} (v_{sq}(t)i_{sd}(t) - v_{sd}(t)i_{sq}(t)).
\]

The control force is implemented by altering the stator current in the PMLG using the back-to-back converter in the MSC. The currents \( i_{sd}(t) \) and \( i_{sq}(t) \) are controlled by regulating the \( v_{sd}(t) \) and \( v_{sq}(t) \), respectively [33]. In order to minimise the copper losses, \( i_{sd} \) is set to zero [47]. The value of \( i_{sq}(t) \) that implements the control force is derived using the following procedure. The captured (mechanical) power, \( P_m \), is formulated as

\[
P_m(t) = f_u(t)\dot{z}(t).
\]

(2.60)

If it is assumed that \( P_e(t) = P_m(t) \), the following is obtained

\[
f_u(t)\dot{z}(t) = \frac{3}{2} p \lambda_{PM} \omega_e(t) i_{sq}(t).
\]
Therefore, the current $i_{sq}(t)$ is

$$i_{sq}(t) = \frac{2f_p(t)\xi(t)}{3p\lambda_PM\omega_e(t)}$$

(2.61)

Practically, there are physical constraints for implementing the current in Equation 2.61, owing to either the rating of the power converter components or the thermal limits in the PMLG [47]. To accommodate the limitations, the constraints are imposed in the voltages and currents. The maximum level of the stator current is dependent on the maximum RMS value of the stator voltage $i_{s}^{\text{max}}$. The stator voltage of the PMLG is controlled by the dc-link voltage $v_{dc}$. In this study, space vector pulse-width modulation (SVPWM) is deployed. Using this method, the relationship between $i_{s}^{\text{max}}$ and $v_{dc}$ can be obtained as follows:

$$v_{s}^{\text{max}} = \frac{v_{dc}}{\sqrt{3}}$$

Then, the maximum currents of $d$-$q$ axis stator is formulated as

$$(-\omega_e(t)L_s i_{sq}(t))^2 + \left(L_s \frac{di_{sq}(t)}{dt} + \omega_e(t)\lambda_PM\right)^2 \leq (i_{s}^{\text{max}})^2.$$  

Expanding the square in the last equation and omitting the $2^{nd}$ order derivative, the following Riccati equation is obtained

$$\frac{di_{sq}(t)}{dt} + \frac{\omega_e(t)L_s}{2\lambda_PM} i_{sq}^2(t) = \frac{v_{s}^{\text{max}} - \omega_e(t)\lambda_PM^2}{2\omega_e(t)\lambda_PM L_s}.$$ 

Solving the last equation, the maximum allowable $i_{sq}$ is given by

$$i_{sq}^{\text{max}}(t) = \lim_{t \to 0} (i_{sq}(t)) \leq \frac{1}{X_s} \sqrt{(v_{s}^{\text{max}})^2 - (\omega_e(t)\lambda_PM)^2}.$$
Chapter 3: Design of Hierarchical Controllers

This work considered mainly hierarchical control strategies (HCSs) rather than the developed control techniques. This strategy is composed of two levels: the lower hierarchical controller (LHC) and higher hierarchical controller (HHC). The upper level, HHC, provides a reference velocity which is in phase with the wave's excitation. This reference signal is utilized by the lower-level controller, LHC, to robustly regulate the velocity of the buoy in the presence of uncertainties and/or disturbances. Configuration of HCS is discussed in section 3.1. Three techniques for designing the LHC are discussed in sections 3.2, 3.3, and 3.4. Moreover, two techniques for HHC are discussed in sections 3.5 and 3.6. In all, five HCS controllers are designed for the heaving WEC. The simulation setup and results of the designed controllers are discussed in chapter 4.

3.1 Control System Configuration

Maximum power absorption of a heaving WEC is achieved if the velocity of the buoy \( v(t) = \dot{z}(t) \) is in phase with the excitation force \( f_{ex}(t) \). This is known as the resonance condition resulting from the impedance matching principle, based on which we can formulate the reference velocity as [48]

\[
\dot{z}_r(t) = \frac{|f_{ex}(t)|}{2R_{im}(\omega)} \cos \theta ,
\]

where \( \theta \) is the phase difference between \( f_{ex}(t) \) and \( \dot{z}(t) \), and it equals zero at resonance. Usually, the constant \( R_{im}(\omega) \) in Equation 3.1 is selected from the maximum value of \( R_{im}(\omega) \) over a range of operating frequencies.

In order to obtain resonance status, the control force generated by the PMLG, \( f_u(t) \), is used. This results in well-known drawbacks of the approach in Equation 3.1. The constant \( R_{im} \) is selected from one frequency only. Thus, a high
level of \( f_d(t) \) due to insufficient damping of \( f_{ex}(t) \) can be caused. In the reactive control strategies, the high-level control is undesirable because it enlarges the size of the PTO and reduces the efficiency of WEC systems. In the worst-case condition, a high level of \( f_d(t) \) can cause power reverse. As a result, we can reach a point where the PTO acts as a motor rather than a generator. To avoid these drawbacks, a bottom-up hierarchical control strategy (BU-HCS) was proposed.

The control system configuration for the BU-HCS is shown in Figure 3.1. The BU-HCS consists of the lower hierarchical controller (LHC) and higher hierarchical controller (HHC). The HHC gives the reference to be followed by the LHC. The LHC follows this reference, despite uncertainties in the model. The HHC ensures that the reference will not cause the control force to exceed its maximum value in the LHC. This can be achieved by calculating the intrinsic impedance in the HHC, using transfer functions in the HHC and LHC. In order to fulfil this objective, the LHC is designed before the HHC.

Two designs were developed for the HHC, and three designs were developed for the main controller of the LHC, as follows:

- **Lower hierarchical controllers (LHCs)**
  - LHC via robust lead-lag compensator
  - LHC via proportional-integral-derivative (PID) augmented sliding mode control (SMC) controller
  - LHC via complex polynomial stabilisation method

- **Higher hierarchical controllers (HHCs)**
  - HHC via transfer functions method
  - HHC via optimum reference generation using quadratic programming (QP)
3.2 LHC via Robust Lead-Lag Compensator

The feedback control system in the LHC, as shown in Figure 3.1, consists of the mechanical model of the WEC indicated by the transfer function, $\mathcal{P}$, which itself consists of the nominal plant’s model, $P(s)$, and input multiplicative uncertainty represented by the transfer function, $\Delta(s)$. The latter represents unknown uncertainties such as parameter perturbation, unmodelled dynamics or forces, etc. The signal $f_{dis}(t)$ represents any possible external disturbance that may alter the buoy’s velocity. Another component of the feedback control system is the controller, $K(s)$.

In this technique, the controller in the LHC is designed using a simple structured lead-lag compensator (LLC) formulated as

$$K(s) = k \frac{s + z}{s + p},$$

(3.2)

where the constants $k$, $z$, and $p$ are the gain, zero, and pole of the LLC, respectively. These parameters are designed to achieve robust stability and tracking performance, taking into the account the limitation of the control force, $f_u(t)$.

The perturbed plant $\mathcal{P}$ can be seen as a set of transfer functions containing all uncertainties, represented by $\Delta(s)$. The controller $K$ provides robust stability if it provides internal stability for every plant in the set $\mathcal{P}$. This is achieved by using the small-signal theorem stated below [49].

**Theorem 3.1** For stable $\Delta(s)$, the closed-loop system is robustly stable if $K(s)$
stabilises the nominal plant and the following holds

$$\left\| \frac{\Delta(s)K(s)P(s)}{1+K(s)P(s)} \right\|_\infty < 1$$

or, in a strengthened form:

$$\left\| \frac{K(s)P(s)}{1+K(s)P(s)} \right\|_\infty < \frac{1}{\|\Delta\|_\infty}$$

In order to stabilise the largest set of perturbation in $\Delta(s)$, the following minimisation problem must be solved:

$$\text{minimise } K(s) \text{stabilising} \left\| \frac{K(s)P(s)}{1+K(s)P(s)} \right\|_\infty$$

(3.3)

Tracking performance requirements can be formulated within the framework of the $H_\infty$ theory, using the nominal closed-loop system of the LHC in Figure 3.1. The sensitivity transfer function is

$$S = \frac{e(t)}{\hat{z}_r(t)} = \frac{1}{1+P(s)K(s)}$$

(3.4)

The sensitivity transfer function represents the tracking performance. Thus, minimising its $\infty$-norm while keeping $K(s)$ stable achieves good tracking performance. This can be expressed by the following optimisation problem:

$$\text{minimise } K(s) \text{stabilising} \left\| \frac{1}{1+P(s)K(s)} \right\|_\infty$$

(3.5)

Similarly, an optimisation problem can be formulated to achieve low control force as one of the performance specifications. The transfer function between $f_u(t)$ and $\hat{z}_r(t)$ is

$$F_u(s) = \frac{f_u(t)}{\hat{z}_r(t)} = \frac{K(s)}{1+P(s)K(s)}$$

hence, the following optimisation problem must be solved to achieve minimum
In order to achieve the previously mentioned specifications of robust stability, tracking performance, and minimum control effort, we need to combine the optimisation problems in 3.3, 3.5, and 3.6, and solve them simultaneously to find the admissible LLC parameters. The combining process results in the following minimisation problem:

$$\text{minimise} \left\| \frac{K(s)}{1 + P(s)K(s)} \right\|_{\infty}$$

The genetic algorithm optimisation toolbox (GAOT) is used to find the parameters of the LLC by solving the optimisation problem in Equation 3.7. The GAOT offers a flexible method to solve optimisation problems, using different structures for the controller. However, the resulting parameters from the GAOT represent a sub-optimal solution, owing to their heuristic properties. The procedure to find the parameters of the LLC using GAOT is illustrated in the flowchart of Figure 3.2.

### 3.3 LHC via PID augmented with SMC

This method considers using a stabilising proportional-integral-derivative (PID) controller in association with a sliding-mode controller (SMC) to achieve stability and robustness.

#### 3.3.1 Sliding Mode Control (SMC)

In the lower control loop, SMC is deployed owing to its remarkable tracking capability and robustness [50]. The proposed SMC is derived using the following
Figure 3.2: Flowchart to find the parameters of the lead-lag compensator using GAOT procedure. Let us define the sliding surface $s(t)$ as

$$s(t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e(t) = 0,$$

where $e(t)$ is the error, $n$ is the order of the system, and $\lambda$ is a positive constant for ensuring the convergence to the sliding surface. In our case, the WEC system is a second-order system. Therefore, the sliding surface is formulated as

$$s(t) = \dot{e}(t) + \lambda e(t) = 0. \quad (3.8)$$

Consider the control signal, $f_u(t)$, of Equation 3.3.2. Owing to mismatches between the nominal model and actual system, the term $k_1 \text{sgn}(s(t))$ is added to this
The control signal for handling model imperfections. The new control signal is

\[ f_u(t) = k_d \dot{e}(t) + k_p e(t) + k_i \int e(t) dt + k_1 \text{sgn}(s(t)) \]  

(3.9)

where \( \text{sgn}(s(t)) \) is the signum function of \( s(t) \):

\[
\text{sgn}(s(t)) = \begin{cases} 
  +1 & \text{if } s(t) > 0 \\
  -1 & \text{if } s(t) < 0.
\end{cases}
\]

The constant \( k_1 \) is a positive real number. A larger value of \( k_1 \) allows a larger set of perturbation. However, a large value of \( k_1 \) will magnify the control force. Therefore, the value of \( k_1 \) is carefully selected by considering the trade-off between perturbations and the allowable level of \( f_u(t) \).

The addition of the signum function in Equation 3.3.1 injects chattering to \( f_u(t) \). In the practical point of view, this chattering is undesirable because the actuator cannot respond to sudden changes in the commanded control force. Moreover, the high frequency components in the chattering might evoke undesirable mechanical vibrations in the system. In order to avoid this, the addition term can be smoothed using the approximation method in [50]. By defining a thin

Figure 3.3: Chattering control force in the sliding surface.
boundary layer surface as:

\[ B(t) = \{ w, |s(w, t)| \leq \Phi \} \quad \Phi > 0 \]

where \( w = [z(t), \dot{z}(t)]^T \), \( B(t) \) is a layer that contains the chattered control force, \( \Phi \) is the boundary layer thickness, and \( \epsilon = \Phi / \lambda^\alpha \) is the boundary layer width as depicted in Figure 3.3. The chattered control input in \( B(t) \) is interpolated by replacing the term \( \text{sgn}(s(t)) \) with the saturation function \( \text{sat}(s(t)) \). The saturation function is defined as follows:

\[
\text{sat}(s(t)) = \begin{cases} 
\text{sign}(s(t)) & \text{if } |s(t)| > \Phi \\
\frac{s(t)}{\Phi} & \text{if } |s(t)| < \Phi.
\end{cases}
\]

Therefore, the robust SMC with continuous control input for the WECs is formulated as

\[
f_u(t) = k_d \dot{e}(t) + k_p e(t) + k_i \int e(t) dt + k_1 \text{sat}(s(t)) \quad (3.10)
\]

In order to obtain the parameters \( k_d, k_p, \) and \( k_i \) of the PID controller, we can use conventional methods for designing the parameters of the PID controllers. Otherwise, we can use a systematic mathematical method that provides sets of stabilising parameters. This method is known as real polynomial stabilisation and is illustrated in the following subsection.

### 3.3.2 PID Stabilisation - Real Polynomials

Conventional ad hoc methods for synthesising a PID controller can be used. Alternatively, an algorithm that defines regions of stabilising parameters \( (k_p, k_i, \) and \( k_d \) for PID controllers in a systematic, mathematical manner can be used. A general form of the theorem of Hermite-Biehler, where the coefficients of a polynomial are real, is used to derive this algorithm. Applying this algorithm leads
to the production of a whole region of PID parameters that guarantee the stability of a nominal system [51]. Mathematical formulations of this algorithm and its steps are presented in this section. A simulation was carried out using Multi-parametric toolbox version 2.0 (MPT 2.0) in the MATLAB® environment.

The standard signum function, \( sgn(x) \), is equal to -1 for negative values of \( x \), 0 when \( x \) is zero, and +1 for positive values of \( x \). We also need to define a real polynomial with degree \( c \) in the form:

\[
\mu(s) = \mu_0 + \mu_1 s + ... + \mu_c s^c
\]

Define \( \mu_e(s^2) \) and \( \mu_o(s^2) \) as even and odd components of \( \mu(s) \), respectively. Substituting \( s = j\omega \) and decomposing the results into real and imaginary components, we obtain

\[
\mu(j\omega) = r(\omega) + ji(\omega)
\]

where \( r(\omega) \) and \( ji(\omega) \) are the real and imaginary components, respectively, and

\[
r(\omega) = \mu_e(-\omega^2) \text{ and } i(\omega) = \omega\mu_o(-\omega^2).
\]

We also define function \( g(\omega) = (1 + \omega^2)^\frac{1}{2} \) and let \( r_g = \frac{r(\omega)}{f(\omega)} \) and \( i_g = \frac{i(\omega)}{f(\omega)} \).

Then, we let

\[
\mu_f(j\omega) = r_g(\omega) + ji_g(\omega)
\]

Let \( z_l(\mu(s)) \) and \( z_r(\mu(s)) \) define the number of zeros of \( \mu(s) \) on the left-half plane and right-half plane, respectively. Moreover, we define the signature of the polynomial \( \mu(s) \) by \( \varrho(\mu(s)) \) as

\[
\varrho(\mu(s)) = z_l(\mu(s)) - z_r(\mu(s)).
\]

Then, a theorem that generalises the theorem of Hermite-Biehler for real-coefficient polynomials can be stated as follows:
Theorem 3.2 Let \( \mu(s) \) be a given real polynomial of degree \( c \) with a root at the origin of multiplicity \( h \).

Let \( 0 = \omega_0 < \omega_1 < \ldots < \omega_{q-1} \) be the real, non-negative, distinct finite zeros of \( r_g(\omega) \) with odd multiplicities. Also define \( \omega_q = \infty \) and denote \( r_h(\omega_0) = \frac{d^h r(\omega)}{d\omega^h} \bigg|_{\omega=\omega_0} \). Then

\[
\varrho(\mu) = \begin{cases} 
\{\text{sgn}[r_h(\omega_0)] - 2\text{sgn}[r_g(\omega_1)] + 2\text{sgn}[r_g(\omega_2)] + \ldots 
+ (-1)^{q-1}2\text{sgn}[r_g(\omega_{q-1})] \cdot (-1)^{q-1}\text{sgn}[i(\infty)] \} 
& \text{if } c \text{ is even.} \\
\{\text{sgn}[r_h(\omega_0)] - 2\text{sgn}[r_g(\omega_1)] + 2\text{sgn}[r_g(\omega_2)] + \ldots 
+ (-1)^{q-1}2\text{sgn}[r_g(\omega_{q-1})] \cdot (-1)^{q-1}\text{sgn}[i(\infty)] \} 
& \text{if } c \text{ is odd.}
\end{cases}
\]

Considering the feedback control system in Figure 3.4, \( r \) is the reference signal, \( y \) is output, and \( P(s) = \frac{P_N(s)}{P_D(s)} \) is the nominal plant to be controlled (\( P_N(s) \) and \( P_D(s) \) are coprime polynomials). \( K(s) \) is the stabilising PID controller, and has a transfer function of the form

\[
K(s) = k_p + \frac{k_i}{s} + k_ds. 
\tag{3.11}
\]

The closed-loop characteristic polynomial equation of the system is

\[
\mu(s,k_p,k_i,k_d) = s P_D(s) + (k_i + k_ds^2) P_N(s) + k_p s P_N(s).
\]
The control objective is to obtain the values of $k_p, k_i, k_d$ that formulate a Hurwitz closed-loop characteristic polynomial, $\mu(s, k_p, k_i, k_d)$. This implies that all the roots of the characteristic polynomial, $\mu$, are in the open left-half plane.

We observe that for $\mu(s, k_p, k_i, k_d)$, controller parameters $k_i, k_d, k_p$ are distributed on $P_N(s)$, so that it causes difficulty in finding all stabilising PID controllers. We will consider the following procedure for solving our design problem. First, we consider the even-odd decomposition of

$$P_N(s) = P_{Ne}(s^2) + sP_{Nd}(s^2)$$

$$P_D(s) = P_{De}(s^2) + sP_{Dd}(s^2)$$

and define

$$P_N^*(s) = P_N(-s) = P_{Ne}(s^2) - sP_{Nd}(s^2).$$

To achieve parameter separation, we multiply $\mu(s, k_p, k_i, k_d)$ by $P_N^*(s)$ to obtain

$$\theta(s) = \mu(s, k_p, k_i, k_d)P_N^*(s)$$

$$= s^2(P_{Ne}(s^2)P_{Dd}(s^2) - P_{De}(s^2)P_{Nd}(s^2)) + (k_i + k_ds^2)(P_{Ne}(s^2)P_{Ne}(s^2))s^2P_{Nd}(s^2)P_{Nd}(s^2)$$

$$+ s[P_{Dd}(s^2)P_{Ne}(s^2) - s^2P_{De}(s^2)P_{Nd}(s^2) + k_p(P_{Ne}(s^2)P_{Nd}(s^2) - s^2P_{Nd}(s^2)P_{Nd}(s^2))].$$

The degrees of the characteristic polynomial, $\mu$, and the numerator of the nominal plant, $P_N(s)$, are denoted by $n$ and $m$, respectively. Next, we substitute $s$ with $j\omega$ in $\theta(s)$, and decompose $\theta(s)$ into a real-imaginary decomposition. We obtain

$$\theta(j\omega) = \mu(j\omega, k_p, k_i, k_d)N^*(j\omega) = r(\omega, k_i, k_d) + ji(\omega, k_p)$$
where

\[ r(\omega, k_i, k_d) = r_1(\omega) + (k_i - k_d \omega^2) r_2(\omega) \]

\[ i(\omega, k_p) = t_1(\omega) + k_p t_2(\omega) \]

\[ r_1(\omega) = -\omega^2 (N_e (-\omega^2) D_o (-\omega^2) - D_e (-\omega^2) N_o (-\omega^2)) \]

\[ r_2(\omega) = N_e (-\omega^2) N_e (-\omega^2) + \omega^2 N_o (-\omega^2) N_o (-\omega^2) \]

\[ t_1(\omega) = \omega (D_e (-\omega^2) N_e (-\omega^2) + \omega^2 D_o (-\omega^2) N_o (-\omega^2)) \]

\[ t_2(\omega) = \omega (N_e (-\omega^2) N_e (-\omega^2) + \omega^2 N_o (-\omega^2) N_o (-\omega^2)). \]

Parameters of the controller are nicely separated in \( \theta(j \omega) \). The proportionality gain (i.e. \( k_p \)) exists only in the imaginary term, \( i \), without any occurrence of other parameters. The other two parameters, \( k_i \) and \( k_d \), are in the real term, \( r \). If the closed right-half plane zeros of \( \theta(s) \), and the same of \( P^*_N(s) \) are of equal number, then the characteristic polynomial, \( \mu(s, k_p, k_i, k_d) \), is said to be Hurwitz.

Let \( \omega_0, \omega_1, \omega_2, \ldots, \omega_{q-1} \) denote the real, non-negative distinct roots of \( i(\omega, k_p) \) of odd multiplicity, where

\[ 0 = \omega_0 < \omega_1 < \omega_2 < \ldots < \omega_{q-1} \]
and define $\omega_q = \infty$. Using Theorem 3.2, the stability condition is that:

$$n = (z_i(P_N(s)) - z_r(P_N(s))) = \begin{cases} 
\{\text{sgn}[r^{(h)}(\omega_0)] - 2\text{sgn}[r_g(\omega_1)] + 2\text{sgn}[r_g(\omega_2)] + \ldots \} 
\times (-1)^{q-1}\text{sgn}[i(\infty)] 
& \text{if summation of } m \text{ and } n \text{ is even} \\
\{\text{sgn}[r^{(h)}(\omega_0)] - 2\text{sgn}[r_g(\omega_1)] + 2\text{sgn}[r_g(\omega_2)] + \ldots \} 
\times (-1)^{q-1}\text{sgn}[i(\infty)] 
& \text{if summation of } m \text{ and } n \text{ is odd} 
\end{cases}$$

From the stability condition in Equation 3.12, a necessary condition is that $i(\omega, k_p)$ has at least

$$i(\omega, k_p) \geq \text{even summation of } m \text{ and } n$$

$$i(\omega, k_p) \geq \text{odd summation of } m \text{ and } n$$

real, non-negative, distinct roots of odd multiplicity. Moreover, "allowable" represents the ranges of $k_p$ that satisfy this condition. For every fixed $k_p$ in the $i$ term, we can determine stabilising values for $k_i$ and $k_d$ in the $r$ term, using the following step:

First, define

$$\text{sgn}[r_g(\omega_b)] = t_b \quad \text{for } b = 0, 1, \ldots, t.$$  

We can construct sequences of number $t_0, t_1, \ldots, t_q$ by the following rule:

- If $N^*(j\omega_b) = 0$ for some $b = 1, 2, \ldots, q-1$, then define

$$t_b = 0.$$
• for all others, \( b = 0, 1, 2, \ldots, q \),

\[ q_b = (-1 \text{ or } 1) \]

With the defined \( t_0, t_1, \ldots \), the set \( A_{(k_p)} \) can be defined as the set of all admissible strings which satisfy the stability condition of Equation 3.12, as

\[
A_{(k_p)} = \begin{cases} 
\{t_0, t_1, \ldots, t_q\} & \text{if the summation } m \text{ and } n \text{ is even} \\
\{t_0, t_1, \ldots, t_{q-1}\} & \text{if the summation } m \text{ and } n \text{ is odd}
\end{cases}
\]  

(3.14)

Next, we determine the admissible strings of \( I = \{t_0, t_1, \ldots, t_{q-1} \text{ or } t_q\} \) in \( A_{(k_p)} \) which satisfy the stability condition of Equation 3.12. We note that each member of the set in \( T \), i.e. \( \{t_0, t_1, \ldots, t_{q-1} \text{ or } t_q\} \) is associated with the sign of \( r \) evaluated at associated frequency \( \omega \); mathematically, we can write this as the following linear inequality:

\[
[r_1(\omega_b) + (k_i - k_d \omega_b^2)r_2(\omega_b)] > 0, \forall b = 0, 1, 2, \ldots, q - 1 \text{ or } q 
\]  

if \( t_b > 0 \)  

(3.15)

\[
[r_1(\omega_b) + (k_i - k_d \omega_b^2)r_2(\omega_b)] < 0, \forall b = 0, 1, 2, \ldots, q - 1 \text{ or } q 
\]  

if \( t_b < 0 \)

The set of stabilising \( k_i, k_d \) is obtained from the intersection of the feasible region of \( k_i \) and \( k_d \) satisfying linear inequalities in 3.15 for the admissible string \( T \). In other words, each string \( T \) in \( A_{(k_p)} \) is one family of the linear inequality problem which can be solved efficiently by linear programming tools. Suppose that we have admissible strings \( T_1, T_2, \ldots, T_v \) in \( A_{(k_p)} \), which are associated with feasible regions of \( k_i, k_d \) denoted by \( S_1, S_2, \ldots, S_v \). The region of all stabilising \( k_i \) and \( k_d \) related to a particular \( k_p \) is given by

\[
S_{(k_p)} = \bigcup_{x=1}^v S_x,
\]  

(3.16)

where \( S_x \) is the set of all stabilising \( k_i \) and \( k_d \) corresponding to the fixed \( k_p \) for each single string in \( A_{(k_p)} \) of 3.14. From this set, we can synthesise a stabilising PID
controller to produce a control signal, \( f_u(t) \), as

\[
f_u(t) = k_d \dot{e}(t) + k_p e(t) + k_i \int e(t) dt \tag{3.17}
\]

where \( e(t) \) is the error between the reference velocity, \( \dot{z}_r(t) \), and the output velocity of the buoy, \( \dot{z}(t) \), and is defined as

\[
e(t) = \dot{z}_r(t) - \dot{z}(t) . \tag{3.18}
\]

### 3.4 LHC via Complex Polynomial Stabilisation

The real polynomial stabilisation algorithm presented in section 3.3 can be extended to provide a robust controller [52, 53]. In this method, weighting transfer functions are incorporated within the algorithm to compensate for the system's uncertainty, and to satisfy design requirements such as tracking performance. This generalisation leads into the problem of complex polynomial stabilisation [53].

![Feedback control system with input multiplicative uncertainty](image)

Figure 3.5: Feedback control system with input multiplicative uncertainty

Referring to the feedback control system shown in Figure 3.5, \( r \) is the reference signal, \( y \) is output, and \( c \) is energy-bounded disturbance. \( P(s) = P_N(s)/P_D(s) \) is the nominal model of the plant subjected to control; the nominator, \( P_N(s) \), and the denominator, \( P_D(s) \) are coprime polynomials. \( \Delta(s) \) represents the perturbation in a form of a proper and stable transfer function with \( \|\Delta\|_\infty \leq 1 \).
The performance specifications and model uncertainty are characterised with two weighting functions in the frequency domain, $W_e(s)$ and $W_T(s)$, respectively. $K(s)$ is a PID controller in the form of Equation 3.11.

The complementary sensitivity function, $T(s)$, and the sensitivity function, $S(s)$, are [54]

$$T(s) = \frac{K(s)P(s)}{1 + K(s)P(s)}$$

$$S(s) = \frac{1}{1 + K(s)P(s)}$$

A design requirement to be considered in this work is disturbance rejection for a plant with input multiplicative uncertainty, as shown in Figure 3.5. This problem can be formulated by the following condition [54]

$$\| W_e(s)S(s) \| + \| W_T(s)T(s) \|_\infty < 1$$  \hspace{1cm} (3.19)

where $W_e(s) = N_e(s)/D_e(s)$ and $W_T(s) = N_T(s)/D_T(s)$, and $N_e(s), D_e(s), N_T(s)$ and $D_T(s)$ are some real polynomials. Moreover, this implies

$$|W_e(\infty)S(\infty)| + |W_T(\infty)T(\infty)| < 1$$  \hspace{1cm} (3.20)

The control objective is to design a PID controller that stabilises the system and satisfies the robust conditions given in Equation 3.19. In order to achieve stability of the nominal system, the characteristic equation must be Hurwitz, i.e. its roots are in the left-half plane. Thus,

$$p(s,k_p,k_i,k_d) = sP_D(s) + (k_i + k_p + k_ds^2)P_N(s)$$  \hspace{1cm} (3.21)

The same procedure considered in the previous section can be applied here to solve the stabilisation problem.
Based on [52, 53], we also define

$$\varpi(s, k_p, k_i, k_d, \theta) \triangleq sD_e(s)D_T(s)P_D(s) + e^{j\theta} sN_e(s)D_T(s)P_D(s)$$

$$+ (k_d s^2 + k_p s + k_i)[D_e(s)D_T(s)P_N(s) + e^{j\theta} D_e(s)N_T(s)P_N(s)].$$

with the following condition:

$$\varpi(s, k_p, k_i, k_d, \theta) \text{ is Hurwitz for all } \theta \text{ and } \in [0, 2\pi). \quad (3.22)$$

Then, for solving our robust PID tuning problem, we must satisfy three conditions simultaneously given in (3.21), (3.20), and (3.22), i.e.

a) $p(s, k_p, k_i, k_d)$ is Hurwitz;

b) $\varpi(s, k_p, k_i, k_d, \theta)$ is Hurwitz for all $\theta$ and $\in [0, 2\pi)$;

c) $|W_e(\infty)S(\infty)| + |W_T(\infty)T(\infty)| < 1.$

The design problem of finding PID controllers that stabilise the system (given the uncertainty in the model) and provide robust performance is solvable if and only if there exists gain value $(k_p, k_i, k_d)$ such that conditions 3.21, 3.20, and 3.22 hold. For a given value of $k_p$, we denote the set of admissible $(k_i, k_d)$ gain values satisfying condition 3.21 to be $\mathcal{F}_{(1,k_p)}$. We also denote the set of admissible $(k_p, k_i, k_d)$ values satisfying condition 3.20 to be $\mathcal{F}_{(2,k_p)}$ where

$$\mathcal{F}_{(2,k_p)} = \cap_{\theta, \phi \in [0,2\pi]} \mathcal{F}_{(2,k_p,\theta)}.$$

All the values of $k_i$ and $k_d$ that satisfy the condition 3.22 are contained in a set denoted by $\mathcal{F}_{(3,k_p)}$. However, for a fixed $k_p$, the set that contains all $k_i$ and $k_d$ values that achieve the robustness specified in condition 3.19, denoted by $\mathcal{F}_{k_p}$, is given by

$$\mathcal{F}_{k_p} = \cap_{i=1}^{3} \mathcal{F}_{(i,k_p)}.$$
The set of all admissible \((k_p, k_i, k_d)\) values that will satisfy the robust performance condition 3.19 can now be found by simply sweeping over the necessary range of \(k_p\) and determining \(\mathcal{F}_{k_p}\) at each stage.

3.5 HHC via Transfer Functions Method

The HHC in Figure 3.1 consists of two transfer functions. The first one from the left, \(F_{ex}\), is the excitation force generator. It converts the wave elevation \(\eta(t)\) to an estimated excitation force \(\hat{f}_{ex}(t)\). This transfer function is represented by the eighth-order transfer function described in subsection 2.5.2 (Equation 2.33) [15]. The second transfer function, \(K_z\), is used to generate the reference velocity \(\dot{z}_r(t)\). The derivation of this transfer function was illustrated in subsection 2.5.4, and it can be simply formulated as

\[
K_z = \frac{\dot{z}_r(t)}{\hat{f}_{ex}(t)} = \frac{1}{R_{tf}^*},
\]

where \(R_{tf}^*\) is the constant of intrinsic resistance to be designed using the transfer functions of the overall HCS.

The intrinsic resistance is designed so that the maximum value of the control force \(f_{um}\) will not exceed its allowable value. This condition occurs in the more energetic sea states of significantly high wave heights, \(H_s\), and low peak frequency, \(\omega_p\). Another design specification for \(R_{tf}^*\) is to keep the ratio between the mean value of the converted electrical power, \(\bar{P}_e\), and the maximum value of the same, \(P_{em}\), below a specified value based on the design requirements. A high \(P_{em}/\bar{P}_e\) ratio indicates higher generated reactive power and hence represents an inefficiency of the PTO. This situation occurs in the less energetic sea states where the sea states have a lower \(H_s\) and a higher \(\omega_p\).

The intrinsic resistance that satisfies the above mentioned constraints can be designed using the transfer functions of \(F_u(s)\) and \(F_{ex}(s)\). Using these transfer
functions, the following equation can be derived

\[ f_u(t) = \frac{K(s)}{1 + P(s)K(s)} \tilde{r}(t) \]

\[ = \frac{K(s)}{1 + P(s)K(s)} \frac{f_p(t)}{R_{if}^*} \]

\[ = \frac{K(s)}{1 + P(s)K(s)} \frac{F_{ex}(s)\eta(t)}{f_{um}} \]  

(3.24)

By setting the control force to its maximum allowable value, \( R_{if}^* \) can be obtained from Equation 3.24 as

\[ R_{if}^*(t) = \frac{K(s)}{1 + P(s)K(s)} \frac{F_{ex}(s)\eta(t)}{f_{um}} \]  

(3.25)

It is obvious that the value of the intrinsic resistance of Equation 3.25 varies with time due to \( \eta(t) \). In monochromatic (sinusoidal) sea states, the values of \( R_{if}^* \) can be found for a specific \( H_s \) and \( \omega_p \) as the magnitude of the Bode plot of Equation 3.25, thus

\[ R_{if}^*(\omega_p) = \left| \frac{K(s)}{1 + P(s)K(s)} \frac{\eta_p}{f_{um}} \right| \]  

(3.26)

where the constant \( \eta_p \) is equal to half of the wave height, \( H_s \). Thus, a look-up table can be generated for a sea state, with specific values for \( H_s \) and \( \omega_p \).

The intrinsic resistance in Equation 3.26 will not violate the design value of \( f_{um} \) in the nominal system. However, any perturbation in the modelling tends to increase the value of \( f_u(t) \) in the LHS to maintain its tracking capability. The more the system deviates from its nominal value, the higher the generated value of \( f_u(t) \).

In order to keep the maximum value of the control force below \( f_{um} \) in all scenarios, while also satisfying the constraint of the ratio \( P_{em}/P_e \), the following procedure was conducted:

1. The highest peak value of \( f_u(t) \) in, theoretically, all scenarios was found by simulating the system in the worst-case scenario of perturbation.

2. At a specific value of \( H_s \), \( f_{um} \) was set to its designed value in Equation 3.26.
This value was reduced if:

- The peak of $f_u(t)$ exceeded the designed $f_{um}$ in the lowest operating $\omega_p$ (i.e. the most energetic sea states in the respected $H_s$).
- The ratio $P_{em}/\overline{P_e}$ exceeded its designed value in the highest operating $\omega_p$ (i.e. the lowest energetic sea states in the respected $H_s$).

This algorithm provided a fast procedure to find the value of $R_{ij}$ that satisfies the design constraints. A look-up table was generated using the magnitude of the Bode plot. The table decides the constant value of $R_{ij}$ based on $H_s$ and $\omega_p$ values. Practically, these values can be obtained by performing a fast Fourier transform over the incoming sea states. This involves a prediction method which is beyond the scope of this study. There is no need to change the value of $R_{ij}$ in each sampling instant because the sea profile (i.e. $H_s$ and $\omega_p$) does not change for a duration of 20-30 minutes [55].

3.6 HHC via QP method

Instead of using Equation 3.23, which has its drawbacks for constant $R_{int}$ as discussed earlier, a constrained optimisation problem is formulated, in which the objective function is to maximise the absorbed energy $E_{abs}$ subject to constraints on the buoy's maximum displacement $z_m$ [56]. This is accomplished by formulating a constrained quadratic programming problem (QP) and solving it across a finite window $n_m$ as follows:

$$\max E_{abs} = T_s \sum_{i=0}^{n_{m}-1} [P_{ex}(k+i|k) - P_r(k+i|k)]$$

(3.27)

where $T_s$ is the sampling time, and $P_{ex}(k+i|k)$ and $P_r(k+i|k)$ are the predicted excitation power and radiation power at time $(k+i)$, respectively. This relation can
be expressed in terms of the buoy velocity,

\[ E_{abs} = T_x \sum_{i=0}^{n_{uv}-1} (f_{r+1}(k+i|k) - f_r(k+i|k)) \tilde{z_r}(k+i|k), \]  

(3.28)

\( f_r(k+i|k) \) is assumed to be equal to the radiation kernel of equation 2.53. Thus, by replacing \( f_r(k+i|k) \) with

\[ f_r(k+i|k) = Cx(k+i|k), \]  

(3.29)

where \( x(k+i|k) \) is the state vector of the overall system at time \( k+i \) and \( C = [0_{1 \times 2} \ C_k] \), and sequentially forward solving \( f_r(k+i|k) \) across an \( n_{uv} \)-sized optimisation window, we obtain

\[ F_r = \Theta_1 x(k) + \Theta_2 V, \]  

(3.30)

where

\[ F_r = \begin{bmatrix} f_r(k|k) \\ \vdots \\ f_r(k+n_{uv}-1|k) \end{bmatrix}, \quad V = \begin{bmatrix} \dot{z}_r(k|k) \\ \vdots \\ \dot{z}_r(k+n_{uv}-1|k) \end{bmatrix}, \]

\[ \Theta_1 = \begin{bmatrix} C & CA & \cdots & CA^{n_{uv}-1} \end{bmatrix}^T \in \mathbb{R}^{n_u \times 6}, \]

\[ \Theta_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ CA^{n_{uv}-1}B & \cdots & CAB & CB \end{bmatrix} \in \mathbb{R}^{n_u \times n_u}. \]

\[ A = \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 4} \\ 0_{4 \times 2} & A_k \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \]

A constrained QP problem with a decision variable, \( V \), can be obtained by
rewriting Equation 3.28 into the vector form and combining it with Equation 3.30 as

\[
\begin{align*}
\text{minimise} & \quad E_{abs} = V^T \Theta_2 V + (\Theta_1 x(k) - \hat{F}_{ex})^T V, \\
\text{subject to} & \quad \Xi V \leq \Omega,
\end{align*}
\tag{3.31}
\]

where

\[
\hat{F}_{ex} = \begin{bmatrix} 
\hat{f}_{ex}(k|k) \\
\vdots \\
\hat{f}_{ex}(k + n_w - 1|k)
\end{bmatrix}, \quad \Xi = \begin{bmatrix} -M & M \end{bmatrix}^T
\]

\[
\Omega = \begin{bmatrix} z_m + P_1(k) & z_m - P_1(k) \end{bmatrix}^T.
\]

The estimated excitation force across \(n_w\) is contained in \(\hat{F}_{ex}\). All the values in the latter are set to be fixed and equal to \(\hat{f}_{ex}(k|k)\) as no prediction algorithm is used. The matrices used in the constraint inequality \(M\) and \(P\) are similar to \(\Theta_2\) and \(\Theta_1\), respectively, except for a minor modification: \(C = [1 \ 0 \ 1 \times 5]\). Equation 3.31 is minimised with respect to the future velocity trajectory \(V\), while limiting the buoy excursion to \(\pm z_m\). The formulated QP is convex because the Hessian matrix in Equation 3.31 is positive-definite [57].

It is a very complicated and challenging task to solve Equation 3.31 in real time. Instead, solving it off-line for various expected peak wave frequency \(\omega_p\) and significant wave height \(H_s\) values is a simpler, cheaper, and less computationally intensive solution. Then, the resultant sub-optimal continuous velocity profile is used to compute the sub-optimal intrinsic resistance, \(R_{qp}^*\), as follows:

\[
\frac{1}{R_{qp}^*} = \text{Re}[\frac{f_{ex}e^{j\omega_t}}{f_{ex}e^{j\omega_t}|_{\omega=\omega_p}}],
\tag{3.32}
\]

where \(|f_{ex}|\) and \(|\dot{z}_r|\) are the complex amplitudes of \(f_{ex}(t)\) and \(\dot{z}_r(t)\), respectively. The reciprocal of \(R_{qp}^*\) is used to avoid \(R_{qp}^* \rightarrow \infty\) at \(\dot{z}_r(t) = 0\). Again, a look-up table is
constructed to adjust $R_{qp}$ in real time. The time-averaged sub-optimal resistance $\bar{R}_{qp}$ is calculated for monochromatic sea states of different $\omega_p$ and $H_s$.

### 3.7 Other Control Strategies

Two existing control strategies were selected for the sake of comparison in the results and analysis in chapter 4.

#### 3.7.1 Resistive Loading (RL)

This is a very simple control strategy for the heaving WEC. It is based on producing a control force that is linearly proportional to the buoy velocity [58]. The optimum power transfer occurs when the PTO reactance cancels that of the intrinsic system, leaving out the resistance terms. A simplified sub-optimal approach is to model the PTO impedance as a pure frequency-dependent resistance, $R_c(\omega)$. Therefore, the control force can be represented as

$$f_u(t) = -R_c(\omega)v(t). \quad (3.33)$$

The easiest approach to determine $R_c(\omega)$ is by computing the magnitude of the system intrinsic impedance, $|Z_{inf}(\omega)|$; thus, we obtain [15]

$$f_u(t) = -|Z_{inf}(\omega)|v(t). \quad (3.34)$$

where

$$|Z_{inf}(\omega)| = \sqrt{(R_{inf}(\omega))^2 + (X_{inf}(\omega))^2}$$
3.7.2 Conventional Hierarchical Control System

Referring to subsection 2.5.4, we can obtain the following formula from Equation 2.46

$$v_r(t) = \ddot{z}_r(t) = \frac{\tilde{f}_{ex}(t)}{2R_r} \cos \theta,$$  \hspace{1cm} (3.35)

where $R_r$ and $\theta$ are the system's resistance and the phase difference between $\tilde{f}_{ex}(t)$ and $\ddot{z}(t)$, respectively.

The conventional hierarchical control system (C-HCS) is an HCS that utilises Equation 3.35 in its higher level controller to generate the reference. The value of $R_r$ is fixed over the entire operating frequency range. This method can be combined with the two HLCs proposed in sections 3.2 and 3.4 for the sake of comparison.
Chapter 4: Simulation and Results

This chapter provides the simulation results and analysis of five hierarchical controllers that are synthesised based on HHC and LHC techniques in chapter 3. Simulation parameters and verification strategies are presented in section 4.1. These strategies are used in section 4.2, the results and analysis section, for each synthesised controller, to demonstrate its performance under nominal conditions in various sea states, and test it under the system's perturbations and external disturbances. In addition, a comparison with existing controller techniques is provided in the same section. The simulation was conducted using MATLAB® Simulink®.

4.1 Simulation Setup and Verification Techniques

The system configuration of Figure 3.1 is considered throughout this chapter for each synthesised hierarchical controller. The heaving spherical wave energy converter of Uppsala University [14] is considered, using the detailed parameter values shown in Table 4.1. The parameters of both mechanical and electrical models are included in the table [15].

The excitation force transfer function, $F_{ex}(s)$ of Figure 3.1 is obtained as

$$\frac{F_{ex}(s)}{H(s)} = \frac{-2145s^7 + 1.2 \times 10^4s^6 + ...}{s^8 + 0.9s^7 + 2.8s^6 + 1.5s^5 + 2.3s^4 + ...}$$

$$-4.9 \times 10^4s^5 + 3.9 \times 10^4s^4 + ...$$

$$0.7s^3 + 0.7s^2 + ...$$

$$4.2 \times 10^4s^3 + 1.3 \times 10^4s^2 - 5979s$$

$$0.1s + 0.06$$

(4.1)

where $F_{ex}$ and $H(s)$ are the Laplace transforms of $f_{ex}(t)$ and $\eta(t)$, respectively.

The state-space mechanical model of the mechanical system of equations 2.57 and 2.58 is used with $\alpha_e = R_{\text{loss}}$, where $R_{\text{loss}}$ is the loss resistance. The mechanical
The simulation strategy can be summarised as:

1. Performance of nominal system in a regular (monochromatic) sea state of specific wave heights, $H_s$, of 1 m and 2 m for a range of peak frequencies (i.e. $0.5 - 1 \text{ rad/s}$). Plots of the values listed below vs. the specified peak frequency range were obtained, for both specific heights.

- the average captured power, $\bar{P}_m$
- the average electrical power, $\bar{P}_e$

---

<table>
<thead>
<tr>
<th>Parameter (symbol)</th>
<th>Value unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buoy's Radius ($r$)</td>
<td>5 m</td>
</tr>
<tr>
<td>Buoy and translator mass ($m_b$)</td>
<td>$2.68 \times 10^5$ kg</td>
</tr>
<tr>
<td>Infinite added mass ($m_m$)</td>
<td>$1.34 \times 10^5$ kg</td>
</tr>
<tr>
<td>Water plane area ($A_w$)</td>
<td>78.54 $m^2$</td>
</tr>
<tr>
<td>Submerged Volume ($V_s$)</td>
<td>261.80 $m^3$</td>
</tr>
<tr>
<td>Sea water density ($\rho$)</td>
<td>1025 $kg/m^3$</td>
</tr>
<tr>
<td>Gravitational acceleration ($g$)</td>
<td>9.81 $m/s^2$</td>
</tr>
<tr>
<td>Seabed depth ($d$)</td>
<td>80 m</td>
</tr>
<tr>
<td>Resonance angular frequency ($\omega_0$)</td>
<td>1.56 $rad/s$</td>
</tr>
<tr>
<td>Buoyancy stiffness coefficient ($S_b$)</td>
<td>$7.89 \times 10^5$ $N/m$</td>
</tr>
<tr>
<td>Nominal restoring stiffness coefficient ($S_{rs0}$)</td>
<td>$2 \times 10^5$ $N/m$</td>
</tr>
<tr>
<td>Nominal losses resistance ($R_{loss0}$)</td>
<td>$0.4 \times 10^5$ $N.s/m$</td>
</tr>
<tr>
<td>PMLG synchronous resistance ($R_s$)</td>
<td>0.29 Ohm</td>
</tr>
<tr>
<td>Permanent magnet flux ($\lambda_{PM}$)</td>
<td>23 $Wb$</td>
</tr>
<tr>
<td>PMLG pole width ($p_w$)</td>
<td>0.05 m</td>
</tr>
<tr>
<td>DC link voltage ($V_{dc}$)</td>
<td>3500 V</td>
</tr>
<tr>
<td>Modulation index ($mu$)</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.1: Mechanical and electrical parameters of the WEC model can be obtained with the following components of the radiation force:

$$
A_r = \begin{bmatrix}
-3.4376 & -6.3533 & -4.9714 & -1.7168 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
$$

$$
B_r = [1 \ 0 \ 0 \ 0]^T
$$

$$
C_r = [0.96 \times 10^5 \ 3.62 \times 10^5 \ 1.57 \times 10^5 \ 0].
$$
• the maximum control force, \( f_{um} \)

• the maximum electrical power, \( P_{em} \)

• the of maximum electrical power to the average electrical power, \( P_{em} \bar{P}_e \)

the percentage of conversion efficiency between \( \bar{P}_e \) and \( P_e \)

2. Time-domain response of nominal system in an irregular (polychromatic) sea state of peak frequency, \( \omega_p = 0.7 \text{ rad/s} \), and wave height, \( H_s = 2 \text{ m} \). Plots of this part include the following measures in time interval \( (20 - 140 \text{ s}) \):

• reference and output velocities, \( \dot{z}_r \) and \( \dot{z}_r \), of the excitation force and heaving buoy, respectively

• excitation force position, \( z \), and output position of the heaving buoy, \( \eta \)

• the control force and excitation force, \( f_u \) and \( f_{ex} \), respectively

• the mechanical captured power, \( P_m \), and its average, \( \bar{P}_m \)

• the \( d-q \) components of the stator voltage, \( v_{sq} \) and \( v_{sd} \)

• the \( q \)-axis current component which implements the control force

• the EMF voltage of the linear generator

• the electrical converted power, \( P_e \), and its average, \( \bar{P}_e \)

where the first four items correspond to the mechanical quantities and the last four correspond to the electrical quantities.

3. Performance of the perturbed system under the presence of a disturbance force in an irregular (polychromatic) sea state of peak frequency, \( \omega_p = 0.7 \text{ rad/s} \), and wave height, \( H_s = 2 \text{ m} \). Results of this test are listed in tables that contain nine cases of perturbations, and a tenth case (the worst case) where an external disturbance force, \( f_{dis}(t) \), is added. The performance in each case is measured in terms of an energy drop percentage, based on the energy calculated for the nominal case. Mean square error (MSE) is also calculated.
In the nominal case, the value of the nominal restoring stiffness coefficient, \( R_{r,0} \), is shown in Table 4.1. The loss resistance, \( R_{loss} \), is set to zero. For cases 1 through 9, the values of the restoring stiffness coefficient and the loss resistance are respectively set to

\[
R_{rs} = R_{rs,0} + \Delta_s
\]  
(4.2)

and

\[
R_{loss} = R_{loss,0} + \Delta_l.
\]  
(4.3)

Then, the values \( \Delta_s \) and \( \Delta_l \) are varied from 0 % up to 50 %.

In the worst case, the perturbation values are the same as those in case 9. Moreover, an external disturbance force, \( f_{dis}(t) \), is added as a third input to the mechanical model (in addition to the excitation force, \( f_{ex}(t) \), and the control force, \( f_u(t) \)) (see Figure 3.1). A disturbance force equal to one-third of the magnitude of the excitation force and the same sea state is generated using Simulink®, as shown in Figure 4.1. This unmodelled disturbance force is formulated as

\[
f_{dis} = -R_{loss} \dot{z}(t) - S_{r,0} \dot{z}(t) - \alpha_d \dot{z}^2(t) \sin(\omega_d t)
\]  
(4.4)

where \( \alpha_d = 0.5 \times 10^4 \) and \( \omega_d = 0.5 \text{ rad/s} \)

### 4.1.1 LHC via Robust Lead-Lag Compensator (LLC)

Using the obtained mechanical model in section 4.1 and the procedure in section 3.2, the transfer function of the lead-lag compensator is formulated as

\[
K(s) = 3 \times 10^6 \left( \frac{s + 180}{s + 2} \right)
\]  
(4.5)

The controller in Equation 4.5 represents the lower-level hierarchical
controller, LHC, which provides excellent tracking performance to the reference signal that is generated in the HHC.

4.1.2 LHC via PID augmented with SMC Controller

Referring to section 3.3, an SMC is synthesised and used as an LHC controller. The parameters of Equation 3.10 are obtained and the following control signal, $f_u(t)$ is formulated

$$f_u(t) = 100\dot{e}(t) + 1100e(t) + 1 \times 10^6 \int_0^\infty e(t)dt + k_1 \text{sat}(s(t))$$

(4.6)

where

$$\text{sat}(s(t)) = \begin{cases} 
\text{sign}(s(t)) & \text{if } |s(t)| > 10 \\
\frac{s(t)}{10} & \text{if } |s(t)| < 10.
\end{cases}$$

4.1.3 LHC via Complex PID Stabilisation

Considering the method of complex PID stabilisation described in section 3.4, a robust PID controller is designed as an LHC. The weights $W_e(s)$ and $W_T(s)$ that describe the frequency-domain characteristics of the tracking performance specifications and input multiplicative uncertainty, respectively, are obtained as follows:

$$W_e(s) = \frac{100}{s + 100}$$

(4.7)
The transfer function, \( W_T(s) \), is chosen as the low-pass filter containing the operating frequencies of the system within its bandwidth. It is also chosen with minimal phase shift within this operating frequency range.

The transfer function, \( W_T(s) \), is obtained by plotting the Bode plots of all input multiplicative perturbations, \( \Delta(s) \), of the plants in set \( P \), and tracing their upper limit so that \( \Delta(s) \) is less than \( W_T(s) \) for all frequencies. Figure 4.2 illustrates this procedure.

\[
W_T(s) = \frac{0.0016s^5 + 0.2113s^4 + 1.778s^3 + 7.6s^2 + 10.81s + 18.7}{s^5 + 4.475s^4 + 10.62s^3 + 33.36s^2 + 25.51s + 59.24}
\] (4.8)

![Figure 4.2: Input multiplicative perturbations (\( \Delta(s) \)) and the weighting transfer function (\( W_T(s) \)).](image)

The algorithm in section 3.4 is then applied using MATLAB®. The following
set of normalised inequalities is formulated:

\[-k_i \leq 0\]
\[1.57 \times 10^{-23} k_i - k_d \leq 32360.7\]
\[-1.658 \times 10^{-24} k_i + k_d \leq 10514.6\]  \hspace{1cm} (4.9)

Choosing the parameters of the controller from the admissible region, the controller \(K(s)\) is obtained as

\[K(s) = \frac{100s^2 + 1 \times 10^6 s + 6.6 \times 10^7}{s}\]  \hspace{1cm} (4.10)

The selected parameters of the controller satisfy the condition

\[\| | W_e(s)S(s) | + | W_T(s)T(s) | \|_\infty < 1\]

as shown in Figure 4.3

![Figure 4.3: Design problem of the robust PID controller satisfying the norm condition: \[\| | W_e(s)S(s) | + | W_T(s)T(s) | \|_\infty < 1\]]
4.1.4 HHC via Transfer Functions Method (TF)

The obtained transfer functions of the controllers in subsections 4.1.1 and 4.1.3 are used to obtain two HHCs via transfer function method controllers. Intrinsic resistance values of each HHC are obtained below.

HHC via TF method for the lead-lag compensator LHC

Using the procedure in section 3.5, a look-up table is obtained for the intrinsic resistance, $R_{tf}^*$ of Equation 3.26, for different sea states of peak frequency $\omega_p$ in ranges from 0.5 – 1 rad/s and wave specific heights, $H_s$, of 1 m, 2 m, and 3 m. Results are shown in Table 4.2 and plotted in Figure 4.4.

<table>
<thead>
<tr>
<th>$\omega_p$ rad/s</th>
<th>$H_s = 1$ m</th>
<th>$H_s = 2$ m</th>
<th>$H_s = 3$ m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.17</td>
<td>1.15</td>
<td>1.73</td>
</tr>
<tr>
<td>0.55</td>
<td>1</td>
<td>0.98</td>
<td>1.48</td>
</tr>
<tr>
<td>0.6</td>
<td>0.86</td>
<td>0.85</td>
<td>1.27</td>
</tr>
<tr>
<td>0.65</td>
<td>0.73</td>
<td>0.72</td>
<td>1.08</td>
</tr>
<tr>
<td>0.7</td>
<td>0.63</td>
<td>0.61</td>
<td>0.92</td>
</tr>
<tr>
<td>0.75</td>
<td>0.54</td>
<td>0.53</td>
<td>0.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.47</td>
<td>0.46</td>
<td>0.69</td>
</tr>
<tr>
<td>0.85</td>
<td>0.4</td>
<td>0.39</td>
<td>0.58</td>
</tr>
<tr>
<td>0.9</td>
<td>0.33</td>
<td>0.33</td>
<td>0.49</td>
</tr>
<tr>
<td>0.95</td>
<td>0.3</td>
<td>0.29</td>
<td>0.43</td>
</tr>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.24</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Table 4.2: Look-up table for the value of $R_{tf}^*$ in MN/m based on transfer functions of the LLC

It is assumed that the maximum level of control force, $f_{um}$, that can be generated by the PTO is 1.5 MN. The ratio $P_{em}/P_e$ is designed to be less than 10. The designed value of $f_{um}$ for the obtained $R_{tf}^*$ is reduced to 0.49 MN, 1 MN, and 1 MN for $H_s = 1$ m, 2 m, and 3 m, respectively.

HHC via TF method for the PID compensator LHC

Similar to the case of TF-LLC, using the procedure in section 3.5, a look-up table is obtained for the intrinsic resistance, $R_{tf}^*$ of Equation 3.26, for different sea states.
of peak frequency $\omega_p$ in ranges from $0.5-1 \text{ rad/s}$ and wave specific heights, $H_s$, of $1 \text{ m}$, $2 \text{ m}$, and $3 \text{ m}$. Results are shown in Table 4.3 and plotted in Figure 4.5.

As explained earlier, it is assumed that the maximum level of control force, $f_{um}$, that can be generated by the PTO is $1.5 \text{ MN}$. The ratio $P_{em}/P_e$ is designed to be less than 10. The designed value of $f_{um}$ for the obtained $R^*_tf$ is reduced to $0.5 \text{ MN}$, $1 \text{ MN}$, and $1 \text{ MN}$ for $H_s = 1 \text{ m}$, $2 \text{ m}$, and $3 \text{ m}$, respectively.

<table>
<thead>
<tr>
<th>$\omega_p \text{ rad/s}$</th>
<th>$H_s = 1 \text{ m}$</th>
<th>$H_s = 2 \text{ m}$</th>
<th>$H_s = 3 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.14</td>
<td>1.14</td>
<td>1.71</td>
</tr>
<tr>
<td>0.55</td>
<td>0.97</td>
<td>0.97</td>
<td>1.46</td>
</tr>
<tr>
<td>0.6</td>
<td>0.84</td>
<td>0.84</td>
<td>1.26</td>
</tr>
<tr>
<td>0.65</td>
<td>0.71</td>
<td>0.71</td>
<td>1.07</td>
</tr>
<tr>
<td>0.7</td>
<td>0.61</td>
<td>0.61</td>
<td>0.91</td>
</tr>
<tr>
<td>0.75</td>
<td>0.52</td>
<td>0.52</td>
<td>0.78</td>
</tr>
<tr>
<td>0.8</td>
<td>0.45</td>
<td>0.45</td>
<td>0.68</td>
</tr>
<tr>
<td>0.85</td>
<td>0.39</td>
<td>0.39</td>
<td>0.58</td>
</tr>
<tr>
<td>0.9</td>
<td>0.33</td>
<td>0.33</td>
<td>0.49</td>
</tr>
<tr>
<td>0.95</td>
<td>0.28</td>
<td>0.28</td>
<td>0.42</td>
</tr>
<tr>
<td>1</td>
<td>0.24</td>
<td>0.24</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 4.3: Look-up table for the value of $R^*_tf$ in $\text{MN s/m}$ based on transfer functions of the PID.
4.1.5 HHC via Quadratic Programming Method (QP)

This HCS control strategy deploys the quadratic programming (QP) process discussed in section 3.6 to design the HHC. The resulting HHC is a look-up table (Table 4.4) that chooses the intrinsic resistance, $R^*_{qp}$, based on the sea state, i.e. the peak wave frequency, $\omega_p$, and the significant wave height, $H_s$. A mesh plot for the $R^*_{qp}$ is shown in Figure 4.6.

<table>
<thead>
<tr>
<th>$\omega_p \text{ rad/s}$</th>
<th>$H - s = 1 \text{ m}$</th>
<th>$H - s = 2 \text{ m}$</th>
<th>$H - s = 3 \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.190</td>
<td>0.3660</td>
<td>0.4360</td>
</tr>
<tr>
<td>0.55</td>
<td>0.1680</td>
<td>0.3300</td>
<td>0.4000</td>
</tr>
<tr>
<td>0.60</td>
<td>0.1560</td>
<td>0.3000</td>
<td>0.3910</td>
</tr>
<tr>
<td>0.65</td>
<td>0.1400</td>
<td>0.2800</td>
<td>0.3740</td>
</tr>
<tr>
<td>0.70</td>
<td>0.1300</td>
<td>0.2600</td>
<td>0.3540</td>
</tr>
<tr>
<td>0.75</td>
<td>0.1190</td>
<td>0.2500</td>
<td>0.3400</td>
</tr>
<tr>
<td>0.80</td>
<td>0.1040</td>
<td>0.2400</td>
<td>0.3300</td>
</tr>
<tr>
<td>0.85</td>
<td>0.0999</td>
<td>0.2200</td>
<td>0.3100</td>
</tr>
<tr>
<td>0.90</td>
<td>0.0960</td>
<td>0.2100</td>
<td>0.2920</td>
</tr>
<tr>
<td>0.95</td>
<td>0.09</td>
<td>0.2000</td>
<td>0.2800</td>
</tr>
<tr>
<td>1</td>
<td>0.0830</td>
<td>0.1900</td>
<td>0.2730</td>
</tr>
</tbody>
</table>

Table 4.4: Look-up table for the value of $R^*_{qp}$ in $MN/s/m$ based on the quadratic programming (QP) method
Using the setup values for each controller in section 4.1, five hierarchical control systems (HCSs) are constructed. For each HCS, a combination of LHC and HHC is selected. The resultant overall systems are:

1. TF-LLC: refers to an HHC via the transfer functions method, combined with an LHC via a robust lead-lag compensator.

2. TF-PID: refers to an HHC via the transfer functions method, combined with an LHC via a complex PID stabilisation method.

3. QP-LLC: refers to an HHC via the quadratic programming method, combined with an LHC via a robust lead-lag compensator.

4. QP-PID: refers to an HHC via the quadratic programming method, combined with an LHC via a complex PID stabilisation method.

5. QP-PID: refers to an HHC via the quadratic programming method, combined with an LHC via a PID augmented with an SMC controller.
Results for each HCS are obtained and analysed in the following sections. Moreover, the performance of some HCSs is compared with that of other control methods in section 3.7.

4.2.1 Results of TF-LLC

System performance under nominal conditions for significant wave heights, $H_s$, of 1 m and 3 m is shown in the graphs of figures 4.7 and 4.8, respectively. The performance of the RL method, discussed in the last section of chapter 3 is added to the graphs for the sake of comparison.

![Graphs showing system performance](image)

Figure 4.7: Simulation results of the TF-LLC and RL using a monochromatic sea state of $H_s = 1$ m

Both control techniques show acceptable performance, for both significant heights ($H_s = 1$ m and $H_s = 3$ m), in terms of maximum control force and maximum ratio of maximum captured mechanical power to its mean. These are 1.5 MN and 10, respectively. The efficiency of the RL is stable at around 62% in all sea states, while it shows a lower amount with a gradual increment of the TF-LLC for both
Figure 4.8: Simulation results of the TF-LLC and RL using a monochromatic sea state of $H_s = 3 \text{ m}$

heights. The efficiency of the TF-LLC ranges from about 30\% to below 60\% with a gradual increment from low frequency to high frequency for both heights. However, the captured mechanical power, $P_m$, and the converted electrical power, $P_e$, are significantly higher with the TF-LLC method than with the RL method for all ranges of the peak frequency, $\omega_p$, and the significant wave height, $H_s$.

The simulation results using the irregular sea state for mechanical and electrical quantities are shown in figures 4.9 and 4.10, respectively. The tracking capability of the LHC is demonstrated in Figure 4.9 (a). The figure shows that the velocity of the buoy $\dot{z}$ is exactly equal to its reference. The value of the mean square error (MSE) is $4.14 \times 10^{-5}$. This indicates nearly perfect tracking capability of the lead-lag compensator. Considering the radius of the buoy, the elevation of the buoy $z$, and the wave elevation $\eta$ in Figure 4.9 (b), we can conclude that there is no extreme excursion of the buoy. Extreme excursion must be avoided, to prevent physical damage on the PTO. Figure 4.9 shows a comparison between the control
force $f_n(t)$ and the excitation force $f_{ex}$. The control force has the same order of magnitude as the excitation force, $f_{ex}(t)$. It also stays below its designed limitation. The instantaneous captured power $P_m$ is shown in Figure 4.9 (c). From the results, an average captured power $\bar{P}_m$ of 0.11 MW is generated. The $d-q$ components of the stator voltage are shown in Figure 4.10 (a). The $q$-axis current component $i_{sq}$ that implements the $f_n(t)$ is shown in Figure 4.10 (b), while Figure 4.10 (c) shows the EMF voltage of the PTO. An average converted power $\bar{P}_e$ of 0.07 MW is produced by the PTO. This is shown in Figure 4.10 (c). This corresponds to 62.4% of the conversion efficiency from $P_m$ to $P_e$.

![Graphs](image)

Figure 4.9: Simulation results showing mechanical quantities produced by the TF-LLC under a polychromatic sea state with $H_s = 2$ m and $\omega_p = 0.7$ rad/s

The main properties of the LHC provide the tracking and maintain the robustness of the controlled system. Perturbation scenarios discussed in section 4.1 are applied in the WEC system to test these properties. Results are summarised in Table 4.5 in terms of energy drop percentage and MSE. The table shows that the energy drop increases slightly with the increase in the restoring stiffness coefficient, $R_{rs}$, while it increases significantly with the loss resistance, $R_{loss}$. Tracking performance is almost conserved, which ensures robustness. However, it slightly
Figure 4.10: Simulation results showing electrical quantities produced by the TF-LLC under a polychromatic sea state with $H_s = 2 \text{ m}$ and $\omega_p = 0.7 \text{ rad/s}$

<table>
<thead>
<tr>
<th>Cases</th>
<th>$\Delta_1$</th>
<th>$\Delta_2$</th>
<th>Energy [kWh]</th>
<th>Energy Drop %</th>
<th>MSE [$\times 10^{-5}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0</td>
<td>0</td>
<td>3.77</td>
<td>7.21</td>
<td>4.07</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>0.25</td>
<td>3.76</td>
<td>7.23</td>
<td>4.11</td>
</tr>
<tr>
<td>Case 3</td>
<td>0</td>
<td>0.5</td>
<td>3.75</td>
<td>7.25</td>
<td>4.16</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.25</td>
<td>0</td>
<td>3.68</td>
<td>9.02</td>
<td>4.05</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.25</td>
<td>0.25</td>
<td>3.68</td>
<td>9.03</td>
<td>4.09</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.25</td>
<td>0.5</td>
<td>3.68</td>
<td>9.05</td>
<td>4.14</td>
</tr>
<tr>
<td>Case 7</td>
<td>0.5</td>
<td>0</td>
<td>3.61</td>
<td>10.82</td>
<td>4.03</td>
</tr>
<tr>
<td>Case 8</td>
<td>0.5</td>
<td>0.25</td>
<td>3.61</td>
<td>10.84</td>
<td>4.08</td>
</tr>
<tr>
<td>Case 9</td>
<td>0.5</td>
<td>0.5</td>
<td>3.61</td>
<td>10.86</td>
<td>4.13</td>
</tr>
<tr>
<td>Worst Case</td>
<td>0.5</td>
<td>0.5</td>
<td>3.61</td>
<td>10.9</td>
<td>4.31</td>
</tr>
</tbody>
</table>

Table 4.5: Performance of TF-LLC under various perturbation scenarios and disturbance forces, and the worst-case scenario

affected the existence of the disturbance force in the worst-case scenario.

4.2.2 Results of TF-PID

System performance under nominal conditions for significant wave heights, $H_s$, of 1 m and 3 m is shown in the graphs of figures 4.11 and 4.12, respectively. The performance of the TF-LLC strategy is added to the graphs for the sake of comparison.
Figure 4.11: Simulation results of the TF-PID and TF-LLC using a monochromatic sea state of $H_s = 1$ m

Figure 4.12: Simulation results of the TF-PID and TF-LLC using a monochromatic sea state of $H_s = 3$ m
Both control techniques show acceptable performance, for both significant heights, \( H_s = 1 \text{ m} \) and \( H_s = 3 \text{ m} \), in terms of maximum control force and maximum ratio of maximum captured mechanical power to its mean. These are 1.5 \( MN \) and 10, respectively. The efficiency of the TF-PID is higher and more stable compared to the TF-LLC. It shows a slightly increasing trend with respect to the peak wave frequency, and ranges from 61% to 64% for all sea states. Powers in the case of \( H_s = 1 \text{ m} \) are generally of the same order. Average captured power, \( \bar{P}_m \), of the TF-PID is lower in low-peak frequencies, while it increases in high peak frequencies, compared to the TF-LLC. On the other hand, the converted power, \( \bar{P}_e \), of the TF-PID is higher through the peak frequency range, as shown in Figure 4.11 (a) and (b), respectively. For all sea states (i.e., both heights), the maximum control force, \( f_{um} \), and the maximum converted power, \( P_{em} \), show interestingly linear trends, decreasing and increasing respectively, with respect to the peak wave frequency, \( \omega_p \).

The simulation results using the irregular sea state for mechanical and electrical quantities are shown in figures 4.13 and 4.14, respectively. The tracking capability of the LHC is demonstrated in Figure 4.13 (a). The figure shows that the velocity of the buoy, \( z \), is almost equal to its reference. The value of the mean square error (MSE) is \( 1.72 \times 10^{-4} \). This indicates very good tracking capability of the PID controller. Considering the radius of the buoy, the elevation of the buoy, \( z \), and the wave elevation, \( \eta \), in Figure 4.13 (b), we can conclude that there is no extreme excursion of the buoy. Figure 4.13 shows a comparison between the control force \( f_u(t) \) and the excitation force \( f_{ex} \). The control force has the same order of magnitude as the excitation force, \( f_{ex}(t) \). It also stays below its designed limitation.

The instantaneous captured power \( P_m \) is shown in Figure 4.13 (c). From the results, an average captured power \( \bar{P}_m \) of 0.11 \( MW \) is generated. The \( d-q \) components of the stator voltage are shown in Figure 4.14 (a). The \( q \)-axis current component \( i_{sq} \) that implements the \( f_u(t) \) is shown in Figure 4.14 (b), while Figure 4.14 (c)
shows the EMF voltage of the PTO. An average converted power $\bar{P}_c$ of 0.07 MW is produced by the PTO. This is shown in Figure 4.14 (c). This corresponds to 62.4% of the conversion efficiency from $P_m$ to $P_c$. The results obtained are similar to the corresponding results in the case of TF-LLC.

Figure 4.13: Simulation results showing mechanical quantities produced by the TF-PID under a polychromatic sea state with $H_s = 2$ m and $\omega_p = 0.7$ rad/s

Perturbation scenarios discussed in section 4.1 are applied to the TF-PID to test its tracking capability and robustness. Results are summarised in Table 4.6 in terms of energy drop percentage and MSE. The TF-PID shows similar attitude to that of the TF-LLC but with a higher MSE, which indicates that the tracking capability of the LLC is superior. The table shows that the energy drop increases slightly with an increase in the restoring stiffness coefficient, $R_{rs}$, while it increases significantly with the loss resistance, $R_{loss}$. Tracking performance is almost conserved, which ensures robustness. However, it slightly affected the existence of the disturbance force in the worst-case scenario.
Figure 4.14: Simulation results showing electrical quantities produced by the TF-PID under a polychromatic sea state with $H_s = 2 \text{ m}$ and $\omega_p = 0.7 \text{ rad/s}$

Table 4.6: Performance of TF-PID under various perturbation scenarios and disturbance forces, and the worst-case scenario

<table>
<thead>
<tr>
<th>Cases</th>
<th>$\Delta_L %$</th>
<th>$\Delta_D %$</th>
<th>Energy [kWh]</th>
<th>Energy Drop %</th>
<th>MSE [$\times 10^{-4}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>0</td>
<td>0</td>
<td>3.75</td>
<td>7.19</td>
<td>1.23</td>
</tr>
<tr>
<td>Case 2</td>
<td>0</td>
<td>0.25</td>
<td>3.75</td>
<td>7.26</td>
<td>1.26</td>
</tr>
<tr>
<td>Case 3</td>
<td>0</td>
<td>0.5</td>
<td>3.75</td>
<td>7.33</td>
<td>1.3</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.25</td>
<td>0</td>
<td>3.68</td>
<td>8.10</td>
<td>1.22</td>
</tr>
<tr>
<td>Case 5</td>
<td>0.25</td>
<td>0.25</td>
<td>3.68</td>
<td>9.05</td>
<td>1.25</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.25</td>
<td>0.5</td>
<td>3.67</td>
<td>9.12</td>
<td>1.28</td>
</tr>
<tr>
<td>Case 7</td>
<td>0.5</td>
<td>0</td>
<td>3.61</td>
<td>10.78</td>
<td>1.21</td>
</tr>
<tr>
<td>Case 8</td>
<td>0.5</td>
<td>0.25</td>
<td>3.60</td>
<td>10.85</td>
<td>1.24</td>
</tr>
<tr>
<td>Case 9</td>
<td>0.5</td>
<td>0.5</td>
<td>3.60</td>
<td>10.91</td>
<td>1.27</td>
</tr>
<tr>
<td>Worst Case</td>
<td>0.5</td>
<td>0.5</td>
<td>3.59</td>
<td>11.11</td>
<td>1.38</td>
</tr>
</tbody>
</table>

4.2.3 Results of QP-LLC

System performance under nominal conditions for significant wave heights, $H_s$, of 1 m and 3 m is shown in the graphs of figures 4.15 and 4.16, respectively. The performance of the TF-LLC method is added to the graphs to compare the effect of
changing the HHC.

As mentioned previously, the TF-LLC shows acceptable performance, for both significant heights \((H_s = 1 \text{ m} \text{ and } H_s = 3 \text{ m})\), in terms of maximum control force and maximum ratio of maximum captured mechanical power to its mean. However, changing the HHC in the QP-LLC results in a slight violation of these constraints. For \(H_s = 1 \text{ m}\), the control force generated by the LHC exceeds its limitation in a very small range at low peak frequencies. This range increases for approximately 40% of the operating range when \(H_s = 3 \text{ m}\). Maximum converted power of the QP-LLC reaches 0.75 \(MN\) at low heights and almost 2 \(MN\) at greater heights. It tends to decrease at higher frequencies for both heights, which makes sense as the sea state becomes less energetic. In addition, the ratio \(P_{em}/P_e\) exceeds its design value, 10, at high frequencies when the significant wave height is low \((H_s = 1 \text{ m})\), i.e. a high energetic sea state. Captured power and converted power are both higher in the case of QP-LLC at all operating points. The efficiency of the QP-LLC shows the same trend (a gradual increase) as in the case of TF-LLC but at a lower rate, which results in a smaller difference between its low and high peak frequency values (52.5 – 59 % at \(H_s = 1 \text{ m}\) and 60 – 65 % at \(H_s = 3 \text{ m}\)).

The simulation results using the irregular sea state for mechanical and electrical quantities are shown in figures 4.17 and 4.18, respectively. The tracking capability of the LHC is demonstrated in Figure 4.17 (a). The figure shows that the velocity of the buoy \(\tilde{z}\) almost equals the reference velocity, \(\tilde{z}_r\). The value of the mean square error (MSE) is \(6.49 \times 10^{-5}\). This indicates the nearly perfect tracking capability of the lead-lag compensator, as mentioned before. Considering the radius of the buoy, the elevation of the buoy \(z\), and the wave elevation \(\eta\) in Figure 4.17 (b), we can see that the buoy's elevation reaches critical values, 1.5 \(m\), at some points of the graph. Extreme excursion must be avoided to prevent physical damage to the PTO. Figure 4.17 shows a comparison between the control force \(f_c(t)\) and the excitation force \(f_{ex}\). The control force has a slightly higher order of magnitude than
Figure 4.15: Simulation results of the QP-LLC and TF-LLC using a monochromatic sea state of $H_s = 1\ m$

Figure 4.16: Simulation results of the QP-LLC and TF-LLC using a monochromatic sea state of $H_s = 3\ m$
the excitation force, \( f_{ex}(t) \). It stays below its designed limitation for the specific sea state used in this test. However, the previous Bode plots show that this constraint is violated in some sea-state conditions. The instantaneous captured power \( P_m \) is shown in Figure 4.17 (c). From the results, an average captured power \( \bar{P}_m \) of 0.13 MW is generated. The \( d-q \) components of the stator voltage are shown in Figure 4.18 (a). The \( q \)-axis current component \( i_{sq} \) that implements the \( f_{sq}(t) \) is shown in Figure 4.18 (b), while Figure 4.18 (c) shows the EMF voltage of the PTO. An average converted power \( \bar{P}_e \) of 0.08 MW is produced by the PTO. This is shown in Figure 4.18 (c). This corresponds to 61.8\% of the conversion efficiency from \( P_m \) to \( P_e \).

![Graphs showing mechanical quantities produced by the QP-LLC](image)

Figure 4.17: Simulation results showing mechanical quantities produced by the QP-LLC under a polychromatic sea state with \( H_s = 2 \text{ m} \) and \( \omega_p = 0.7 \text{ rad/s} \)

Perturbation scenarios discussed in section 4.1 are applied in the WEC system to test the tracking capability and robustness. Results are summarised in Table 4.7 in terms of energy drop percentage and MSE. The table shows that the energy drop
Figure 4.18: Simulation results showing electrical quantities produced by the QP-LLC under a polychromatic sea state with $H_s = 2 \text{ m}$ and $\omega_p = 0.7 \text{ rad/s}$

increases slightly with an increase in the restoring stiffness coefficient, $R_s$, while it increases significantly with the loss resistance, $R_{loss}$. Tracking performance is satisfactory to some extent, but the MSE increases significantly in the case of 50% perturbation in the restoring stiffness coefficient, $R_s$. Performance in the worst-case scenario is similar to that in case 9.

4.2.4 Results of QP-PID

The QP method is deployed in the HHC along with PID controller in the LHC level to produce a new strategy. This strategy is referred to as QP-PID.

System performance under nominal conditions for significant wave heights, $H_s$, of 1 m and 3 m is shown in the graphs of figures 4.19 and 4.20, respectively. The performance of the TF-PID and the QP-LLC strategies are added to the graphs for the sake of comparison.
Table 4.7: Performance of QP-LLC under various perturbation scenarios and disturbance forces, and the worst-case scenario

The performance of the QP-PID is almost typical to that of the QP-LLC. However, the QP-PID does not violate the ratio constraint as the QP-LLC does in the case of \( H_s = 1 \text{ m} \), as shown in Figure 4.19 (e). In addition, the QP-PID shows slightly higher maximum converted power, \( P_{em} \), and slightly lower efficiency in the case of \( H_s = 1 \text{ m} \), as shown in figures 4.20 (d) and (f), respectively. Both violate the designed maximum control force, \( f_{um} \), at low peak wave frequency, \( \omega_p \).

For the sake of comparison, the Bode plots of the conventional method discussed in the last section of chapter 3 are shown in figures 4.21 and 4.22, along with those of the TF-PID and QP-PID. The control strategy of using the conventional method in the HHC along with the LLC in the LHC is referred to in the figures as CHCS-LLC.

In the case of \( H_s = 1 \text{ m} \), the performance of the CHCS-LLC is very similar to the QP-PID in terms of average captured power (\( \bar{P}_m \)), average converted power (\( \bar{P}_e \)), and maximum control force (\( f_{um} \)), as shown in figures 4.21 (a), (b), and (c), respectively. However, the CHCS-LLC violates the designed control force control for wider ranges of the peak wave frequency, \( \omega_p \). The CHCS-LLC significantly exceeds its design value, 10, for the ratio \( P_{em}/\bar{P}_e \) as shown in Figure 4.21 (e), while other strategies do not. The efficiency of the CHCS-LLC is lower than the efficiency of the other two strategies (see Figure 4.21 (f)).
Figure 4.19: Simulation results of the QP-PID, TF-PID, and QP-LLC using a monochromatic sea state of $H_s = 1 \text{ m}$

Figure 4.20: Simulation results of the QP-PID, TF-PID, and TF-LLC using a monochromatic sea state of $H_s = 3 \text{ m}$
The CHCS-LLC experiences very poor performance in the case of \( H_s = 3 \text{ m} \). In low peak-wave frequencies, the reversed direction of the average captured and converted powers indicates higher reactive content in the electrical power. This performance is completely undesired as it indicates that the PTO works as a motor instead of generator (see figures 4.22 (a) and (b)). The maximum control force, \( f_{\text{um}} \), of the CHCS-LLC in Figure 4.22 (c) exceeds the designed value, 1.5 \( MN \), for the entire peak wave frequency range, \( \omega_p \).

![Graphs showing simulation results](image)

**Figure 4.21:** Simulation results of the QP-PID, TF-PID, and CHCS-LLC using a monochromatic sea state of \( H_s = 1 \text{ m} \)

The simulation results using the irregular sea state for mechanical and electrical quantities are shown in figures 4.23 and 4.24, respectively. The tracking capability of the LHC is demonstrated in Figure 4.23 (a). The figure shows that the velocity of the buoy \( \dot{z} \) almost equals the reference velocity, \( \dot{z}_r \), with an MSE of \( 1.72 \times 10^{-4} \). This value is close to the TF-PID (1.28 \( \times 10^{-4} \)). However, it is noticeable that the tracking capability of the LLC is better in cases in which it has an MSE value of \( 4.14 \times 10^{-5} \) and \( 6.49 \times 10^{-6} \) for the TF-LLC and QP-LLC.
Figure 4.22: Simulation results of the QP-PID, TF-PID, and CHCS-LLC using a monochromatic sea state of $H_s = 3$ m

respectively. Considering the radius of the buoy, the elevation of the buoy $z$, and the wave elevation $\eta$ in Figure 4.23 (b), we can see that the buoy’s elevation reaches critical values, $1.5$ m, at some points of the graph. Extreme excursion must be avoided to prevent physical damage on the PTO. Figure 4.23 shows a comparison between the control force $f_u(t)$ and the excitation force $f_{ex}$. The control force has a slightly higher order of magnitude than the excitation force, $f_{ex}(t)$. It stays below its designed limitation for the specific sea state used in this test. However, the previous Bode plots show that this constraint is violated in some sea-state conditions. The instantaneous captured power $P_m$ is shown in the Figure 4.23 (c). From the results, an average captured power $\bar{P}_m$ of $0.13$ MW is generated, the same as that generated by the QP-LLC. The $d-q$ components of the stator voltage are shown in Figure 4.24(a). The $q$-axis current component $i_{sq}$ that implements the $f_u(t)$ is shown in Figure 4.24(b), while Figure 4.24(c) shows the EMF voltage of the PTO. An average converted power $\bar{P}_e$ of $0.08$ MW, also similar to the QP-LLC, is produced by the PTO. This is shown in Figure 4.24 (c). This corresponds to
61.8% of the conversion efficiency from $P_m$ to $P_e$.

![Graphs showing velocity, position, force, and mechanical power over time.](a) Velocity [m/s] vs. Time [s], (b) Position [m] vs. Time [s], (c) Force [MN] vs. Time [s], (d) Mechanical power [MW] vs. Time [s].

Figure 4.23: Simulation results showing mechanical quantities produced by the QP-PID under a polychromatic sea state with $H_s = 2 m$ and $\omega_p = 0.7 \text{ rad/s}$.

Similar to previous strategies, we apply perturbation scenarios discussed in section 4.1 to the QP-PID to test the tracking capability and robustness. Results are summarised in Table 4.8 in terms of energy drop percentage and MSE.

<table>
<thead>
<tr>
<th>Cases</th>
<th>$\Delta f$ [%]</th>
<th>$\Delta x$ [%]</th>
<th>Energy [kWh]</th>
<th>Energy Drop [%]</th>
<th>MSE [$10^{-4}$]</th>
</tr>
</thead>
<tbody>
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<td>Case 1</td>
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<td>0</td>
<td>4.24</td>
<td>8.56</td>
<td>1.65</td>
</tr>
<tr>
<td>Case 2</td>
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<td>0.25</td>
<td>4.24</td>
<td>8.62</td>
<td>1.7</td>
</tr>
<tr>
<td>Case 3</td>
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<td>0.5</td>
<td>4.24</td>
<td>8.75</td>
<td>4.83</td>
</tr>
<tr>
<td>Case 4</td>
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<td>0</td>
<td>4.15</td>
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<td>1.64</td>
</tr>
<tr>
<td>Case 5</td>
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<td>0.25</td>
<td>4.14</td>
<td>10.76</td>
<td>1.68</td>
</tr>
<tr>
<td>Case 6</td>
<td>0.25</td>
<td>0.5</td>
<td>4.14</td>
<td>10.88</td>
<td>4.26</td>
</tr>
<tr>
<td>Case 7</td>
<td>0.5</td>
<td>0</td>
<td>4.05</td>
<td>12.83</td>
<td>1.62</td>
</tr>
<tr>
<td>Case 8</td>
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<td>0.25</td>
<td>4.04</td>
<td>12.90</td>
<td>1.67</td>
</tr>
<tr>
<td>Case 9</td>
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<td>0.5</td>
<td>4.0381</td>
<td>13.00</td>
<td>3.77</td>
</tr>
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<td>Worst Case</td>
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<td>0.5</td>
<td>4.03</td>
<td>13.183</td>
<td>3.62</td>
</tr>
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</table>

Table 4.8: Performance of QP-PID under various perturbation scenarios and disturbance forces, and the worst-case scenario.

The table shows similar results to those of the strategies using the quadratic
programming technique (QP) to obtain the optimum reference. The energy drop increases slightly with an increase in the restoring stiffness coefficient, $R_s$, while it increases significantly with the loss resistance, $R_{loss}$. Tracking performance is satisfactory to some extent, but the MSE increases significantly for the case of 50% perturbation in the restoring stiffness coefficient, $R_s$. Performance in the worst-case scenario is similar to that in case 9. We can conclude that the QP HHCs show poor results in terms of robustness at 50% perturbation in the restoring stiffness coefficient, $R_s$.

4.2.5 Results of QP-SMC

System performance under nominal conditions for significant wave heights, $H_s$, of 1 m and 3 m is shown in the graphs of figures 4.25 and 4.26, respectively. The previously analysed performance of the QP-LLC method is added to the graphs for comparison.
Figure 4.25: Simulation results of the QP-SMC and QP-LLC using a monochromatic sea state of $H_s = 1 \text{ m}$

Figure 4.26: Simulation results of the QP-SMC and QP-LLC using a monochromatic sea state of $H_s = 3 \text{ m}$
The performance of the QP-SMC is very similar to that of the QP-LLC. The noticeable difference is that the ratio $P_{em}/P_e$ does not exceed its design value, 10, in the case of QP-SMC compared to the QP-LLC for a significant wave height $H_s = 1 \, m$. The maximum converted electrical power $P_{em}$ is slightly lower in the QP-SMC. For significant height $H_s = 3 \, m$, the performance results are still close to each other. However, the maximum converted electrical power $P_{em}$ and the ratio $P_{em}/P_e$ are low in low peak frequencies. Hence, the efficiency is also lower in this range (i.e. $0.5 - 0.7 \, rad/s$).

The simulation results using the irregular sea state for mechanical and electrical quantities are shown in figures 4.27 and 4.28, respectively. The tracking capability of the SMC is demonstrated in Figure 4.27 (a). The figure shows that the velocity of the buoy $\dot{z}$ perfectly matches the reference velocity, $\dot{z}_r$. The value of the mean square error (MSE) is $2.19 \times 10^{-5}$. This indicates that the SMC provides perfect tracking capability. Considering the radius of the buoy, the elevation of the buoy $z$, and the wave elevation $\eta$ in Figure 4.27 (b), we can see that the buoy's elevation reaches critical values, $1.5 \, m$ at some points of the graph. Figure 4.27 shows a comparison between the control force $f_u(t)$ and the excitation force $f_{ex}$. The control force has a slightly higher order of magnitude than the excitation force, $f_{ex}(t)$. It stays below its designed limitation for the specific sea state used in this test. The instantaneous captured power $P_m$ is shown in Figure 4.27 (c). From the results, an average captured power $\bar{P}_m$ of $0.13 \, MW$ is generated. The $d-q$ components of the stator voltage are shown in Figure 4.28 (a). The $q$-axis current component $i_{sq}$ that implements the $f_u(t)$ is shown in Figure 4.28 (b), while Figure 4.28 (c) shows the EMF voltage of the PTO. An average converted power $\bar{P}_e$ of $0.08 \, MW$ is produced by the PTO. This is shown in Figure 4.28 (c). This corresponds to $61.8 \%$ of the conversion efficiency from $P_m$ to $P_e$.

In general, the performance of the QP-SMC is almost identical to that of the QP-LLC for this sea state. This is expected, based on the Bode plots of these
HCSs. In lower frequencies (less than 0.7 \( \text{rad/s} \)), differences are expected in the average powers and efficiency. However, the MSE of the QP-SMC (2.19 \( \times 10^{-5} \)) is significantly lower than that of the QP-LLC (6.49 \( \times 10^{-5} \)). This reflects the higher tracking capability of the QP-SMC.

![Figure 4.27: Simulation results showing mechanical quantities produced by the QP-SMC under a polychromatic sea state with \( H_s = 2 \text{ m} \) and \( \omega_p = 0.7 \text{ rad/s} \)](image)

Perturbation scenarios discussed in section 4.1 are applied in the WEC system to test the tracking capability and robustness. Results are summarised in Table 4.9 in terms of energy drop percentage and MSE. A trend similar to that of the QP-LLC is noted for the QP-AMC. The table shows that the energy drop increases slightly with an increase in the restoring stiffness coefficient, \( R_s \), while it increases significantly with the loss resistance, \( R_{loss} \). Tracking performance is satisfactory to some extent, but the MSE increases significantly for the case of 50 % perturbation in the restoring stiffness coefficient, \( R_s \). Performance in the worst-case scenario is similar to that in case 9.
Figure 4.28: Simulation results showing electrical quantities produced by the QP-SMC under a polychromatic sea state with $H_s = 2 \, m$ and $\omega_p = 0.7 \, rad/s$

<table>
<thead>
<tr>
<th>Cases</th>
<th>$\Delta_l$ %</th>
<th>$\Delta_s$ %</th>
<th>Energy [kWh]</th>
<th>Energy Drop %</th>
<th>MSE [$\times 10^{-5}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
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<td>0</td>
<td>4.27</td>
<td>8.63</td>
<td>2.18</td>
</tr>
<tr>
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<td>0.25</td>
<td>4.27</td>
<td>8.67</td>
<td>2.28</td>
</tr>
<tr>
<td>Case 3</td>
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<td>0.5</td>
<td>4.27</td>
<td>8.76</td>
<td>1.33</td>
</tr>
<tr>
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<td>10.80</td>
<td>2.18</td>
</tr>
<tr>
<td>Case 5</td>
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<td>0.25</td>
<td>4.17</td>
<td>10.82</td>
<td>2.24</td>
</tr>
<tr>
<td>Case 6</td>
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<td>0.5</td>
<td>4.17</td>
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<td>1.16</td>
</tr>
<tr>
<td>Case 7</td>
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<td>0</td>
<td>4.07</td>
<td>12.95</td>
<td>2.18</td>
</tr>
<tr>
<td>Case 8</td>
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<td>0.25</td>
<td>4.07</td>
<td>12.98</td>
<td>2.23</td>
</tr>
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<td>Case 9</td>
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<td>0.5</td>
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<td>1.01</td>
</tr>
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<td>0.5</td>
<td>4.06</td>
<td>13.11</td>
<td>1.00</td>
</tr>
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Table 4.9: Performance of QP-LLC under various perturbation scenarios and disturbance forces, and worst-case scenario
Chapter 5: Conclusion

In this study, several hierarchical control strategies (HCSs) are proposed for controlling heaving wave energy converters (WECs). Two methods are proposed for the HHC and three for the LHC. Five combinations are formulated to assess their performance and find out the best combination in terms of enhancing captured and converted power, achieving best tracking performance and disturbance rejection and maintaining design constraints. All the controllers are model-based reference-based controllers. The principle of maximum power transfer is used to design all the control strategies. A computer simulation is used to assess the proposed controllers in terms of the amount of captured energy, system limitation handling, and the sensitivity to model uncertainties and external disturbances.

5.1 Research Findings Summary

The core research output and findings are summarised as:

- A novel higher hierarchical Controller (HHC) is developed for maximising the captured power with constraints on the buoy's elevation. A constrained optimisation problem is synthesised and solved using quadratic programming (QP).
- A simple algorithm is used in the HHC to obtain the reference signal for the lower hierarchical controller (LHC). The objective is to effectively improve the captured power without violating the constraints on the maximum value of the control force and the PTO utilisation index. The transfer function of the controller in the latter is utilised to carry out the calculations of the intrinsic resistance in the HHC.
- A novel LHC of a PID controller augmented with sliding mode control is
designed. The dynamic model in the LHC is not considered in the design process of this method.

- A robust PID controller is designed as an LHC using complex PID stabilisation to provide tracking performance and robustness against modelled uncertainties.

- Another LHC is designed using $H_{\infty}$ theory as a novel lead-lag compensator. Tracking capability and minimised control force are achieved by this LHC.

- The developed novel HHCs and LHCs are combined to produce complete hierarchical control systems (HCSs) strategies. Each combination is analyzed and assessed.

- The proposed control strategies are validated using a full wave-to-wire time-domain model, and proper performance indices that measure the effectiveness of the controller in all aspects, mechanical and electrical.

- TF-PID and TF-LLC show the best performance in terms of maintaining constraints, enhancing captured and converted powers and achieving satisfying tracking performance and robustness. However, TF-PID achieves better efficiency while TF-LLC achieves better tracking performance.

- Using QP as HHC dominates the overall performance of the HCS. It achieves higher captured and converted powers but violates constraints in certain sea-states.

- TF-PID and TF-LLC are recommended in general. However, QP methods can be used for specific sea-states.

5.2 Future Work

Many research efforts related to WECs are proposed, and the WEC research team at the UAEU plans to carry them out in the near future. Below is a summary
of these topics:

- The topic of integrating the WECs to the grid will be considered. This includes investigating the reaction of the WECs against sudden changes in the point of common coupling (PCC), such as faults and rapid load variations. The power laboratory of the UAEU is equipped with a wave energy test rig and power systems simulator that facilitate this research topic.

- The concept of the hardware-in-the-loop (HIL) simulation will be utilised to validate the developed control strategies. The test rig available in the UAEU labs represents the PMLG, while the wave-buoy interactions can be simulated in the computer. Both parts interact with each other. This eliminates the need to conduct a wet test using a scaled-down model of the WEC and reduces the experimental cost.

- The possibilities of developing intelligent controllers utilising neural networks (NNs) will be investigated. Such controllers can carry out online system identification for the WEC by collecting data from the device and the surroundings during operation.

- Supervisory control strategies will be designed, in which better coordination between the higher-level and lower-level control loops will be achieved. This produces control actions that create a compromise between the mechanical and electrical sides of the system.

- Assessment studies that consider the sea state of the eastern coast of the UAE will be carried out, and the feasibility of implementing WEC farms there will be investigated.
Bibliography


List of Publications

Published Articles


Articles Submitted to Journals


Articles Under Preparation

Appendix A: MATLAB® Codes

Complex polynomials stabilization algorithm and files of MATLAB® are in the CD.
Appendix A: MATLAB® Codes

Complex polynomials stabilization algorithm and files of MATLAB® are in the CD.