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Entitled

ON THE PROJECTIONS AND UNITARY GROUPS OF UNITAL C-ALGEBRAS*

by

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Abstract

H. Dye proved that the unitary group in a factor determines the algebraic type of that factor. Al-Rawashdeh, Booth and Giordano established that, for a large class of simple unital C*-algebras, an isomorphism between the unitary groups, induces an isomorphism between their K_0-ordered group and K_1-groups. Then using the results of Dadarlat-Elliot-Gong and Kirchberg-Phillips, the C*-algebras are isomorphic. Dye introduced special projections $P_{\{i,j\}}(a)$ of the matrix algebra $M_n(A)$, and he used it as a main tool to establish his results in the case of von Neumann factors. Precisely, in case of von Neumann algebra, he proved that if θ is an orthoisomprhism which fixes all the $P_{\{i,j\}}(a)$, then it is the identity mapping on the projections. We discuss these projections and we give more properties in the case of C*-matrix algebras. Using Dye's approach, we prove that for a unital C*-algebra A, if θ is an orthoisomorphism on $P(M_n(A))$ which fixes the $P_{\{i,j\}}(a)$, then θ fixes all the projections on class *D*, consisting of some decomposition of $P_{\{i,j\}}(a)$. We introduce the invariant unitary groups property (IUG-P), the orthogonal IUG-P and the topological IUG-P. We investigate that some properties are IUG-P, orthogonal IUG-P or topological IUG-P, for certain C*-algebras. If the general linear groups (GL(A)) are isomorphic, we prove that the induced mapping between idempotents preserves the orthogonality, for a large class of unital C*-algebras, including certain type of UHF-algebras, 2-divisible K_0-groups, Cuntz algebras O_n, $2 \leq n \leq \infty$, and for simple unital purely infinite C*-algebras having 2-divisible K_0- groups. We prove that if N is a normal subgroup of $GL(O_n)$, then N contains all the symmetries of O_n. Also, we show that if N is any normal subgroup of unitary groups of compact operator K, which contains some certain type of involution, then N contains all the involutions of K.