[Emirates Journal for Engineering Research](https://scholarworks.uaeu.ac.ae/ejer)

Volume 27 | Issue 1 Article 3

2-3-2022

Numerical Treatment for Special Type of Mixed Linear Delay Volterra Integro-Differential Equations

Atheer J. Kadhim
University of Technology,Iraq, Atheer.j.kadhim@uotechnology.edu.iq

Follow this and additional works at: https://scholarworks.uaeu.ac.ae/ejer

Part of the Ordinary Differential Equations and Applied Dynamics Commons, Statistical, Nonlinear, and Soft Matter Physics Commons, and the Theory and Algorithms Commons

Recommended Citation

Kadhim, Atheer J. (2022) "Numerical Treatment for Special Type of Mixed Linear Delay Volterra Integro-Differential Equations," *Emirates Journal for Engineering Research*: Vol. 27 : Iss. 1 , Article 3.
Available at: https://scholarworks.uaeu.ac.ae/ejer/vol27/iss1/3

This Article is brought to you for free and open access by Scholarworks@UAEU. It has been accepted for inclusion in Emirates Journal for Engineering Research by an authorized editor of Scholarworks@UAEU. For more information, please contact EJER@uaeu.ac.ae.

NUMERICAL TREATMENT FOR SPECIAL TYPE OF MIXED LINEAR DELAY VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

Atheer J. Kadhim

University of Technology, Applied Science Department, Iraq.

Atheer.j.kadhim@uotechnology.edu.iq

(Received 26th October 2021 and Accepted 3rd February 2022)

المعالجة العددية

لنوع خاص من معادلات فولتيرا التكاملية التفاضلية التباطئية الخطية المختلطة

ملخص

طريقة مقترحة تم تقديمها لحل نوع خاص من معادلات فولتيرا التكاملية التفاضلية المختلطة التباطئية الخطية (LMDVIDE) عدديًا باستخدام طريقة البلوك مع طريقة Simpson 3/8. كما قمنا بتقديم مقارنة بين النتائج العددية والتحليلية لبيان دقة نتائج الطريقة المقترحة.

Abstract

The idea of research is a representation of the nonlinear pseudo-random generators using state-space equations that is not based on the usual description as shift register synthesis but in terms of matrices. Different types of nonlinear pseudo-random generators with their algorithms have been applied in order to investigate the output pseudo-random sequences. Moreover, two examples are given for conciliated the results of this representation.

1. INTRODUCTION

One of the important and applicable subjects of applied mathematics is the integro-differential equation which is an equation that involves both integrals and derivatives of a function. The names of many modern mathematicians notably, Volterra, Fredholm, Cauchy and others are associated with this topic. The integral and integro-differential equations formulation of physical problems are more elegant and compact than the differential equation formulation, since the boundary conditions can be satisfied and embedded in the integral or integro-differential equation. [1,2]. Salih [3] Solve nth order retarded delay Volterra integro-differential equations utilizing block and Weddle methods while Hassan [4] solve special type of nonlinear delay Volterra integro-differential equations using block and Bool methods. [5] used B-Spline Functions for Solving n th Order Linear Delay Integro-Differential Equations of Convolution Type. [6] presented an approximated solutions for nth order linear delay integro-differential equations of convolution type. [7] Solved nth order nonlinear differential-difference equations of hybrid-time systems using Nystrom method.

In this work we solve a special type of LMDVIDE numerically using block and Simpson3/8 rule which is utilized for treating the integral of LMDVIDE.

2. Linear mixed delay Volterra integro-differential equations (LMDVIDE) [3,4,5]

The LMDVIDE given by:

$$\sum_{i=0}^{n} p_i(x)\frac{d^i u(x)}{dx^i} + \sum_{i=1}^{n} q_i(x)\frac{d^i u(x-\tau_i)}{dx^i} + \sum_{i=0}^{n} r_i(x)u(x-\tau_i) = g(x) + \lambda\int_a^x k(x,t)u(t-\tau)dt \qquad x \in [a, b(x)] \quad \dots (1)$$

with initial functions:

$$\left.\begin{aligned} u(x) &= \varphi(x) \\ u'(x) &= \varphi'(x) \\ &\vdots \\ u^{(n-1)}(x) &= \varphi^{(n-1)}(x) \end{aligned}\right\}$$

$$for \quad x_0 - max(\tau, \tau_i) \le x \le x_0 ,$$

$i = 0,1,\dots,n$. … (8)

where $g(x)$, $p_i(x)$, $q_i(x)$, $k(x,t)$ are known functions of x, $u(x)$ is the unknown function.

3. BLOCK METHOD and SIMPSON RULE (BS).

The following section explained two methods are utilized for solving LMDVIDE.

Simpson 3/8 Rule

Simpson formula approximates the function on the interval [t_0 ,t_3] by a curve that possesses through four points as [2]

$$\int_a^b f(t)dt = \frac{3H}{8}[f_0 + 3f_1 + 3f_2 + f_3] \quad \dots (2)$$

where $H = \frac{(b-a)}{N}$, N is the number of intervals.

Block Method

Consider the following first order differential equation [3,4,5] :

$$y' = f(t, y(t)) \quad \text{with initial condition} \quad y(t_0) = y_0 \quad \dots(3)$$

Let

$$\left.\begin{array}{l} B_1 = f(t_n, y(t_n)) \\ B_2 = f(t_n + h, y(t_n) + hB_1) \\ B_3 = f\left(t_n + h, y(t_n) + \frac{h}{2}B_1 + \frac{h}{2}B_2\right) \\ B_4 = f(t_n + 2h, y(t_n) + 2hB_3) \\ B_5 = f\left(t_n + h, y(t_n) + \frac{h}{12}(5B_1 + 8B_3 - B_4)\right) \\ B_6 = f\left(t_n + 2h, y(t_n) + \frac{h}{3}(B_1 + B_4 + 4B_5)\right) \end{array}\right\} \quad \dots (4)$$

Then the 4th order block method may be written in the form:

$$y_{n+1} = y_n + \frac{h}{12}(5B_1 + 8B_3 - B_4) \quad \dots (5)$$

$$y_{n+2} = y_n + \frac{h}{3}(B_1 + 4B_5 + B_6) \quad \dots (6)$$

4. The SOLUTION of LMDVIDE

The general form of LMDVIDE in eq.(1) can be written as:

$$\frac{d^n u(x)}{dx^n} = f\begin{pmatrix} x, p_0(x)u(x), p_1(x)u'(x), \dots, p_{n-1}(x)u^{(n-1)}(x), q_1(x)u'(x-\tau_1), \dots, \\ q_n(x)u^{(n)}(x-\tau_n), r_0(x)u(x-\tau_0), \dots, r_n(x)u(x-\tau_n), g(x), \\ I[Q(x,t)] \end{pmatrix} \quad \dots (7)$$

where $I[Q(x,t)]$ is the finite integral on $[a,x]$,$\boldsymbol{x \geq a}$

Equation (7) was replaced by a system of nth-equation of first order equations

Let

$$v_1(x) = u(x)$$
$$v_2(x) = u'(x)$$
$$\vdots$$
$$v_{n-1}(x) = u^{(n-2)}(x)$$
$$v_n(x) = u^{(n-1)}(x)$$

Then, the above system can be treated numerically by using block and Simpson methods as follows:

$$v_i(x_{j+1}) = v_i(x_j) + \frac{h}{12}(5B_{1i} + 8B_{3i} - B_{4i}) \quad \dots (9)$$

$$v_i(x_{j+2}) = v_i(x_j) + \frac{h}{3}(B_{1i} + 4B_{5i} + B_{6i}) \quad \dots (10)$$

where

$$B_{1i} = f_i\begin{pmatrix} x_j, p_0(x_j)v_1(x_j), \dots, p_{n-1}(x_j)v_n(x_j), q_1(x_j) \\ \varphi'(x_j-\tau_1), \dots, q_n(x_j)\varphi^{(n)}(x_j-\tau_n), \\ r_0(x_j)\varphi(x_j-\tau_0), \dots, r_n(x_j)\varphi(x_j-\tau_n), \\ g(x_j), Simp(Q(x_j,t), a, x_j, N) \end{pmatrix}$$

$$B_{2i} = f_i\begin{pmatrix} x_j+h, p_0(x_j+h)v_1(x_j) + hB_{11}, \dots, \\ p_{n-1}(x_j+h)v_n(x_j) + hB_{1n}, q_1(x_j+h) \\ \varphi'(x_j+h-\tau_1), \dots, q_n(x_j+h)\varphi^{(n)}(x_j+h-\tau_n), \\ r_0(x_j+h)\varphi(x_j+h-\tau_0) \\ , \dots, r_n(x_j+h)\varphi(x_j+h-\tau_n), g(x_j+h), \\ Simp(Q(x_j+h,t), a, x_j+h, N) \end{pmatrix}$$

$$B_{3i} = f_i\begin{pmatrix} x_j+h, p_0(x_j+h)v_1(x_j) + \frac{h}{2}B_{11} + \frac{h}{2}B_{21}, \dots, \\ p_{n-1}(x_j+h)v_n(x_j) + \frac{h}{2}B_{1n} + \frac{h}{2}B_{2n}, \\ q_1(x_j+h)\varphi'(x_j+h-\tau_1), \dots, q_n(x_j+h)\varphi^{(n)}(x_j+h-\tau_n), \\ r_0(x_j+h)\varphi(x_j+h-\tau_0) \\ , \dots, r_n(x_j+h)\varphi(x_j+h-\tau_n), g(x_j+h), \\ Simp(Q(x_j+h,t), a, x_j+h, N) \end{pmatrix}$$

$$B_{4i} = f_i\begin{pmatrix} x_j+2h, p_0(x_j+2h)v_1(x_j) + 2hB_{31}, \dots, p_{n-1}(x_j+2h)v_n(x_j) + \\ 2hB_{3n}, q_1(x_j+2h) \\ \varphi'(x_j+2h-\tau_1), \dots, q_n(x_j+2h)\varphi^{(n)}(x_j+2h-\tau_n), \\ r_0(x_j+2h)\varphi(x_j+2h-\tau_0) \\ , \dots, r_n(x_j+2h)\varphi(x_j+2h-\tau_n), g(x_j+2h), \\ Simp(Q(x_j+2h,t), a, x_j+2h, N) \end{pmatrix}$$

$$B_{5i} = f_i\begin{pmatrix} x_j+h, p_0(x_j+h)v_1(x_j) + \frac{h}{12}(5B_{11} + 8B_{31} - B_{41}), \dots, \\ p_{n-1}(x_j+h)v_n(x_j) + \\ \frac{h}{12}(5B_{1n} + 8B_{3n} - B_{4n}), q_1(x_j+h)\varphi'(x_j+h-\tau_1), \dots, \\ q_n(x_j+h)\varphi^{(n)}(x_j+h-\tau_n) \\ , r_0(x_j+h)\varphi(x_j+h-\tau_0), \dots, r_n(x_j+h)\varphi(x_j+h-\tau_n), \\ g(x_j+h), \\ Simp(Q(x_j+h,t), a, x_j+h, N) \end{pmatrix}$$

$$B_{6i} = f_i\begin{pmatrix} x_j+2h, p_0(x_j+2h)v_1(x_j) + \frac{h}{3}(B_{11} + B_{41} + 4B_{51}), \dots, \\ p_{n-1}(x_j+2h)v_n(x_j) + \\ \frac{h}{3}(B_{1n} + B_{4n} + 4B_{5n}), q_1(x_j+2h)\varphi'(x_j+2h-\tau_1), \dots, \\ q_n(x_j+2h) \\ \varphi^{(n)}(x_j+2h-\tau_n), r_0(x_j+2h)\varphi(x_j+2h-\tau_0), \dots, \\ r_n(x_j+2h)\varphi(x_j+2h-\tau_n), \\ g(x_j+2h), Simp(Q(x_j+2h,t), a, x_j+2h, N) \end{pmatrix}$$

for each i=1,2,…,n. and j=0,1,…,m where $Simp(Q(x,t), a, x, N)$

$$= \frac{3H}{8}\begin{bmatrix} Q(x,t_0) + 3Q(x,t_1) + 3Q(x,t_2) + 2Q(x,t_3) + \\ 3Q(x,t_4) + 3Q(x,t_5) \\ + \dots + 2Q(x,t_{N-3}) + 3Q(x,t_{N-2}) + \\ 3Q(x,t_{N-1}) + Q(x,t_N) \end{bmatrix}$$

where $t_k = a + kH$, $H = \frac{(b-a)}{N}$ and k = 0,1,…,N.

5. Test Example

Consider LMDVIDE of third order:

$$\frac{d^3u(x)}{dx^3} + \frac{d^2u(x-2)}{dx^3} + x\frac{du(x-2)}{dx} + 2\frac{du(x)}{dx} - u(x) + u(x-\tfrac{1}{2}) = \left(-\tfrac{5}{6}x^3 - \tfrac{9}{4}x^2 + x + \tfrac{3}{2}\right) + \int_0^x (x+t)u(t-\tfrac{1}{2})dt \quad x \geq 0 \qquad \dots (12)$$

with initial functions:

$u(x) = x + 2 \qquad -0.5 \leq x \leq 0$

$u'(x) = 1 \qquad -2 \leq x \leq 0$

$u''(x) = 0 \qquad -2 \leq x \leq 0$

The exact solution is:

$u(x) = x + 2 \qquad 0 \leq x \leq 0.5$

Table (1) presents the comparison between the analytic and numerical solutions of eq.(12) using BS methods for m=10, h=0.05, $x_j = jh$, $j = 0,1,\dots,m$ and m=100, h=0.005 , depending on (L.S.E.).

Table (1) The solution of LMDVIDE

x	$Exact_1$	BS Methods $v_1(x)$		$Exact_2$	BS Methods $v_2(x)$		$Exact_3$	BS Methods $v_3(x)$	
		h=0.05	h=0.005		h=0.05	h=0.005		h=0.05	h=0.005
0	2.0000	2.0000	2.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.05	2.0500	2.0500	2.0500	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.10	2.1000	2.1000	2.1000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.15	2.1500	2.1500	2.1500	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.20	2.2000	2.2000	2.2000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.25	2.2500	2.2500	2.2500	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.30	2.3000	2.3000	2.3000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.35	2.3500	2.3500	2.3500	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.40	2.4000	2.4000	2.4000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.45	2.4500	2.4500	2.4500	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
0.50	2.5000	2.5000	2.5000	1.0000	1.0000	1.0000	0.0000	0.0000	0.0000
L.S.E.		0.12e-31	0.0000	L.S.E.	0.0000	0.0000	L.S.E.	0.15e-45	0.0000

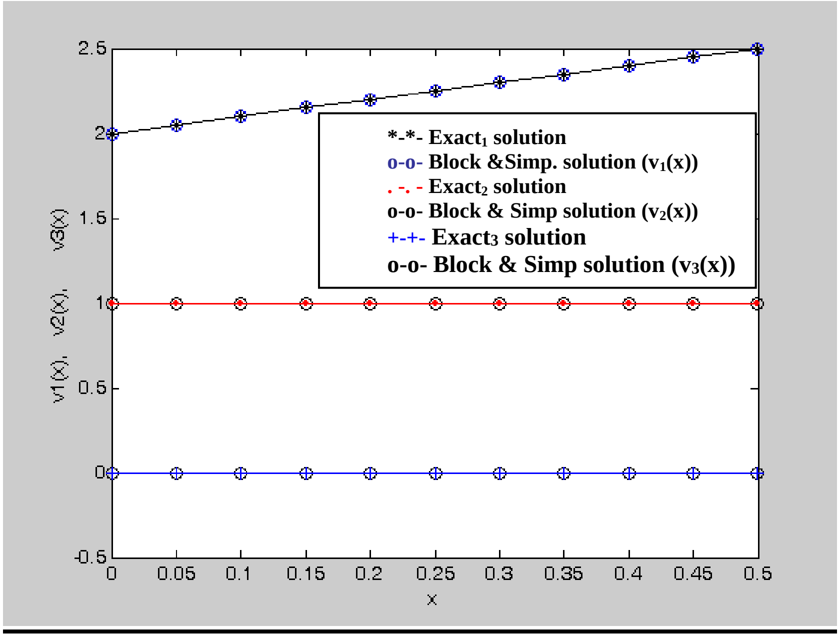

Figure (1) The comparison depending on the table(1).

CONCLUSIONS

1- BS methods proved their effectiveness in solving LMDVIDE as see in the above example.

2- The good solution depends on choosing h small enough. Different values of h and the corresponding L.S.E. are listed in table (2) where $\mathbf{0} \le \boldsymbol{x} \le \mathbf{1}$.

Table(2) The L.S.E. where h=0.05 ,0.02 and 0.001

The BS solutions		
h	***m***	***L.S.E.***
0.05	20	0.323e-10
0.02	50	0.12e-11
0.001	1000	0.509e-26

REFRENCES

1. Burgestaller, R.H. Integral and Integro-Differential Equation Theory Methods and Applications, 3rd edition, p.340, (2000) Edit by Agarwal R.P. Oregun D. Gordon and Breach Science Publisher, N.Cliffs.
2. Jerri, A.J. Introduction to Integral Equations with Applications, 1st edition, p.254, (1985) Marcel Dekker Inc., USA.
3. Salih, R.K., 2020. Block and Weddle Methods for Solving nth Order Linear Retarded Volterra Integro-Differential Equations. Emirates Journal for Engineering Research, 25(2), p.3.
4. Hassan, I. H.,,Block and Bool Methods for Solving Special Type of nth Order Nonlinear Delay Volterra Integro-Differential Equations, J. of the College of Basic Education, Al-Mustansiriyah University, No.71, 2011.
5. Salih, R.K., Kadhim, A.J. and Al-Heety, F.A., 2010. B-Spline Functions for Solving n th Order Linear Delay Integro-Differential Equations of Convolution Type. Engineering and Technology Journal, 28(23), pp.6801-6813.
6. Salih RK, Hassan IH, Kadhim AJ. An Approximated Solutions for nth Order Linear Delay Integro-Differential Equations of Convolution Type Using B-Spline Functions and Weddle Method. Baghdad Science Journal. 2014;11(1).
7. Salih, R.K., 2009. Nyström's Method for Solving nth Order Nonlinear Differential-Difference Equations of Hybrid-Time Systems. journal of the college of basic education, Al-Mustansiriyah University, 12(57).