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MATHEMATICAL MODELING OF COMMUNICABLE IMPORTED DISEASES SCREENING IN THE UNITED ARAB EMIRATES

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MATHEMATICAL MODELING OF COMMUNICABLE IMPORTED
DISEASES SCREENING IN THE UNITED ARAB EMIRATES

Lahbib Ben Ahmadi

This thesis is submitted in partial fulfillment of the requirements for the degree of
Master of Science in Mathematics

Under the Supervision of Dr. Abdessamad Tridane

May 2015

Declaration of Original Work

I, Lahbib Ben Ahmadi, the undersigned, a graduate student at the United Arab Emirates University (UAEU), and the author of this thesis entitled, *"Mathematical Modeling of Communicable Imported Diseases Screening in the United Arab Emirates"*, hereby, solemnly declare that this thesis is an original research work that has been done and prepared by me under the supervision of Dr. Abdessamad Tridane, in the College of Science at UAEU. This work has not been previously formed as the basis for the award of any academic degree, diploma or a similar title at this or any other university. The materials borrowed from other sources and included in my thesis have been properly cited and acknowledged.

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Approval of the Master Thesis

This Master Thesis is approved by the following Examining Committee Members:

1) Advisor (Committee Chair): Abdessamad Tridane

Title: Assistant Professor

Department of Mathematical Sciences

College of Science

Signature ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   Date ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

2) Member: Mohamed Ali Hajji

Title: Associate Professor

Department of Mathematical Sciences

College of Science

Signature ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   Date ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

3) Member (External Examiner): Abdul Salam Jarrah

Title: Associate Professor

Department of Mathematics and Statistics

Institution: American University of Sharjah

Signature ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   Date ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

This Master Thesis is accepted by:

Dean of the College of Science: Professor Frederick Chi-Ching Leung

Signature ───────────────────────────── Date ─────────────────

Dean of the College of Graduate Studies: Professor Nagi T. Wakim

Signature ───────────────────────────── Date ─────────────────

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Abstract

The United Arab Emirates (UAE), as one of the countries with high numbers of expatriates in the world, is expected to face public health challenges. The reason for this situation is that the majority of those expatriates belong to regions where health issues are usually left behind. This may create the possibility of having imported communicable diseases. However, screening policy should be tested and adapted to protect the population from any imported communicable disease.

This study aims at identifying an approach and method to deal with these imported diseases via a set of differential equations. The spread of a communicable disease is examined by taking in consideration the nature of the expatriates in the UAE. The population of expatriates is divided into high risk and low risk groups.

The study concluded to the possibility of the persistence of the diseases under seven possible scenarios. Each of these scenarios represents the endemic level of the disease. To clarify the case simulations of two types of diseases are examined: HIV and Tuberculosis (TB).

Keywords: Basic reproduction number, stability analysis, local sensitivity analysis, HIV, TB.

Title and Abstract (in Arabic)

النمذجة الرياضية للفحص على الأمراض المعدية الوافدة على دولة الامارات العربية المتحدة

الملخص

تعتبر دولة الإمارات العربية المتحدة واحدة من الدول ذات أكبر عدد من الوافدين في العالم ولذلك قد تواجه مجموعة من التحديات في مجال الصحة العامة ويعود ذلك إلى أن معظم الوافدين قادمون من مناطق ذات أمراض وبائية . ويؤدي هذا إلى احتمال انتقال هذه الأوبئة إلى الدولة مما يتطلب أن تكون الدولة في حالة استعداد تام بوضع مجموعة من السياسات لمواجهة هذه الأخطار. ويجب مراجعة هذه السياسات وتعديلها لحماية المجتمع من أي أمراض معدية . والهدف من هذا البحث هو تقديم مجموعة من المقترحات للتعامل مع أخطار هذه الأوبئة، وذلك من خلال عرض مجموعة من المعادلات التفاضلية وما قمنا به من دراسة انتشار الأمراض المعدية في الامارات مع الأخذ بعين الإعتبار طبيعة الوافدين في دولة الإمارات العربية المتحدة. وقد صنفنا الوافين إلى فئتين : فئة قادمة من مناطق فيها أوبئة عالية وفئة من مناطق ذات أوبئة منخفضة. مع التوصية بضرورة عزل أو إبعاد الحالات الإيجابية من هذه الاوبئة من خلال الفحوصات الطبية. ومن خلال الدراسة تم توقع سبع سيناريوهات محتملة كل منها يمثل مستوى من مستويات هذه الأوبئة. لتوضيح ذلك ، قمنا بعمل محاكاة لنوعين من الأمراض التي يتم فحصها في الإمارات العربية المتحدة : مرض نقص المناعة البشرية (الإيدز) ومرض السل .

كلمات البحث: عدد الاستنساخ الأساسي، تحليل الاستقرار، تحليل الحساسية المحلية، فيروس نقص المناعة البشرية (الإيدز) والسل.

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My deep and sincere thanks and appreciation go to my parents, my brothers, sisters, my children and my big family for their constant prayers, love and support.

Last but not least, I would like to thank my wife, Aisha. She is always beside me through the bad and good times.

Dedication

To my beloved parents and family

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Chapter 1: Introduction

One of the effective measures that the health authorities are implementing to reduce the spread of diseases, particularly communicable ones, is the screening of the new arrival. The screening takes place under two conditions: first, by the arrival of the person for the first time. Second, every time the residence needs renewal. Despite the efficacy of these procedures, the UAE population is vulnerable to possible disease spread from the inflow of expatriates coming to the country. This problem arises from the fact that some diseases have relatively long incubation period. This is defined in [6] as the time that an infection happens by a microorganism to the time that the symptoms of disease appear allowing the imported infected disease person, to stay under the radar for a period of time and be infectious to the general population. The best example of this scenario is the case of imported Malaria to the UAE in 2011.

Evidently some studies have attempted to analyse the effects of the incubation period of the diseases dynamics [21]. However, this procedure of screening the new arrival may be considered as insecure. The aim of this study is to investigate possible mechanisms of enhancing the efficiency of the screening measure by studying a mathematical model of a general imported communicable disease.

There are several mathematical modelings that study the efficacy disease screening. For example [26] examined the relationship between the spread of HIV and the screening of infective expatriates in populations of varying sizes. By dividing the population into susceptible, unaware infective, aware infective, and AIDS patients, the model investigated the impact of screening and whether or not the dis-

ease became endemic. One of these studies argued that the screening greatly has reduced the spread of HIV by keeping the expatriates under control. That is to say, the screening of unaware infective both within the population and expatriates must be intensified. Another study in [23] showed the importance of screening, treatment, and education on the transmission of HIV infection within a population. In fact, the analysis of this mathematical model showed that screening and treatment have reduced the transmission of the disease. A similar study [16] showed the benefits of screening and testing in the battle against HIV using surveillance data. It is also important to mention that reducing the time between screening has a higher impact on reducing the disease spread compared to increasing the proportion of the population screened, which was the cases of the mathematical model of Syphilis [31]. This approach was confirmed also in the study by [34] of mathematical model of Chlamydia transmission between specific populations. In fact, the model showed that the Chlamydia screening is more effective in reducing HIV incidences with more frequent screening and with higher participation of the most risky population in the screening program. Another mathematical modeling of the expatriate Tuberculosis (TB) screening in Canada [18] showed the need of extending screening the permanent residents to control the burden of the TB on the population.

All these studies have showed the efficacy of screening as a measure to control the spread of the communicable diseases. Particularly if the population is composed of expatriates from different countries which is the case in the UAE, the legitimate question to ask is how to improve the health control policy measure, such as screening, to protect the public health from any pandemic that can be caused by expatriates in the UAE. To answer this question, a mathematical model is proposed in attempt to take different approaches of screening expatriates and the citizens. The model is not specific to any type of disease, but it will consider the case of some communicable diseases that can be of concern to the UAE.

1.1 The Structure of the Study

This study falls into seven chapters. Chapter one is the introduction whereas Chapter two presents a lengthy description of the screening policies in the UAE. A statistics of disease screening of some expatriates populations for specific communicable diseases are given. Also in this chapter the data that shows the variation of the prevalence of diseases among expatriates after and before screening are presented. The focus will be mainly on HIV and TB. This may help contextualizing this study within the communicable diseases screening issue in the UAE.

Chapter 3 consists of the literature review of all existing mathematical models diseases screening. This review looks at the mathematical models of diseases screen-ing of sexually transmitted diseases including HIV and syphilis. The mathematical models of airborne diseases such TB and SARS are reviewed.

Chapter 4 presents a core model of screening of imported diseases in the UAE. The model takes into consideration the nature of the population in the UAE, both the expatriates and the local, in relation to the health policy. The basic reproduction number $\mathscr{R}_0$ and the basic mathematical analysis of the model are introduced as well as the different scenarios of the model depending on the value $\mathscr{R}_0$.

As an illustration of this work, the cases of two possible important diseases HIV and TB are considered.
In chapter 5 the parameters of the model in the case of HIV are estimated. Using these parameters, the impact of the scenarios of the model in the case of HIV and fatality of the disease on the UAE population is examined. Using sensitivity analysis, the parameter that has a high impact on the size of the epidemic is presented.

The TB case will be presented in Chapter 6. Similar to the previous chapter, the parameters of the model in this case are estimated. Time series simulation will show the outcome of the disease in population of the UAE. The sensitivity analysis will examine in this case the parameters that make the model sensitive and the time series simulation.

Finally, the conclusion will be presented in Chapter 7, where the findings of the thesis are summarized, with recommendations that may help in improving the quality of the screening in the UAE. In addition, some perspective of possible extensions of this work will be proposed.

Chapter 2: Screening Policies in the UAE

The UAE federation is formed of seven emirates in the second of December 1971 and has an area of almost 83,600 square kilometers. The estimated population is 9.44 million in 2014 with an increasing level of population that is expected to grow up and adding about 7.9 million by 2020. As a country depends on oil and gas (the fifth largest in the Organization of Petroleum Exporting Country - OPEC), the UAE has a developing economy based on the diversify which opens the country's door wiedly open to expatriates. Still the UAE is in need for more workforces. This leads the UAE to be a multicultural country as people keep invading it from the earth four corners. Expectedly this poses a challenge for public health strategies (See figure 2.2 for the total population of the UAE).
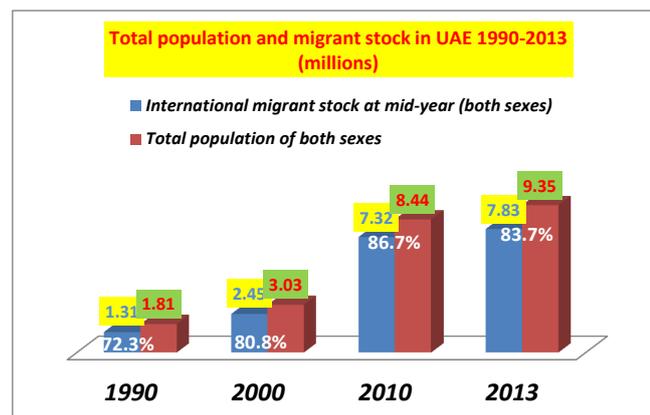
Figure 2.1: Total population and migrant stock in UAE 1990-2013 (millions)
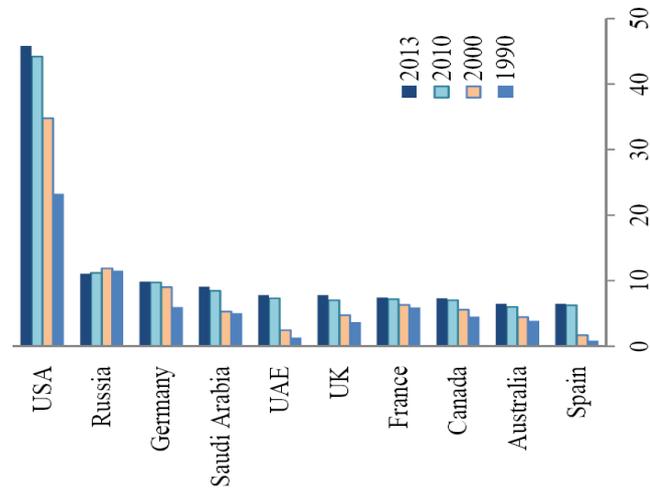
Figure 2.2: The ten countries with the largest number of international migrants, 1990-2013 (millions)

2.1 Communicable Diseases in the UAE

Statistics shows that more than 200 foreign nationalities are living in the UAE with 7.83 million expatriates from the total population of 9.35 million in 2013, and 90% of them work in the private sector. These numbers have created a great challenge for the country, particularly as a large group of these arrivals came from areas where infectious diseases such as tuberculosis, HIV, TB are widely spread. In 2008, 1518 expatriates were deported, namely those who are infected with HIV, hepatitis types B and C, and tuberculosis via visa screening program.
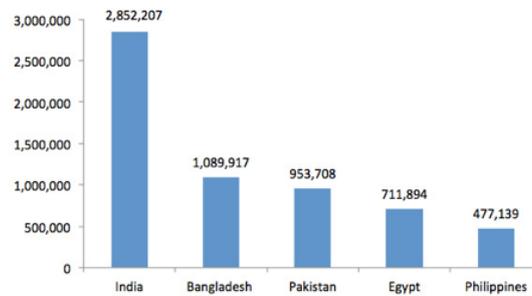
Figure 2.3: Top Five Origin Countries of the Foreign-Born Population in the United Arab Emirates 2013

The UAE is one of the developed countries, which always strives to be the best at all levels. Since it relies on expatriate labor in the development of its economy, it faces a major challenge in fighting the arrival infectious diseases. The health policy in the UAE aims to educate people about the seriousness of the disease and prevention, provision of health facilities and the development of service level throughout the UAE.

2.2 Policies and Cooperation Strategy

The health sector is administered by the ministry of health and the emirates health authorities, (Health Authority Abu Dhabi (HAAD), Dubai Health Authority (DHA)). Health care is provided for all nationals and health care insurance is obligatory for expatriates. A number of laws, strategies and health policies exist at health sector to improve heath situation at all levels. In the health sector, The UAE collaborates with UNDP, UNICEF, UNAIDS and a number of universities and famous hospitals and clinics from around the world [1]. According to the strategy agenda (2012-2017) with WHO, the government enhances to prevent and control communicable diseases, and looks forward to developing a national center for disease control to provide technical support for disease control programmes within primary health care

especially in relation to epidemic and pandemics [33]. Gulf Cooperation Council (GCC) countries, including the United Arab Emirates, is in need of the labor force in order to assist in the continued development and construction activity in the region. In 1995, the Executive Board in GCC issued a decision to build some medical centers to take appropriate measures to check the expatriate workers in their home country before they get a visa to the Gulf States [12]. According to the UAE federal law No 27 in 1981 all visa applicants for residence or work for persons older than 18 years should be screened against specific communicable diseases. This is done at the time of arrival to the country and at the time of visa renewal. As stated in Chapter I Article 2, a Communicable Disease, Infectious Disease or Contagious Disease means any disease that can be transmitted to others through humans, animals, insects, food, places or other objects and substances that may be contaminated by the microbes or toxins of the communicable disease. In Chapter II, Article 3, Enumeration and Reporting of Communicable Diseases has defined and listed the diseases in sections A, B and C which are considered communicable diseases [3]. All Competent Health Authorities in the medical zones shall immediately notify the Health Department upon detecting any of the diseases listed in section A.

Section A:

1. Plague

2. Smallpox

3. Cholera

4. Typhus

5. Intermittent fever

6. Yellow fever

Section B:

1. Measles

2. Diphtheria

3. Chicken pox

4. Typhoid fever

5. Paratyphoid fever

6. Poliomyelitis

7. Scarlet fever

8. Epidemic Hepatitis

9. Hepatitis

10. Pertussis

11. Epidemic Parotitis

12. Tetanus

13. Rabies

14. Influenza

15. Acute Encephalitis

16. Cerebrospinal meningitis

17. Eye diseases in newborns

18. Puerperal fever

19. Food poisoning

20. Syphilis

21. Amoebic dysentery

22. Bacillary dysentery

23. Other venereal diseases

Section C:

1. Tuberculosis

2. Malaria

3. Leprosy

4. Anthrax

5. Trachoma

6. Scabies

7. Intestinal and urine parasites

On October 1st 2011 a home screening program was launched in the UAE to make a home-country pre-visa health screening; this program started with two countries Indonesia and Sri Lanka to minimize the number of cases entering with infectious diseases [8]. The below figures compare the output of visa screening before and after the implementation of home screening among applicants from Indonesia and Sri Lanka.
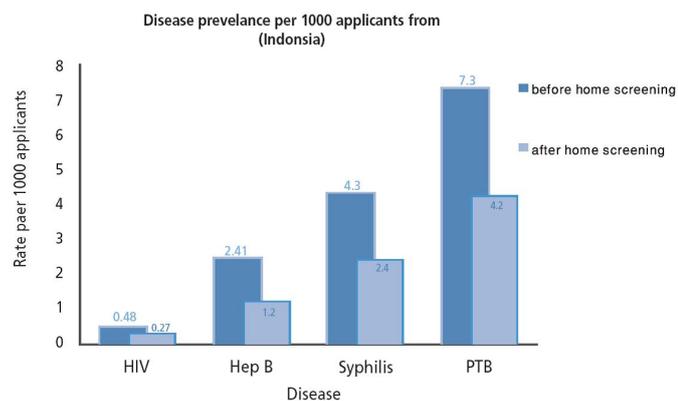
Figure 2.4: Disease prevalence per 1000 applicants from (Indonesia)
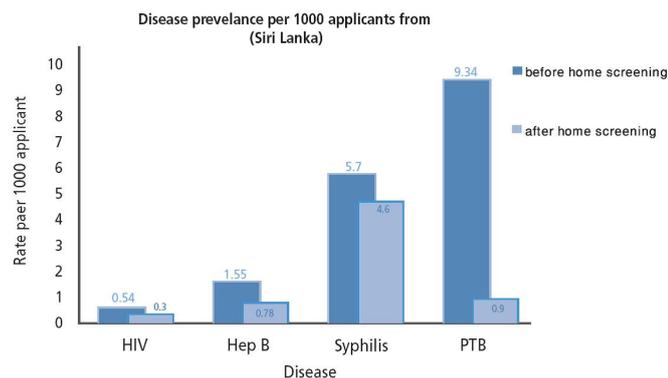
Figure 2.5: Disease prevalence per 1000 applicants from (Sri Lanka)

The first assessment of the program showed that the program was effective in reducing the number of cases that screen positive for the tested infectious diseases among new visa applicants and these results support the continuation of this program.

2.3 Visa Screening in Abu Dhabi Emirate

The regulatory body of the health care sector in the Emirate of Abu Dhabi is the HAAD which was established by Law No. 1 of (2007) [20]. Imported diseases from expatriates are a major health problem. More than 80 % of the UAE population are expatriates, and about half million of new expatriates entered Abu Dhabi emirate only in 2011. Therefore, visa screening is mandatory for all expatriates applying for residence or work in the UAE. In the Emirate of Abu Dhabi, screening consists of Human Immunodeficiency Virus (HIV), leprosy and pulmonary tuberculosis as well as Hepatitis B and syphilis for limited occupational categories. Infectious Diseases Prevention and Control is one of the top 10 public health priorities in the Emirate of Abu Dhabi strategic plans. Through the Residence-Visa Medical Check-up any expatriate who is over the age of eighteen years can obtain a medical check-up certificate which is necessary to apply for or renew a residence visa application.

Type of test	Screening Categories
Physical examination	All screening Categories
Screening to detect pulmonary tuberculosis by chest x-ray	All screening Categories
Blood test for HIV	All screening Categories
Blood test for hepatitis B	Workers in nurseries Domestic workers including Housemaids Private drivers Food handlers workers in restaurants Workers Saloons, barbers health clubs
Blood test for syphilis	Workers in nurseries Domestic workers including Housemaids Private drivers Food handlers workers in restaurants Workers Saloons barbers health clubs Pregnancy test for females Domestic workers including Housemaids

Table 2.1: Types of Tests Required for each Category [3]

2.4 Data Collection and Analysis

The following tables below show the number of positive results and overall prevalence per 100000 screened applicants among new visa applicants and those who applied for visa renewal quarterly in the Emirate of Abu Dhabi [8].

2.4.1 HIV/AIDS

The first case of AIDS was reported in the US in 1981. There were about 35.3 million people infected with HIV in the world at the end of 2012, and since the beginning of the spread of this epidemic 75 million were infected and 36 million people have died of HIV. This epidemic continues to spread causing the most important health issue of the 21st century; it is one of the most significant causes of illness and death in human history. In the UAE despite the very low spread of HIV, expatriate has an obvious impact on the HIV prevalence. Until the end of 2012, a total of 780 HIV cases was reported among nationals in both sexes [22], including 55 cases among citizen in that year (See figure2.6).

	2012			
	Male	**Female**	**Total**	**% of Total**
Abu Dhabi	6	3	9	16.4
Dubai	20	2	22	40
Sharjah	17	3	20	36.4
Ajman	0	0	0	0
Umm Al-Quwain	0	0	0	0
Ras Al-Khajmah	1	0	1	1.8
Fujairah	2	1	3	5.5
TOTAL	**46**	**9**	**55**	100
% of Total (male-female)	83.6	16.4		

Figure 2.6: New reported HIV cases among UAE nationals, by Emirate, 2012

Also the Communicable Diseases Section of the HAAD provided us with the following figure about HIV screening in Emirates of Abu Dhabi (See figure 2.7).
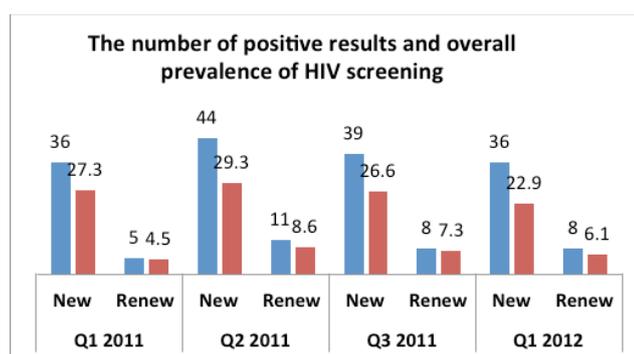
Figure 2.7: The number of positive results and overall prevalence of HIV screening in the Emirate of Abu Dhabi 2011/2012

2.4.2   Tuberculosis (TB)

Tuberculosis (TB) is second only to HIV/AIDS as the greatest killer worldwide. Movements of people have increased in recent decades between the countries and communities in order to either trade or get better job opportunities, which contributed to the spread of tuberculosis at the global level. This epidemic causes the death of about two million people a year, globally. In Emirate of Abu Dhabi the table shown below reflects how this disease is controlled by the specialized authorities. It is noticed that this disease recorded the highest level among the other

diseases.
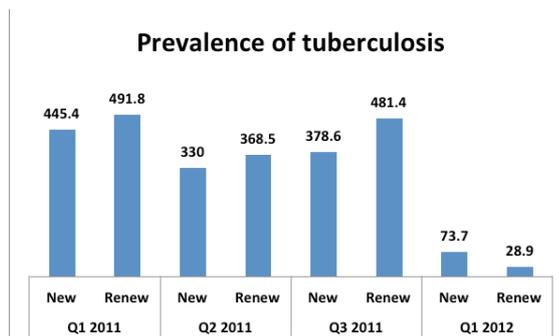
Figure 2.8: Quarterly Tuberculosis prevalence (per 100000 people) in Emirate of Abu Dhabi 2011/2012

The bar graph in 2.8 shows the first, second, and third quarter of the year 2011, the number of the individuals who have tuberculosis during the visa renewal is always more than the new cases. These results are caused from either direct contact with other unaware expatriate or by travelling back to home land during vacations. However, in the first quarter of 2012 a fewer cases of TB were noticed in the visa renewal. This can be explained by the measures that the health authorities took to reduce the prevalence of TB either by limiting acceptance of visas from these epidemic regions or by screening prior coming to the UAE.

2.5 The Need for Efficient Screening Policies

As shown in this chapter, although the UAE has screening policies to protect the general population from the imported communicable diseases, the country is susceptible to have diseases that are in the list of must screen. There is always a possibility of having these diseases. The possibility increases when incubation period of the disease is long, which delays the appearance of the symptoms at the time of screening.

Chapter 3: Literature Review and Theoretical

Some of the literature related to the present study will be reviewed, by investigating the existing papers on mathematical modeling screening of communicable diseases. It is noticed that, the majority of these papers has either focused on HIV, TB or respiratory diseases (SARS, flu).

3.1 Mathematical Models of HIV Screening

This section introduces the existing studies on mathematical modelling of HIV. In [23], the researchers have studied the effect of screening and treatment in the spread of HIV/AIDS infection. A nonlinear mathematical model for the problem is proposed and analysed qualitatively using the stability theory of the differential equations. The model in this paper is as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= \lambda N - \frac{c_1\beta_1 I_1 S}{N} - \frac{c_2\beta_2 I_2 S}{N} - \frac{c_3\beta_3 T S}{N} - \mu S \\[2mm]
\frac{dI_1}{dt} &= \frac{c_1\beta_1 I_1 S}{N} + \frac{c_2\beta_2 I_2 S}{N} + \frac{c_3\beta_3 T S}{N} - (\theta - \delta_1 + \mu)I_1 \\[2mm]
\frac{dI_2}{dt} &= \theta I_1 - (\gamma_1 + \delta_2 + \mu)I_2 \\[2mm]
\frac{dT}{dt} &= \gamma_1 I_2 + \gamma_2 A + (\sigma + \mu)T \\[2mm]
\frac{dA}{dt} &= \delta_1 I_1 + \delta_2 I_2 + \sigma T - (\gamma_2 + \alpha + \mu)A
\end{aligned}
\tag{3.1}
$$

With S stands for susceptibles, I_1 are unaware infectives, I_2 are screened infectives, T treated class and A are the AIDS patients.

The parameters used her are: β_1 is the contact rate of unaware HIV infective with susceptible, γ_1 is the rate at which screened HIV infective are treated with ARV therapy (anti retrovirus), γ_2 is the rate at which full blown AIDS can be treated with ARV infective are treated with ARV therapy, λ recruitment rate, θ the rate at which unaware HIV infectives θ are screened, δ_1 the rate at which unaware infectives develop full blown AIDS, δ_2 the rate at which screened HIV infective develop full blown AIDS, β_2 contact rate of screened HIV infective with susceptible, β_3 contact rate of treated infective with susceptible, σ the rate at which treated infectives develop full down AIDS, μ mortality rate, λ morbidity rate due to AIDS,the term $\frac{(c_1\beta_1 I_1 + c_2\beta_2 I_2 + c_3\beta_3 T)}{N}$ represents the force of infection. By performing the sensitivity analysis the researchers found out that the local stability analysis, showed the persistence of the virus in the population when $R_e > 1$ (R_e is reproduction number). This study showed the effect of screening as measurement to reduce the impact of HIV among the population. Hence, educating the population about the importance of testing for HIV will reduce incidence of unsafe sex and reduce number of AIDS cases.The analysis of this study noted that, the absence of screening and treatment lead to increase endemic diseases and thus increase the number of people living with HIV/AIDS. Therefore the best way to minimize the transmission of HIV/AIDS is by getting people value the need for a medical examination and lightening them about the dangerous consequences of unsafe sex. In addition to that, appropriate treatment facilities should be provided in order to control and reduce the spread of HIV/AIDS.

The study [34], has focused on showing how chlamydia could increase the affectivity of HIV and the susceptibility to HIV infection.
The model presented in paper is as follows:

$$\frac{dX_{00i}}{dt} = -\lambda_{Hi}X_{00i} + \lambda_{Si}X_{00i} + \gamma X_{01i} + (\delta + \sigma_{0i})X_{02i} + \mu\xi_i N_0 - \mu X_{00i}$$

$$\frac{dX_{01i}}{dt} = p\lambda_{Si}X_{00i} - (\gamma + \varphi_1\lambda_{Hi} + \mu)X_{01i}$$

$$\frac{dX_{02i}}{dt} = (1-p)\lambda_{Si}X_{00i} - (\delta + \sigma_{0i} + \varphi_2\lambda_{Hi} + \mu)X_{02i}$$

$$\frac{dX_{10i}}{dt} = p\lambda_{Hi}X_{00i} - (\theta + \lambda_{Si} + \mu + \mu_1)X_{10i} + \gamma X_{11i} + (\delta + \sigma_{1i})X_{12i}$$

$$\frac{dX_{20i}}{dt} = \theta X_{10i} - (\lambda_{Si} + \mu + \mu_2)X_{20i} + \gamma X_{21i} + (\delta + \sigma_{2i})X_{22i}$$

$$\frac{dX_{11i}}{dt} = p\lambda_{Si}X_{10i} - (\gamma + \theta + \mu + \mu_1)X_{11i} + \phi_1\lambda_{Hi}X_{01i}$$

$$\frac{dX_{12i}}{dt} = (1-p)\lambda_{Si}X_{10i} - (\delta + \sigma_{1i} + \theta + \mu + \mu_1)X_{12i} + \phi_2\lambda_{Hi}X_{02i}$$

$$\frac{dX_{21i}}{dt} = p\lambda_{Si}X_{20i} - (\gamma + \mu + \mu_2)X_{21i} + \theta X_{11i}$$

$$\frac{dX_{22i}}{dt} = (1-p)\lambda_{Si}X_{20i} - (\delta + \sigma_{2i} + \sigma'_{2i} + \mu + \mu_2)X_{22i} + \theta X_{12i}$$

(3.2)

Where λ_{Hi} and λ_{Si} are the rates of HIV transmission and of chlamydia transmission respectively. the size of the MSM population is N_0 and ξ_i is the fraction of the MSM population in sexual risk group (I=1,2,3,4).

The study by (Boiley et al) in [4], shows another type of preventive approaches HIV by the use of condoms, screening of gonorrhea. By using a mathematical model of gonorrhoea this study relayed on data from Cotonou (Benin). The population of heterosexual participants are divided into low sexual activity class

(low risk) and high sexual activity class (high risk). The results of this study in Cotonou confirm that, all the conditions appear to have favoured a successful intervention and stages of the epidemic affecting the transmission and prevention, as well as in the evaluation of interventions. The paper showed that, stage of epidemic could play a crucial role in efficacy of screening and in preventing more infected people.

Another study [25] observes the spread of HIV/AIDS in a specific community in relation to the flow of migrants and screening procedure. The population size is divided into four subclasses: susceptible, unaware infective, aware infective and AIDS patient with natural morality rate in all of them. Therefore the total population is given by $N(t) = S(t) + I_1(t) + I_2(t) + A(t)$ at any time t. The model obtained by the following system of nonlinear ODEs:

$$\frac{dS}{dt} = Q_0 - S(\frac{\beta_1 I_1 + \beta_2 I_2}{N}) - \mu S$$

$$\frac{dI_1}{dt} = P_1 I_1 S(\frac{\beta_1 I_1 + \beta_2 I_2}{N}) - (\theta + \delta + \mu)I_1$$

$$\tag{3.3}$$

$$\frac{dI_2}{dt} = \theta I_1 + (\delta + \mu)I_2$$

$$\frac{dA}{dt} = \delta I_2 + (\alpha + \mu)A$$

Where The recruitment rate into the susceptible class is represented by Q_0, the transmission rate of diseases by unaware and aware infective are given as β_1 and β_2, respectively. While θ is the detection rate for the unaware infective, δ is the rate of movement from infectious class to AIDS class and α is the AIDS related . The results show that screening greatly reduces the spread of HIV/AIDS; to keep this under control the screening of unaware infective both within the population and immigrants should be increased.

As continuation of the previous work, the author presented in [30] a study of the effect of screening of unaware infective on the spread of HIV/AIDS in a regular population with constant immigration of susceptible. A mathematical model proposed for this problem and the population size $N(t)$ at time t divided into four subgroups of susceptible $S(t)$, unaware infected $I_1(t)$, aware infected $I_2(t)$ and that of AIDS population $A(t)$. The model is represented by the same approach in previous model.

where $\beta_i (i = 1, 2)$ are the per capita contact rates for susceptible with unaware infective and with aware infective, θ the rate at which unaware infective become infected by screening, Q_0 is the rate of constant immigration of susceptible, d the natural morality rate, α is the AIDS related death rate, δ is the rate at which both types of infective develop AIDS.

It has been shown, in this paper, that if the flow of immigrants as well as the rate of interaction then there is an increase of the number of infected, and the disease will remain latent.

The paper [2] proposed a nonlinear mathematical delay model to study the effect of time delay in the employment of infected persons with HIV/AIDS. The model represented by change the two first equation of the paper [25] by

$$\frac{dS}{dt} = Q_0 - \frac{\beta_1 S(t-\tau)I_1(t-\tau)S}{N(t-\tau)}e^{-\mu\tau} - \frac{\beta_2 S(t-\tau)I_2(t-\tau)S}{N(t-\tau)}e^{-\mu\tau} - \mu S$$

$$\frac{dI_1(t)}{dt} = \frac{\beta_1 S(t-\tau)I_1(t-\tau)S}{N(t-\tau)}e^{-\mu\tau} + \frac{\beta_2 S(t-\tau)I_2(t-\tau)S}{N(t-\tau)}e^{-\mu\tau} - (\theta + \delta + \mu)I_1(t)$$

$$(3.4)$$

The parameters are, $S(t)$ is susceptible , $I_1(t)$ infectives, $I_2(t)$ HIV positives that know there are infected and $A(t)$ that of AIDS population . θ is the rate at which

unaware infective become infected by screening. $\tau > 0$ is the latent time delay that make a susceptible become infected by contact with unaware and/or aware infective, μ the naturale morality rate, δ the rate at which both types of infectives develop their own AIDS.

This model divided the population and analyzed by the same approach as in [30] . As a result of this study, when the delay τ passes over the critical value then stability switches and Hopf bifurcation occurs.

3.2 Mathematical Models of TB Screening

This paper by (Samuel) [5] studied an endemic model of TB in Cameroon. The main feature of the model is the use of heterogeneities to study progress of the disease. The screening measurement and contact tracing were tested in the model as control measurement of the disease spread. The model constructed as follow:

$$\dot{S} = \Lambda - \beta SI - \mu S$$

$$\dot{E} = \beta SI + \gamma I - (\mu + \alpha)E \tag{3.5}$$

$$\dot{I} = \alpha E - (\mu + d + \alpha)I$$

Where susceptible (S), latently infected (E) and infectious (I). The parameters used in the model are : Λ is the recruitment into the population, β is the probability that a latently infected individual becomes infectious, γ is the probability of infectious will recovered, μ is the probability of natural death and d is the probability of morality caused by the disease.

As a result of this latter after performing numerical simulations it is found that, the parameters in the model can be identified only the number of infectious is measured.

A differential parametrisation obtained to the output to determine the remaining states and the unknown parameters. Finally, the simulations results also confirm the theoretical identifiability analysis.

Due to the fact that most foreign-born TB cases in Canada could be reactivated as Latent tuberculous infection (LTBI). The work of [32] studied, via mathematical model, the frequency of active TB in immigrants to Canada. This is presented in the following model.

$$\frac{dE_{1i}}{dt} = (1-a)b_{1i}(t)E_{1i}(t) - \mu_1 E_{1i}(t) - \xi_E E_{1i}(t) - \varepsilon E_{1i}(t) - \gamma_{1i}(t)E_{1i}(t)$$

$$\frac{dM_{1i}}{dt} = \varepsilon E_{1i}(t) - \mu_1 M_{1i}(t) - \xi_M M_{1i}(t) - \theta_{1i}(t)M_{1i}(t) - \nu(t)M_{1i}(t)$$

$$\frac{dL_{1i}}{dt} = -\nu(t)M_{1i}(t) - \mu_1 L_{1i}(t) - \xi_L L_{1i}(t) - \psi_{1i}(t)L_{1i}(t)$$

$$\frac{dT_{1i}}{dt} = \gamma_{1i}E_{1i}(t) - \phi_{1i}1M_{1i}(t) - \psi 1iL_{1i}(t) - \alpha T_{1i}(t) - \mu_1 T_{1i}$$

$$\frac{dE_{2i}}{dt} = (1-a)b_{2i}(t)E_{2i}(t) - \mu_2 E_{2i}(t) - \xi_E E_{1i}(t) - \varepsilon E_{2i}(t) - \gamma_{2i}(t)E_{2i}(t)$$

$$\frac{dM_{2i}}{dt} = \varepsilon E_{2i}(t) - \mu_2 M_{2i}(t) - \xi_M M_{1i}(t) - \theta_{2i}(t)M_{2i}(t) - \nu(t)M_{2i}(t)$$

$$\frac{dL_{2i}}{dt} = -\nu(t)M_{2i}(t) - \mu_2 L_{2i}(t) - \xi_L L_{1i}(t) - \psi_{2i}(t)L_{2i}(t)$$

$$\frac{dT_{2i}}{dt} = \gamma_{2i}E_{2i}(t) - \phi_{2i}1M_{2i}(t) - \psi 2iL_{2i}(t) - \alpha T_{2i}(t) - \mu_2 T_{2i}$$

(3.6)

The system above which represents foreign-born individuals who did not undergo screening, (*i*) is a model stratified by country of birth group: low (LIC), medium (MIC) and high incidence counties (HIC). This model showed that the most effective way to reduce the burden of TB in Canada is by screening and treating LTBI expatriates in their original country.

An another study focused on assessing the impact of TB control strategies and the prediction of the spread of tuberculosis in Canada was presented in [18], the following mathematical model was proposed:

$$
\begin{aligned}
\frac{dS_M}{dt} &= \pi - \beta_1 S_M I_M - \beta' S_M I_L - \mu S_M \\[2mm]
\frac{dE_M}{dt} &= \beta_1 S_M I_M + \beta' S_M I_L - (k_1 + \mu + \beta' I_M) E_M \\[2mm]
\frac{dI_M}{dt} &= K_1 E_M - (r_1 + \mu + \mu_{IM}) I_M \\[2mm]
\frac{dR_M}{dt} &= r_1 I_M - \mu R_M
\end{aligned}
\tag{3.7}
$$

with

$$
N_M = S_M + E_M + I_M + R_M
$$

and

$$\frac{dS_L}{dt} = \Lambda - \beta_2 S_L I_L - \beta^* I_M S_L - \mu S_L$$

$$\frac{dE_L}{dt} = \beta_2 S_L I_L + \beta^* I_M S_L + q\beta^* I_M R_L - (k_2 + \mu + p\beta^* I_M)E_L$$

(3.8)

$$\frac{dI_L}{dt} = p\beta^* I_M E_L + K_2 E_L - (r_2 + \mu + \mu_{IL})I_L$$

$$\frac{dR_L}{dt} = r_2 I_L - q\beta^* I_M R_L - \mu R_L$$

with

$$N_L = S_L + E_L + I_L + R_L$$

The population is divided into permanent residents N_L and migrants N_M and that give eight subgroups S_M, E_M, I_M, R_M, and S_L, E_L, I_L, R_L. The parameters used are S_i susceptible, lancet L_i, infectious I_i, and recovery R_i (i= L, M). The numerical simulation to analyse the sensitivity of R_0 showed the disease can be eradicated from the population only if $R_{M0} < 1$, $R_{L0} < 1$; otherwise, the disease will persist as an endemic. This simulation showed that the prevalence of TB in the population is sensitive to TB control strategies β^* and β', which is in agreement with analytical results. The finding of this work suggested to improve the screening as control measure for the resident in Canada due to their great impact on the spread of TB.

Chapter 4: Mathematical Model of Imported Communicable Diseases in UAE

4.1 Model Formulation

The model investigated is taking in consideration the population of expatriates with high risk of importing an infectious diseases. As shown in figure 4.1, the model is divided into the three categorises of sub-populations. The high risk importing diseases expatriates population M_1, the general expatriates population M_2 and the citizen population M_3. Each sub-population M_1 and M_2 are divided into susceptible S_{M_i}, unaware infected $I_{M_i}^1$ and aware infected $I_{M_i}^2$ with $i = 1, 2$. By considering only two types of the unaware infected should leave the country immediately after screening. The local group is divided to susceptible S_{M_3}, infected I_{M_3}, treated T_{M_3} and recovered R_{M_3}. This categorisation has a goal to study the burden of the infection on the public. In addition, this will allow us to study different cases where the disease is treated but not controlled.

The equations of the risk infectious population are
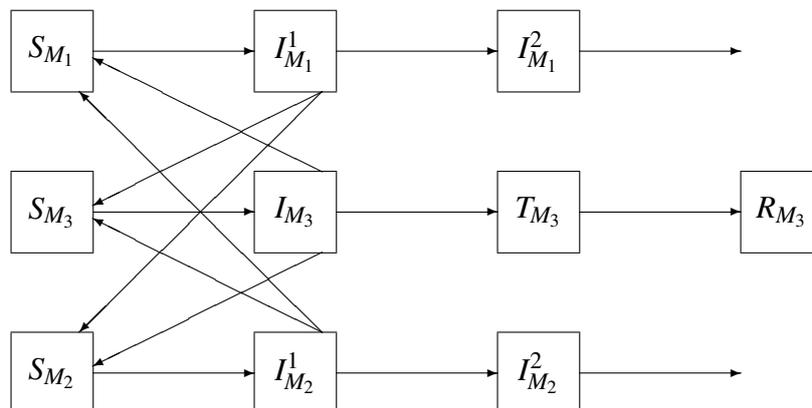
Figure 4.1: The flow chart of the model: the population is divided to three sub-population: M_1 the high risk expatriate population, M_2 the general expatriate population and M_3 the citizen population.

$$\dot{S}_{M_1} = \Lambda_{M_1} - \mu_S S_{M_1} - S_{M_1}\left(\sum_{i=1}^{3}\beta_{i1}I^1_{M_i}\right)$$

$$\dot{I}^1_{M_1} = S_{M_1}\left(\sum_{i=1}^{3}\beta_{i1}I^1_{M_i}\right) - (\mu_I + \alpha)I^1_{M_1}$$

(4.1)

$$\dot{I}^2_{M_1} = \alpha I^1_{M_1} - \mu_I I^2_{M_1}$$

The equations of the general expatriate population are

$$\dot{S}_{M_2} = \Lambda_{M_2} - \mu_S S_{M_2} - S_{M_2}\left(\sum_{i=1}^{3}\beta_{i2}I^1_{M_i}\right)$$

$$\dot{I}^1_{M_2} = S_{M_2}\left(\sum_{i=1}^{3}\beta_{i2}I^1_{M_i}\right) - (\mu_I + \gamma)I^1_{M_2}$$

(4.2)

$$\dot{I}^2_{M_2} = \gamma I^1_{M_2} - \mu_I I^2_{M_2}$$

The equations of the local population are

$$\dot{S}_{M_3} = \Lambda_{M_3} - \mu_S S_{M_3} - S_{M_3}\left(\sum_{i=1}^{3}\beta_{i3}I^1_{M_i}\right) + \delta R_{M_3}$$

$$\dot{I}^1_{M_3} = S_{M_3}\left(\sum_{i=1}^{3}\beta_{i3}I^1_{M_i}\right) - (\mu_I + \theta)I^1_{M_3}$$

(4.3)

$$\dot{T}_{M_3} = \theta I^1_{M_3} - (\xi + \mu_T)T_{M_3}$$

$$\dot{R}_{M_3} = \xi T_{M_3} - (\delta + \mu_R)R_{M_3}$$

β_{ij} is taken as follow

$$\beta_{ij} = \frac{\bar{\beta}_{ij}}{\Sigma}$$

with Σ is the total population of the country and $\bar{\beta}_{ij}$ are the infection rates among the populations M_1, M_2 and M_3. It should be noticed that $\Sigma = M_1 + M_2 + M_3$

The parameters used in this model are defined with their units in table 4.1

Parameters	Description	Unit
M_1	The high risk expatriate population	
M_2	The general expatriate population	
M_3	The local population	
Λ_{M_1}	Constant rate of immigration of M_1	Human $\times$ year^{-1}
Λ_{M_2}	Constant rate of immigration of M_2	Human $\times$ year^{-1}
$\bar{\beta}_{ij}(i,j=1,2,3)$	matrix of transmission rates between susceptible and infectious individuals in(M_1,M_2,M_3)	year^{-1}
Λ_{M_3}	Birth of M_3	year^{-1}
μ_S	The death rate susceptible population	year^{-1}
μ_I	The death rate of the infected population	year^{-1}
μ_T	The death rate of the treated population	year^{-1}
μ_R	The death rate of the recovered population	year^{-1}
α	The rate of unaware infected become aware infected in M_1	year^{-1}
γ	The rate of unaware infected become aware infected in M_2	year^{-1}
θ	The rate of treatment of local infected in M_3	year^{-1}
ξ	The rate of recovered	year^{-1}
δ	The rate of losing immunity	year^{-1}

Table 4.1: Parameters Description

4.2 Basic Properties of the Model

First, the initial conditions for the model is assumed to be positive. Which means that

$$X_{M_i}(0) = X_{M_i}^0 > 0 \quad \text{where } X = S, I^1, I^2, T, R \text{ and } i = 1,2,3 \tag{4.4}$$

4.2.1 Positivity and Boundedness

It should be, in first place, proved that all the variables of the model are non-negative, and they are biologically acceptable. For this, all solutions of (4.1-4.2-4.3) are shown with initial condition in $\mathbb{R}^{10}_+$, are non-negative and bounded. For the non-negativity the standard argument [29] is followed. And we have the following result

Proposition 4.2.1.

Let $\mathbb{R}^{10}_+ = \{(s_1, s_2, ..., s_{10}) \in \mathbb{R}^{10} : s_i \geq 0, \forall i \in \{1, ..., 10\}\}$. *Then* $\mathbb{R}^{10}_+$ *is positively invariant under the flow induced by model (4.1-4.2-4.3).*

To prove the boundedness, from equation (4.1) we can get

$$\dot{S}_{M_1} + \dot{I}^1_{M_1} + \dot{I}^2_{M_1} = \Lambda_{M_1} - \mu_s S_{M_1} - \mu_I I^1_{M_1} - \mu_I I^2_{M_1}$$

let

$$X = S_{M_1} + I^1_{M_1} + I^2_{M_1}$$

and

$$\mu = max(\mu_s, \mu_I)$$

then

$$\dot{S}_{M_1} + \dot{I}^1_{M_1} + \dot{I}^2_{M_1} \leq \Lambda_{M_1} - \mu(S_{M_1} + I^1_{M_1} + I^2_{M_2})$$

$$\dot{X} \leq \Lambda_{M_1} - \mu X$$

$$\dot{X} + \mu X \leq \Lambda_{M_1}$$

we multiply both sides by

$$e^{\mu t}$$

therefore

$$e^{\mu t} \dot{X} + e^{\mu t} \mu X \leq \Lambda_{M_1} e^{\mu t}$$

$$\int_0^t (e^{\mu t} X) dt \le \int_0^t \Lambda_{M_1} e^{\mu t} dt$$

$$e^{\mu t} X(t) - X(0) \le \frac{\Lambda_{M_1}}{\mu} (e^{\mu t} - 1)$$

$$e^{\mu t} X(t) \le X(0) + \frac{\Lambda_{M_1}}{\mu} (e^{\mu t} - 1)$$

$$X(t) \le e^{-\mu t} X(0) + \frac{\Lambda_{M_1}}{\mu} (1 - e^{-\mu t})$$

since

$$\left[(1 - e^{-\mu t}) \le 1 \quad and \quad e^{-\mu t} \le 1 \right]$$

then

$$X(t) \le e^{-\mu t} X(0) + \frac{\Lambda_{M_1}}{\mu}$$

$$\le X(0) + \frac{\Lambda_{M_1}}{\mu}$$

$$\le \frac{\Lambda_{M_1}}{\mu}.$$

Therefore, we have the following

$$S_{M_1} + I_{M_1}^1 + I_{M_1}^2 \le S_{M_1}(0) + I_{M_1}^1(0) + I_{M_1}^2(0) + \frac{\Lambda_{M_1}}{\mu}$$

and

$$\limsup_{t \to \infty} X(t) \le \frac{\Lambda_{M_1}}{\mu}$$

. Following the same steps from equation (4.2) we can get

$$\dot{S}_{M_2} + \dot{I}_{M_2}^1 + \dot{I}_{M_2}^2 = \Lambda_{M_2} - \mu_s S_{M_2} - \mu_I (I_{M_2}^1 + \mu_I I_{M_2}^2)$$

let

$$Y = S_{M_2} + I_{M_2}^1 + I_{M_2}^2$$

and

$$\mu = max(\mu_s, \mu_I)$$

then

$$\dot{S}_{M_2} + \dot{I}^1_{M_2} + \dot{I}^2_{M_2} \leq \Lambda_{M_2} - \mu(S_{M_2} + I^1_{M_2} + I^2_{M_2})$$

$$\dot{Y} \leq \Lambda_{M_2} - \mu Y$$

$$\dot{Y} + \mu Y \leq \Lambda_{M_2}.$$

Therefore

$$e^{\mu t}\dot{Y} + e^{\mu t}\mu Y \leq \Lambda_{M_2} e^{\mu t}$$

$$\int_0^t (e^{\mu t} Y)dt \leq \int_0^t \Lambda_{M_2} e^{\mu t} dt$$

$$e^{\mu t} Y(t) - Y(0) \leq \frac{\Lambda_{M_2}}{\mu}(e^{\mu t} - 1)$$

$$e^{\mu t} Y(t) \leq Y(0) + \frac{\Lambda_{M_2}}{\mu}(e^{\mu t} - 1)$$

$$Y(t) \leq e^{-\mu t} Y(0) + \frac{\Lambda_{M_2}}{\mu}(1 - e^{-\mu t})$$

$$\leq e^{-\mu t} Y(0) + \frac{\Lambda_{M_2}}{\mu}$$

$$\leq Y(0) + \frac{\Lambda_{M_2}}{\mu}$$

$$\leq \frac{\Lambda_{M_2}}{\mu}$$

$$S_{M_2} + I^1_{M_2} + I^2_{M_2} \leq S_{M_2}(0) + I^1_{M_2}(0) + I^2_{M_2}(0) + \frac{\Lambda_{M_2}}{\mu}$$

and

$$\limsup_{t \to \infty} Y(t) \leq \frac{\Lambda_{M_2}}{\mu}$$

also from equation (4.3) we can get

$$\dot{S}_{M_3} + \dot{I}^1_{M_3} + \dot{T}_{M_3} + \dot{R}_{M_3} = \Lambda_{M_3} - \mu_s S_{M_3} - \mu_I I^1_{M_3} - \mu_T T_{M_3} - \mu_R R_{M_3}$$

let

$$Z = S_{M_3} + I_{M_3}^1 + T_{M_3} + R_{M_3}$$

and

$$\mu = max(\mu_s, \mu_I, \mu_T, \mu_R)$$

then

$$\dot{Z} \leq \Lambda_{M_3} - \mu(S_{M_3} + I_{M_3}^1 + T_{M_3} + R_{M_3})$$

$$\dot{Z} \leq \Lambda_{M_3} - \mu Z$$

$$\dot{Z} + \mu Z \leq \Lambda_{M_3} .$$

Therefore

$$e^{\mu t}\dot{Z} + e^{\mu t}\mu Z \leq \Lambda_{M_3} e^{\mu t}$$

$$\int_0^t (e^{\mu t}Z)dt \leq \int_0^t \Lambda_{M_3} e^{\mu t} dt$$

$$e^{\mu t}Z(t) - Z(0) \leq \frac{\Lambda_{M_3}}{\mu}(e^{\mu t} - 1)$$

$$e^{\mu t}Z(t) \leq Z(0) + \frac{\Lambda_{M_2}}{\mu}(e^{\mu t} - 1)$$

$$Z(t) \leq e^{-\mu t}Z(0) + \frac{\Lambda_{M_3}}{\mu}(1 - e^{-\mu t})$$

$$\leq e^{-\mu t}Z(0) + \frac{\Lambda_{M_3}}{\mu}$$

$$\leq Z(0) + \frac{\Lambda_{M_3}}{\mu}$$

$$\leq \frac{\Lambda_{M_3}}{\mu}$$

hence

$$S_{M_3} + I_{M_3}^1 + T_{M_3} + R_{M_3} \leq S_{M_3}(0) + I_{M_3}^1(0) + T_{M_3}(0) + R_{M_3}(0) + \frac{\Lambda_{M_3}}{\mu}$$

that is

$$Z(t) \leq Z(0) + \frac{\Lambda_{M_3}}{\mu}$$

$$\limsup_{t \to \infty} Z(t) \leq \frac{\Lambda_{M_3}}{\mu}$$

.

4.3   Local Stability

The aim of this section is to calculate the reproduction number of the disease spread. We follow the classical approach given in [10].

4.3.1   Reproduction Number

First, from the model system the vector $\mathscr{F}$ that represent rates of secondary infections is defined as follows

$$\mathscr{F} = \begin{pmatrix} S_{M_1}\left(\sum_{i=1}^{3} \beta_{i1} I_{M_i}^1\right) \\[2em] S_{M_2}\left(\sum_{i=1}^{3} \beta_{i2} I_{M_i}^1\right) \\[2em] S_{M_3}\left(\sum_{i=1}^{3} \beta_{i3} I_{M_i}^1\right) \\[1em] 0 \\ 0 \\ 0 \end{pmatrix}$$

and the vector $\mathscr{V}$ that represent rates of disease progression, which is defined by

$$
\mathscr{V} = \begin{pmatrix}
(\mu_I + \alpha)I_{M_1}^1 \\[2em]
(\mu_I + \gamma)I_{M_2}^1 \\[2em]
(\mu_I + \theta)I_{M_3}^1 \\[2em]
-\alpha I_{M_1}^1 + \mu_I I_{M_1}^2 \\[2em]
-\gamma I_{M_2}^1 + \mu_I I_{M_2}^2 \\[2em]
-\theta I_{M_3}^1 + (\xi + \mu_T)T_{M_3}
\end{pmatrix}.
$$

The matrices F and V are defined as follows:

$$
F = \frac{\partial \mathscr{F}}{\partial x_j}(E_0), \quad V = \frac{\partial \mathscr{V}}{\partial x_j}(E_0).
$$

Hence,

$$
F = \begin{pmatrix}
\frac{\Lambda_{M_1}}{\mu_s}\beta_{11} & \frac{\Lambda_{M_1}}{\mu_s}\beta_{21} & \frac{\Lambda_{M_1}}{\mu_s}\beta_{31} & 0 & 0 & 0 \\[1.5em]
\frac{\Lambda_{M_2}}{\mu_s}\beta_{12} & \frac{\Lambda_{M_2}}{\mu_s}\beta_{22} & \frac{\Lambda_{M_2}}{\mu_s}\beta_{32} & 0 & 0 & 0 \\[1.5em]
\frac{\Lambda_{M_3}}{\mu_s}\beta_{13} & \frac{\Lambda_{M_3}}{\mu_s}\beta_{23} & \frac{\Lambda_{M_3}}{\mu_s}\beta_{33} & 0 & 0 & 0 \\[1em]
0 & 0 & 0 & 0 & 0 & 0 \\[1em]
0 & 0 & 0 & 0 & 0 & 0 \\[1em]
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

and

$$V = \begin{pmatrix} \alpha + \mu_I & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma + \mu_I & 0 & 0 & 0 & 0 \\ 0 & 0 & \theta + \mu_I & 0 & 0 & 0 \\ -\alpha & 0 & 0 & \mu_I & 0 & 0 \\ 0 & -\gamma & 0 & 0 & \mu_I & 0 \\ 0 & 0 & -\theta & 0 & 0 & \xi + \mu_I \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\alpha + \mu_I} & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{\gamma + \mu_I} & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{\theta + \mu_I} & 0 & 0 & 0 \\ \dfrac{\alpha}{\mu_I(\alpha + \mu_I)} & 0 & 0 & \dfrac{1}{\mu_I} & 0 & 0 \\ 0 & \dfrac{\gamma}{\mu_I(\gamma + \mu_I)} & 0 & 0 & \dfrac{1}{\mu_I} & 0 \\ 0 & 0 & \dfrac{\theta}{(\xi + \mu_I)(\theta + \mu_I)} & 0 & 0 & \dfrac{1}{\xi + \mu_I} \end{pmatrix}.$$

Therefore

$$F * V^{-1} = \begin{pmatrix} \dfrac{\beta_{11}\Lambda_{M_1}}{\mu_s(\alpha+\mu_I)} & 0 & 0 & 0 & 0 & 0 \\[2em] 0 & \dfrac{\beta_{22}\Lambda_{M_2}}{\mu_s(\gamma+\mu_I)} & 0 & 0 & 0 & 0 \\[2em] 0 & 0 & \dfrac{\beta_{33}\Lambda_{M_3}}{\mu_s(\theta+\mu_I)} & 0 & 0 & 0 \\[2em] 0 & 0 & 0 & 0 & 0 & 0 \\[2em] 0 & 0 & 0 & 0 & 0 & 0 \\[2em] 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is concluded that

$$\mathscr{R}_0 = \rho(F * V^{-1})$$

is given by

$$\mathscr{R}_0 = \max\left\{\frac{\beta_{11}\Lambda_{M_1}}{\mu_s(\alpha+\mu_I)}, \frac{\beta_{22}\Lambda_{M_2}}{\mu_s(\gamma+\mu_I)}, \frac{\beta_{33}\Lambda_{M_3}}{\mu_s(\theta+\mu_I)}\right\}. \tag{4.5}$$

It is noted that

$$\mathscr{R}_0^1 = \frac{\beta_{11}\Lambda_{M_1}}{\mu_s(\alpha+\mu_I)}, \quad \mathscr{R}_0^2 = \frac{\beta_{22}\Lambda_{M_2}}{\mu_s(\gamma+\mu_I)}, \quad \mathscr{R}_0^3 = \frac{\beta_{33}\Lambda_{M_3}}{\mu_s(\theta+\mu_I)}. \tag{4.6}$$

Then it can be conclude that

$$\mathscr{R}_0 = \max(\mathscr{R}_0^1, \mathscr{R}_0^2, \mathscr{R}_0^3). \tag{4.7}$$

This finding showed that spread of the infection depend on the detection of the infection among the risk group and non-risk group as well as the rate of treatment of the locals. This evidence is exactly what is aimed out in this work. The fast the surveillance of the expatriates is increased the more the general public is protected.

Case 1: we assume that

$$S_{M_2} = I_{M_2}^1 = I_{M_2}^2 = 0$$

The vector $\mathscr{F}$ is defined as follows:

$$\mathscr{F} = \begin{pmatrix} S_{M_1}(\beta_{11}I_{M_1}^1 + \beta_{31}I_{M_3}^1) \\ \\ S_{M_3}(\beta_{13}I_{M_1}^1 + \beta_{33}I_{M_3}^1) \\ \\ 0 \\ \\ 0 \end{pmatrix}$$

and also the vector $\mathscr{V}$ is defined as follows:

$$
\mathscr{V} = \begin{pmatrix} (\mu_I + \alpha)I^1_{M_1} \\\\ (\mu_I + \theta)I^1_{M_3} \\\\ -\alpha I^1_{M_1} + \mu_I I^2_{M_1} \\\\ -\theta I^1_{M_3} + (\xi + \mu_T)T_{M_3} \end{pmatrix}.
$$

Now the matrices F and V is defined as follows:

$$
F = \frac{\partial \mathscr{F}}{\partial x_j}(E_0), \quad V = \frac{\partial \mathscr{V}}{\partial x_j}(E_0).
$$

Hence,

$$
F = \begin{pmatrix} \frac{\Lambda_{M_1}}{\mu_s}\beta_{11} & \frac{\Lambda_{M_1}}{\mu_s}\beta_{31} & 0 & 0 \\\\ \frac{\Lambda_{M_3}}{\mu_s}\beta_{13} & \frac{\Lambda_{M_3}}{\mu_s}\beta_{33} & 0 & 0 \\\\ 0 & 0 & 0 & 0 \\\\ 0 & 0 & 0 & 0 \\\\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

and

$$
V = \begin{pmatrix} \alpha + \mu_I & 0 & 0 & 0 \\\\ 0 & \theta + \mu_I & 0 & 0 \\\\ -\alpha & 0 & \mu_I & 0 \\\\ 0 & -\theta & 0 & \xi + \mu_I \end{pmatrix}
$$

$$V^{-1} = \begin{pmatrix} \dfrac{1}{\alpha + \mu_I} & 0 & 0 & 0 \\[2mm] 0 & \dfrac{1}{\theta + \mu_I} & 0 & 0 \\[2mm] \dfrac{\alpha}{\mu_I(\alpha + \mu_I)} & 0 & \dfrac{1}{\mu_I} & 0 \\[2mm] 0 & \dfrac{\theta}{(\mu_I + \theta)(\mu_I + \xi)} & 0 & \dfrac{1}{\mu_I + \xi} \end{pmatrix}$$

$$F * V^{-1} = \begin{pmatrix} \dfrac{\beta_{11}\Lambda_{M_1}}{\mu_s(\alpha + \mu_I)} & \dfrac{\beta_{31}\Lambda_{M_1}}{\mu_s(\mu_I + \theta)} & 0 & 0 \\[4mm] \dfrac{\beta_{13}\Lambda_{M_3}}{\mu_s(\alpha + \mu_I)} & \dfrac{\beta_{33}\Lambda_{M_3}}{\mu_s(\mu_I + \theta)} & 0 & 0 \\[4mm] 0 & 0 & 0 & 0 \\[4mm] 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is concluded that the basic reproduction number of disease in the presence of the high risk expatriate and local population define by $\mathscr{R}_0^h$ is

$$\mathscr{R}_0^h = \rho(F * V^{-1})$$

is given by

$$\mathscr{R}_0^h = \max\{\frac{\beta_{11}\Lambda_{M_1}}{\mu_s(\alpha + \mu_I)}, \frac{\beta_{33}\Lambda_{M_3}}{\mu_s(\mu_I + \theta)}\} \tag{4.8}$$

or similarly

$$\mathscr{R}_0^h = \max(\mathscr{R}_0^1, \mathscr{R}_0^3). \tag{4.9}$$

Case 2: we assume that

$$S_{M_1} = I^1_{M_1} = I^2_{M_1} = 0.$$

The vector $\mathscr{F}$ is defined as follows:

$$\mathscr{F} = \begin{pmatrix} S_{M_2}(\beta_{22}I^1_{M_2} + \beta_{32}I^1_{M_3}) \\ \\ S_{M_3}(\beta_{23}I^1_{M_2} + \beta_{33}I^1_{M_3}) \\ \\ 0 \\ \\ 0 \end{pmatrix}$$

and also the vector $\mathscr{V}$ is defined as follows:

$$\mathscr{V} = \begin{pmatrix} (\mu_I + \gamma)I^1_{M_2} \\ \\ (\mu_I + \theta)I^1_{M_3} \\ \\ -\gamma I^1_{M_2} + \mu_I I^2_{M_2} \\ \\ -\theta I^1_{M_3} + (\xi + \mu_T)T_{M_3} \end{pmatrix}.$$

Now the matrices F and V is defined as follows:

$$F = \frac{\partial \mathscr{F}}{\partial x_j}(E_0), \quad V = \frac{\partial \mathscr{V}}{\partial x_j}(E_0).$$

Hence,

$$F = \begin{pmatrix} \frac{\Lambda_{M_2}}{\mu_s}\beta_{22} & \frac{\Lambda_{M_2}}{\mu_s}\beta_{32} & 0 & 0 \\ \frac{\Lambda_{M_3}}{\mu_s}\beta_{23} & \frac{\Lambda_{M_3}}{\mu_s}\beta_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} \gamma+\mu_I & 0 & 0 & 0 \\ 0 & \theta+\mu_I & 0 & 0 \\ -\alpha & 0 & \mu_I & 0 \\ 0 & -\theta & 0 & \xi+\mu_I \end{pmatrix}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\gamma+\mu_I} & 0 & 0 & 0 \\ 0 & \frac{1}{\theta+\mu_I} & 0 & 0 \\ \frac{\gamma}{\mu_I(\gamma+\mu_I)} & 0 & \frac{1}{\mu_I} & 0 \\ 0 & \frac{\theta}{(\mu_T+\theta)(\mu_I+\xi)} & 0 & \frac{1}{\mu_T+\xi} \end{pmatrix}$$

$$F*V^{-1} = \begin{pmatrix} \frac{\beta_{22}\Lambda_{M_2}}{\mu_s(\gamma+\mu_I)} & \frac{\beta_{32}\Lambda_{M_2}}{\mu_s(\mu_I+\theta)} & 0 & 0 \\ \frac{\beta_{23}\Lambda_{M_3}}{\mu_s(\gamma+\mu_I)} & \frac{\beta_{33}\Lambda_{M_3}}{\mu_s(\mu_I+\theta)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

It can be concluded that, the basic reproduction number of the disease in the presence of the general expatriate population and local population denoted by $\mathscr{R}_0^g$

$$\mathscr{R}_0^g = \rho(F * V^{-1})$$

is given by

$$\mathscr{R}_0^g = \max\{\frac{\beta_{22}\Lambda_{M_2}}{\mu_s(\gamma + \mu_I)}, \frac{\beta_{33}\Lambda_{M_3}}{\mu_s(\mu_I + \theta)}\}. \tag{4.10}$$

Which is similar to

$$\mathscr{R}_0^g = \max(\mathscr{R}_0^2, \mathscr{R}_0^3). \tag{4.11}$$

It is found from the model that in order for the infection to be transmitted in the whole population, It must first build ground within the group. By looking to the $\mathscr{R}_0$ in a maximum of three holds each one of them represent transmission of the disease within one of the three groups. The β is the infection rate by an infected person within the same group. Moreover, α, γ and θ represent the rate of detecting the diseases by screening.

4.3.2 Equilibria points

In this section, all the possible equilibria of the system occur at the state which all the rates of changes equal zero.

The first case is to find the disease free-equilibria. This means

$$\dot{I}_{M_1}^1 = \dot{I}_{M_2}^1 = \dot{I}_{M_3}^1 = 0,$$

which implies

$$I^2_{M_1} = I^2_{M_2} = T_{M_3} = R_{M_3} = 0.$$

Hence, we have

$$S_{M_1} = \frac{\Lambda_{M_1}}{\mu_S}, \quad S_{M_2} = \frac{\Lambda_{M_2}}{\mu_S}, \quad S_{M_3} = \frac{\Lambda_{M_3}}{\mu_S}.$$

Therefore, we have the following disease free equilibria

$$E_0 = (\frac{\Lambda_{M_1}}{\mu_S}, 0, 0, \frac{\Lambda_{M_2}}{\mu_S}, 0, 0, \frac{\Lambda_{M_3}}{\mu_S}, 0, 0, 0)$$

Based on the result of [11] the results are shown in the following

Proposition 4.3.1. *The system describe by the equation (4.1-4.2-4.3) has disease free equilibria E_0. This equilibria is locally asymptotically stable if and only if $\mathcal{R}_0 < 1$ and it is unstable if $\mathcal{R}_0 > 1$*

It is easy to see that if $\mathcal{R}_0^h < 1$ and $\mathcal{R}_0^g < 1$ is equivalent to $\mathcal{R}_0 < 1$. On the other hand if $\mathcal{R}_0 > 1$, then that could lead to seven possible scenarios as follows

1. $\mathcal{R}_0^1 > 1$ and $\mathcal{R}_0^2 < 1$ and $\mathcal{R}_0^3 < 1$

2. $\mathcal{R}_0^2 > 1$ and $\mathcal{R}_0^1 < 1$ and $\mathcal{R}_0^3 < 1$

3. $\mathcal{R}_0^3 > 1$ and $\mathcal{R}_0^1 < 1$ and $\mathcal{R}_0^2 < 1$

4. $\mathcal{R}_0^1 > 1$ and $\mathcal{R}_0^2 > 1$ and $\mathcal{R}_0^3 < 1$

5. $\mathcal{R}_0^1 > 1$ and $\mathcal{R}_0^3 > 1$ and $\mathcal{R}_0^2 < 1$

6. $\mathcal{R}_0^2 > 1$ and $\mathcal{R}_0^3 > 1$ and $\mathcal{R}_0^1 < 1$

7. $\mathcal{R}_0^1 > 1$ and $\mathcal{R}_0^2 > 1$ and $\mathcal{R}_0^3 > 1$

if $I^1_{M_i} \neq 0$ for $i = 1, 2, 3$. Then

$$I^2_{M_1} = \frac{\alpha}{\mu_I} I^1_{M_1}$$

$$I_{M_2}^2 = \frac{\gamma}{\mu_I} I_{M_2}^1$$

$$T_{M_3} = \frac{\theta}{\xi + \mu_T} I_{M_3}^1$$

$$R_{M_3} = \left(\frac{\xi}{\alpha + \mu_R}\right)\left(\frac{\theta}{\xi + \mu_T}\right) I_{M_3}^1$$

from equation (4.1)

$$(\mu_I + \alpha) I_{M_1}^1 = S_{M_1}\left(\sum_{i=1}^{3} \beta_{i1} I_{M_i}^1\right)$$

$$\Lambda_{M_1} - \mu_s S_{M_1} = S_{M_1}\left(\sum_{i=1}^{3} \beta_{i1} I_{M_i}^1\right)$$

from these two equations we get

$$(\mu_I + \alpha) I_{M_1}^1 = \Lambda_{M_1} - \mu_s S_{M_1}.$$

Therefore

$$S_{M_1} = \frac{\Lambda_{M_1}}{\mu_s} - \frac{(\mu_I + \alpha) I_{M_1}^1}{\mu_s}. \tag{4.12}$$

Following the similar steps, it can be founded

$$S_{M_2} = \frac{\Lambda_{M_2}}{\mu_s} - \frac{(\mu_I + \gamma) I_{M_2}^1}{\mu_s}. \tag{4.13}$$

Similarly

$$S_{M_3} = \frac{\Lambda_{M_3}}{\mu_s} - \left(\frac{(\mu_I + \theta)}{\mu_s} - \frac{\delta}{\mu_s}\left(\frac{\xi}{\alpha + \mu_R}\right)\left(\frac{\theta}{\xi + \mu_T}\right)\right) I_{M_3}^1. \tag{4.14}$$

From the equation of $I_{M_1}^1$ in (4.1) we get

$$\left(\Lambda_{M_1} - (\mu_I + \alpha)I^1_{M_1}\right)\left(\beta_{11}I^1_{M_1} + \beta_{21}I^1_{M_2} + \beta_{31}I^1_{M_3}\right) = \mu_s(\mu_I + \alpha)I^1_{M_1}. \quad (4.15)$$

Also from the equation of $I^1_{M_2}$ in (4.2) we get

$$\left(\Lambda_{M_2} - (\mu_I + \gamma)I^1_{M_2}\right)\left(\beta_{12}I^1_{M_1} + \beta_{22}I^1_{M_2} + \beta_{32}I^1_{M_3}\right) = \mu_s(\mu_I + \gamma)I^1_{M_2} \quad (4.16)$$

Also from the equation of $I^1_{M_3}$ in (4.3) we get

$$\left(\Lambda_{M_3} - \left((\mu_I + \theta) - \delta\left(\frac{\xi}{\alpha + \mu_R}\right)\left(\frac{\theta}{\xi + \mu_T}\right)\right)I^1_{M_3}\right)\left(\beta_{13}I^1_{M_1} + \beta_{23}I^1_{M_2} + \beta_{33}I^1_{M_3}\right) = \mu_s(\mu_I + \theta)I^1_{M_3}.$$

$$(4.17)$$

The system of equations (4.15) -(4.16)-(4.17) can be written as a system of polynomial equations in the following form

$$a_{11}(I^1_{M_1})^2 + a_{12}I^1_{M_1} + (a_{13}I^1_{M_2} + a_{14}I^1_{M_3})I^1_{M_1} + a_{15}I^1_{M_2} + a_{16}I^1_{M_3} = 0$$

$$a_{21}(I^1_{M_2})^2 + a_{22}I^1_{M_2} + (a_{23}I^1_{M_1} + a_{24}I^1_{M_3})I^1_{M_2} + a_{25}I^1_{M_1} + a_{26}I^1_{M_3} = 0$$

$$a_{31}(I^1_{M_3})^2 + a_{32}I^1_{M_3} + (a_{33}I^1_{M_1} + a_{34}I^1_{M_2})I^1_{M_3} + a_{35}I^1_{M_1} + a_{36}I^1_{M_2} = 0$$

with

$$a_{11} = -\beta_{11}(\mu_I + \alpha) \qquad\qquad a_{12} = \beta_{11}\Lambda_{M_1} - \mu_s(\mu_I + \alpha)$$

$$a_{13} = -(\mu_I + \alpha)\beta_{21} \qquad\qquad a_{14} = -(\mu_I + \alpha)\beta_{31}$$

$$a_{15} = \Lambda_{M_1}\beta_{21} \qquad\qquad a_{16} = \Lambda_{M_1}\beta_{31}$$

$$a_{21} = -\beta_{22}(\mu_I + \gamma) \qquad\qquad a_{22} = \beta_{22}\Lambda_{M_2} - \mu_s(\mu_I + \gamma)$$

$$a_{23} = -(\mu_I + \gamma)\beta_{12} \qquad\qquad a_{24} = -(\mu_I + \gamma)\beta_{32}$$

$$a_{25} = \Lambda_{M_2}\beta_{12} \qquad\qquad a_{26} = \Lambda_{M_2}\beta_{32}$$

$$a_{31} = -\beta_{33}\left(\left((\mu_I + \theta) - \delta(\tfrac{\xi}{\alpha+\mu_R})(\tfrac{\theta}{\xi+\mu_T})\right)\right) \quad a_{32} = \Lambda_{M_3}\beta_{23} - \mu_s(\mu_I + \theta)$$

$$a_{33} = -\beta_{13}\left(\left((\mu_I + \theta) - \delta(\tfrac{\xi}{\alpha+\mu_R})(\tfrac{\theta}{\xi+\mu_T})\right)\right) \quad a_{34} = -\beta_{23}\left(\left((\mu_I + \theta) - \delta(\tfrac{\xi}{\alpha+\mu_R})(\tfrac{\theta}{\xi+\mu_T})\right)\right)$$

$$a_{35} = \Lambda_{M_3}\beta_{13} \qquad\qquad a_{36} = \Lambda_{M_3}\beta_{23}$$

the previous system can be written as follows:

$$a_{11}x^2 + a_{12}x + (a_{13}y + a_{14}z)x + a_{15}y + a_{16}z = 0$$

$$a_{21}y^2 + a_{22}y + (a_{23}x + a_{24}z)y + a_{25}x + a_{26}z = 0$$

$$a_{31}z^2 + a_{32}z + (a_{33}x + a_{34}y)z + a_{35}x + a_{36}y = 0$$

Hence

$$\begin{pmatrix} a_{11}x^2 + a_{12}x + (a_{13}y + a_{14}z)x + a_{15}y + a_{16}z \\ a_{21}y^2 + a_{22}y + (a_{23}x + a_{24}z)y + a_{25}x + a_{26}z \\ a_{31}z^2 + a_{32}z + (a_{33}x + a_{34}y)z + a_{35}x + a_{36}y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \qquad (4.18)$$

and we have the following result

Theorem 4.3.2. *If the equation (4.18) has a solution then the system defined by (4.1)-(4.2)-(4.3) has an endemic solution.*

Chapter 5: Numerical Simulation and Sensitivity Analysis of the Model for HIV

5.1 Sensitivity Analysis

Infectious disease models provide a mathematical representation of the dynamic transmission cycle, involving interactions between infected and susceptible host that are expressed as a set of different equations. Sensitivity analysis techniques are used to explore the parameter sensitivity. The sensitivity analysis is used to identify the important of parameters in a model. It can identify the parameters that can be the focus of calibration. [17].

5.2 Parameter Estimation Strategy

In order to estimate the different parameters of the model, it has to be kept in mind the effect of each sub-population in the disease spread. First, note that each $\bar{\beta}_{ij}$ is a transfer rate from state i to state j, so $\bar{\beta}_{21}$ is a transfer rate from general expatriate to high risk expatriate. $S_{M_1} I_{M_2}^1$ represent the contact among susceptible in M_1 and infected individual in M_2. Hence, there is a certain order to follow: The infection rate of the high risk population to other subpopulation is higher than any other infection rate between the two other subpopulations. Therefore, $\bar{\beta}_{1k} \geq \max(\bar{\beta}_{2k}, \bar{\beta}_{3k})$. Moreover, the infection of low risk population to the other subpopulation is higher than the infection rate of the local to the other subpopulation. This implies that $\bar{\beta}_{2k} \geq \bar{\beta}_{3k}$. I conclude that $\bar{\beta}_{1k} \geq \bar{\beta}_{2k} \geq \bar{\beta}_{3k}$.

Now, if consider the high risk population is only considered, it may clear that the infection rate among this subpopulation is higher than the infection rates for the high risk to the others. Hence, $\bar{\beta}_{11} \geq \max(\bar{\beta}_{21}, \bar{\beta}_{31})$. Add to that the fact that the infection rate from the high risk subpopulation to low risk subpopulation is higher

than from the high risk subpopulation to local subpopulation because of the high contact between M_1 and M_2. therefore $\bar{\beta}_{11} \geq \bar{\beta}_{21} \geq \bar{\beta}_{31}$.

It is clear also in M_2 that $\bar{\beta}_{12} \geq \bar{\beta}_{22} \geq \bar{\beta}_{32}$, on the other hand in M_3, $\bar{\beta}_{13} \geq \bar{\beta}_{23} \geq \bar{\beta}_{33}$.

So according to [24] the values of all $\bar{\beta}_{ij}$ are estimated. The total population in the UAE in 2013 is given by around 9350000, so the initial conditions $S_{M_1} = 3131600$, $S_{M_2} = 4697700$ and $S_{M_3} = 1519900$ are supposed.

5.3 Estimation of Parameters for HIV

Since the HIV level of the infection in the UAE is very low and because of lack of any official data on the HIV in the UAE among the expatriates and the nationals, the estimation on the parameters of the dynamic of the disease will be used, since the dynamic of the disease transmission does not change much from country to another. Add to that the fact that the other parameters are mainly demographic and has nothing to do with the nature of the disease.

A numerical simulation using the parameter values given in Table 5.1 is carried out and according to [24] the values of all $\bar{\beta}_{ij}$ is estimated.

Notation	Parameter description	Range	Source
Λ_{M_1}	Constant rate by birth or immigration of M_1	44756	[13],estimated
Λ_{M_2}	Constant rate by birth or immigration of M_2	67134	[13],estimated
$\bar{\beta}_{ij}(i,j=1,2,3)$	Matrix of transmission rates between		
	susceptible and infectious individuals in(M_1, M_2, M_3)	$[0.02-1]$	[24]
Λ_{M_3}	Birth of M_3	21721	[13]
μ_S	The death rate susceptible population	0.01429	[13]
μ_I	The death rate of the infected population	0.02	[15],estimated
μ_T	The death rate of the treated population	0.02	[19]
μ_R	The death rate of the recovered population	0	
α	The rate of unaware infected becomes aware infected in M_1.	0.015	[24],estimated
γ	The rate of unaware infected becomes aware infected in M_2.	0.03	[24],estimated
θ	The rate of treatment of local infected in M_3	0.01	[24],estimated
ξ	The rate of chronic HIV (disease under control)	0	[27]
δ	The rate of losing immunity	0	No recovery from of HIV

Table 5.1: Parameters values for HIV

5.4   Numerical Simulations of the Different Endemic Cases

The theoretical investigation shown in the previous chapter will be transformed into numerical simulation. This will be done carefully into the following cases.

5.4.1   Endemic Case 1

It is referred to the case where $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 < 1$. First, the time series is presented in this case with parameters choice as follow

$\beta_{1\,1} = 0.125,\ \beta_{2\,1} = 0.11,\ \beta_{3\,1} = 0.105$ and $\mathscr{R}_0^1 = 1.196325594$

$\beta_{1\,2} = 0.095,\ \beta_{2\,2} = 0.092,\ \beta_{3\,2} = 0.0905$ and $\mathscr{R}_0^2 = 0.924520419$

$\beta_{1\,3} = 0.090,\ \beta_{2\,3} = 0.082,\ \beta_{3\,3} = 0.081$ and $\mathscr{R}_0^3 = 0.438934523$

The following results are shown in the figure 5.1.

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
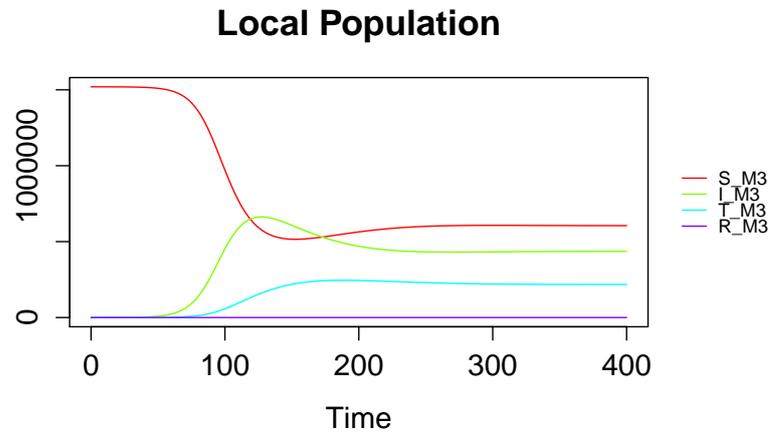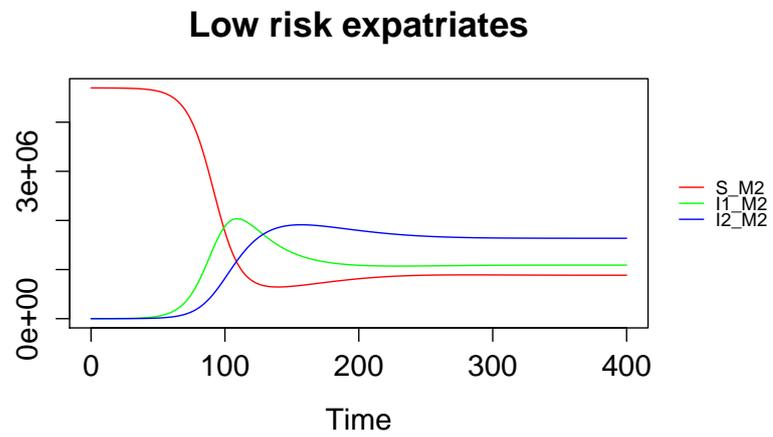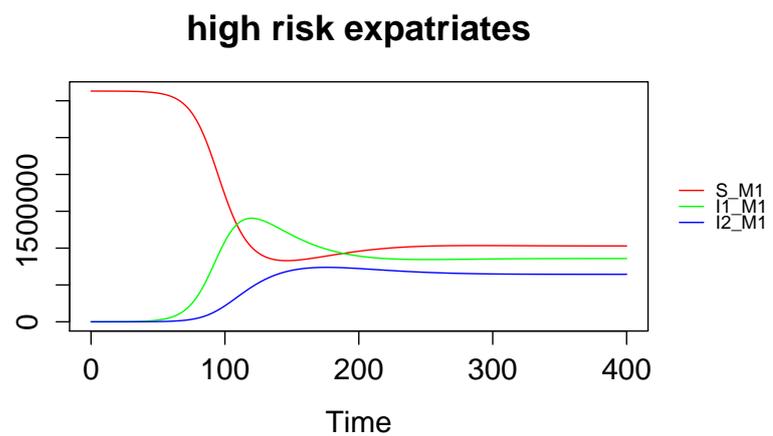
(c) high-risk population

Figure 5.1: The time series of the model compartments in case $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$, and $\mathscr{R}_0^3 < 1$

The time series of this model case showed that, in this endemic situation, the susceptible local population drop to 39.8% with fatality 17%, due to death of the disease. The infected local, treated represent 28.7% and 14.3% respectively. The high risk population, where the disease is endemic with $\mathscr{R}_0^1 > 1$, the fatality is higher, compared to the local population, which is 19.2%. In this subpopulation, the susceptible compartment is reduced to 32.9% and infected unaware is higher than the aware with values 27.4% and 20.6%, respectively.

For the general expatriate population, the fatality is the highest among all the sub-population, which is 23.2%. The burden of the disease reduces the susceptible to the lowest value, 18.8%. Moreover, the aware infected is much higher than unaware population; the values are 34.8% and 23.2%, respectively.

This is a situation because it shows that even when the disease is endemic in the high risk expatriate population, $\mathscr{R}_0^1 > 1$, the disease is more fatal in the low risk (general) expatriate subpopulation.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.2.

Sensitivity of infected locals
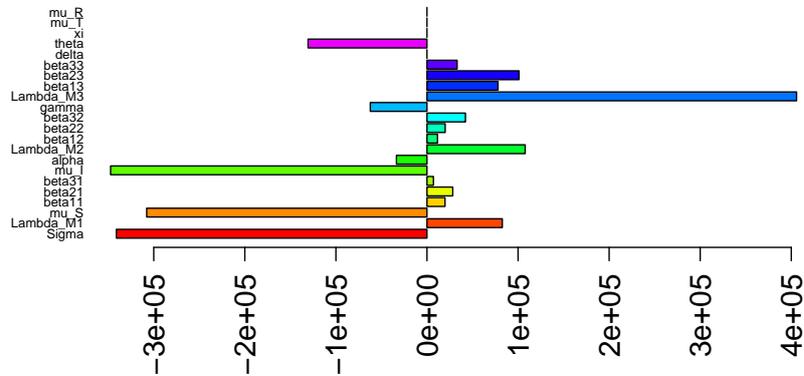
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
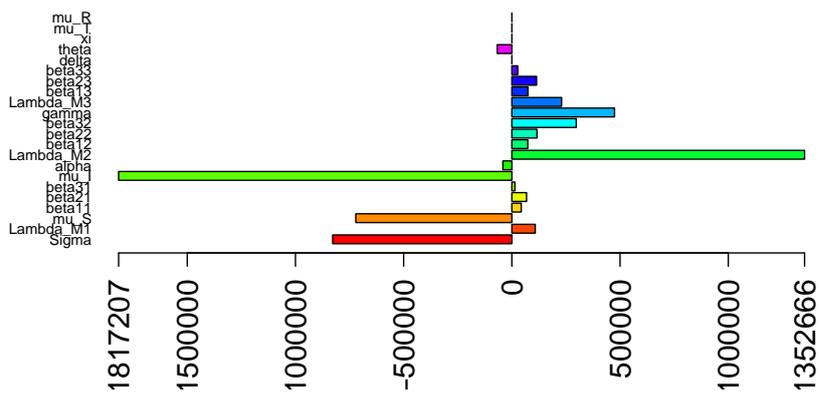
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
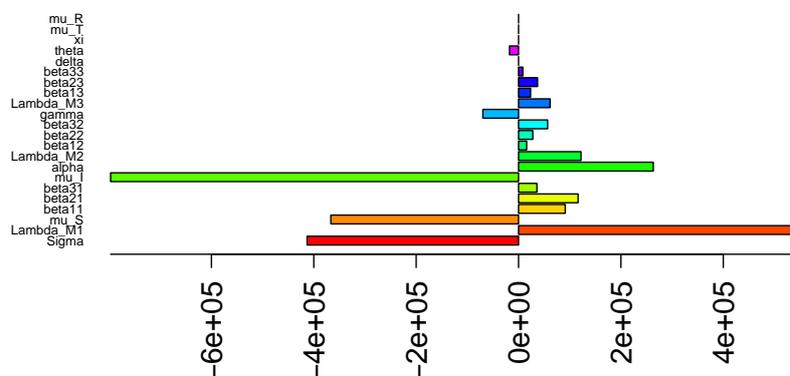
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.2: Sensitivity of the infected populations with respect to all parameters

In the sensitivity analysis of case 1, one can see from the local sensitivity of $I_{M_1}^2, I_{M_1}^2$ and $I_{M_3}^1$ that some parameters are always very sensitive for the three variables. On the other hand , other parameters are not sensitive in these three variables. The parameters that are not sensitive in these three plots are μ_R, μ_T, ξ and δ. This means μ_R and μ_T are of concern in this analysis. Since $\xi = \delta = 0$ in HIV case, this is normal for HIV studies. Some parameters make the variable sensitive in similar way for example β_{ij} (i,j=1,2,3), Λ_{M3}, Λ_{M2} and μ_s. The γ ,Σ and α can change the sensitivity of the parameter in both directions. For example, $I_{M_1}^2$ and $I_{M_3}^1$ are negatively sensitive to γ and Σ, but $I_{M_2}^2$ is positively sensitive to both of them. Also, $I_{M_2}^2$ and $I_{M_3}^1$ are negatively sensitive to α, but $I_{M_1}^2$ is positively sensitive to α.

5.4.2 Endemic Case 2

Referring to $\mathscr{R}_0^1 < 1$, $\mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 < 1$ case. First, the time series of this case with parameters is presented as follow

$\beta_{11} = 0.101, \quad \beta_{21} = 0.10099, \quad \beta_{31} = 0.1009$ and $\mathscr{R}_0^1 = 0.96663108$

$\beta_{12} = 0.01, \beta_{22} = 0.0999999, \beta_{32} = 0.0999$ and $\mathscr{R}_0^2 = 1.004912494$

$\beta_{13} = 0.090, \beta_{23} = 0.082, \beta_{33} = 0.081$ and $\mathscr{R}_0^3 = 0.438934523$

The following results are shown in the figure 5.3
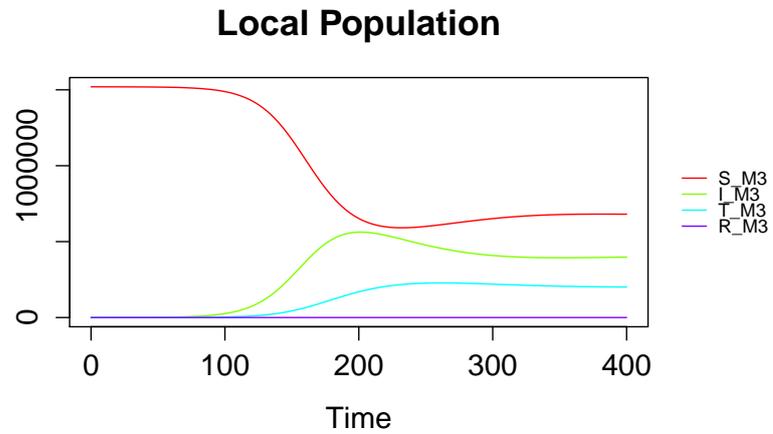
(a) Local population
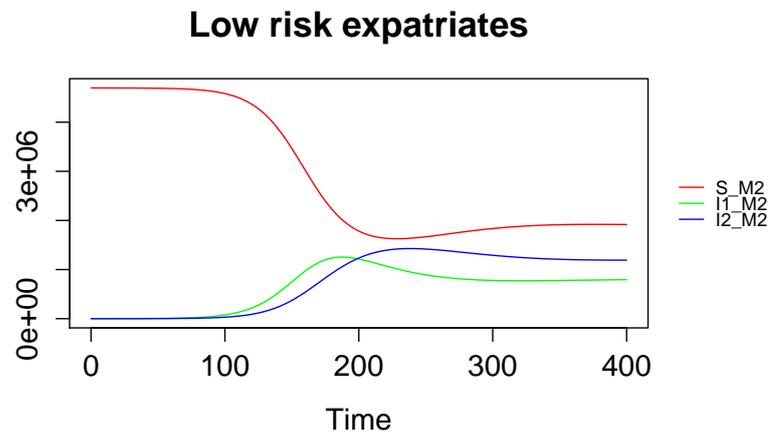
(b) low-risk population
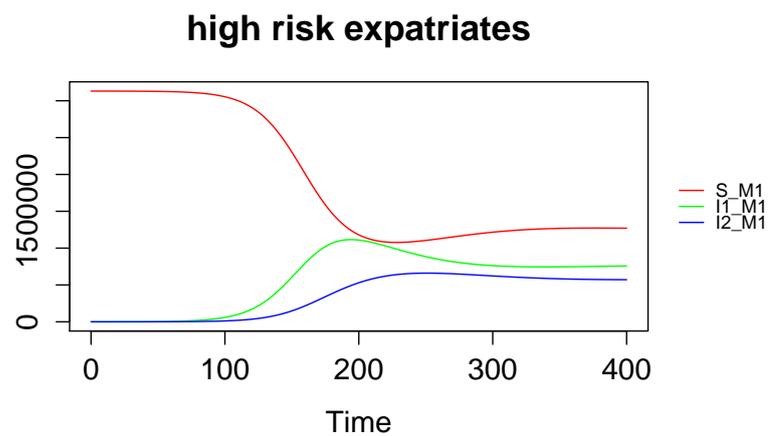
(c) high-risk population

Figure 5.3: The model compartments at $\mathscr{R}_0^1 = 0.97$, $\mathscr{R}_0^2 = 1.005$, $\mathscr{R}_0^3 = 0.44$

The time series of this model in this endemic case 2 showed that, the susceptible local population drop to 44.5% with lowest fatality 15.8% compared with other subpopulation , due to death of the disease. The infected local represents 26.5%, and 13.2% is the treated local.

For the general expatriate population, where the disease is endemic with $\mathscr{R}_0^2 > 1$, the fatality is 17%, and the susceptible compartment is reduced to 40.5%. On the other hand, the infected unaware is lower than the aware with values 25.5% and 17%, respectively.

In the high risk population, the fatality represents 17.1% as the highest percentage among all the subpopulation. Moreover, the aware infected is much higher than the unaware population, the values 24.4% and 18.3%, respectively.

In this situation, despite the fact that the disease is endemic in the low risk expatriate population, $\mathscr{R}_0^2 > 1$, the disease is a little more fatal in the high risk expatriate subpopulation.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.4.
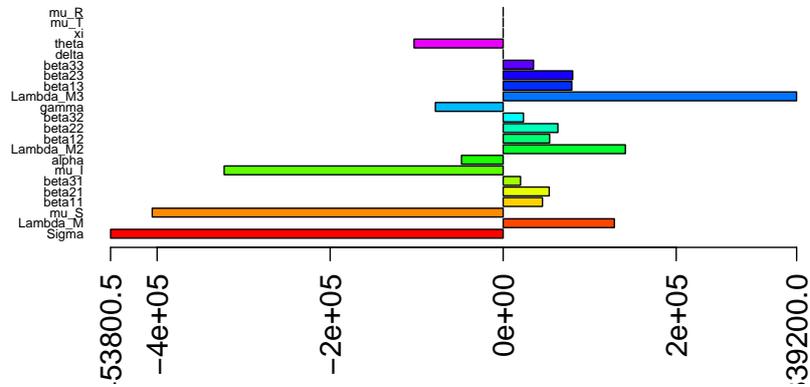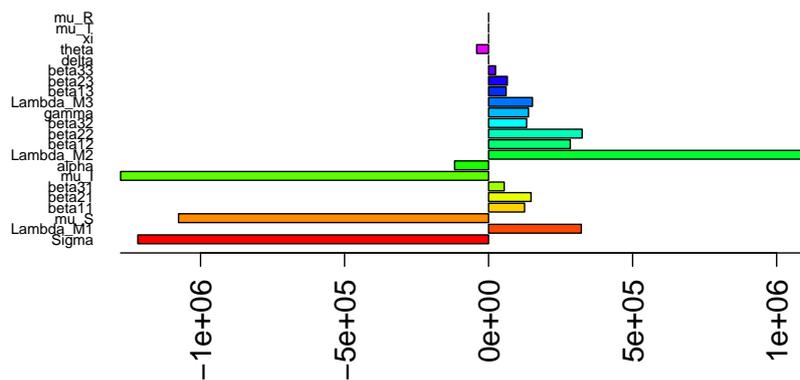
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model
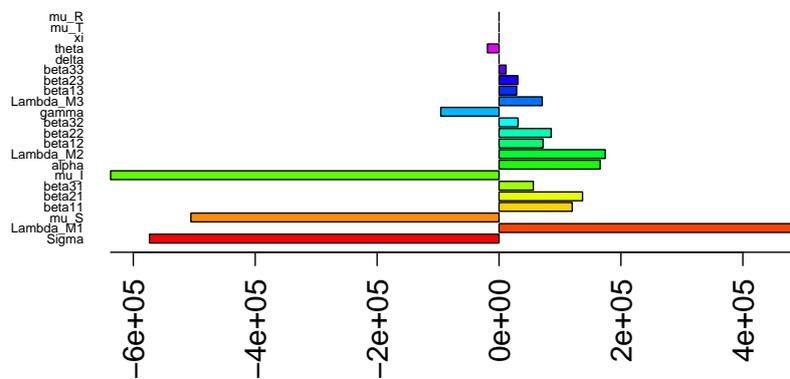
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.4: Sensitivity of the infected populations with respect to all parameters

The parameters β_{ij} and Λ_{Mi}, (i,j=1,2,3) make the variable sensitive in a positive direction. Some parameters are making the variable sensitive to a negative direction i.e. μ_s, μ_i, θ and Σ. The γ and α can change the sensitivity of the parameter in both directions. For example, $I^2_{M_1}$ and $I^1_{M_3}$ are negatively sensitive to γ, but $I^2_{M_2}$ is positively sensitive to γ. Also, $I^2_{M_2}$ and $I^1_{M_3}$ are negatively sensitive to α, but $I^2_{M_1}$ is positively sensitive to α.

5.4.3 Endemic Case 3

Referring to $\mathscr{R}^1_0 < 1$, $\mathscr{R}^2_0 < 1$ and $\mathscr{R}^3_0 > 1$ case. First, the time series of this case with parameter is shown as follows

$$\beta_{11} = 0.132, \quad \beta_{21} = 0.13198, \quad \beta_{31} = 0.13191 \text{ and} \quad \mathscr{R}^1_0 = 0.982582088$$

$$\beta_{12} = 0.1319, \quad \beta_{22} = 0.13175, \quad \beta_{32} = 0.13165 \text{ and} \quad \mathscr{R}^2_0 = 0.945695382$$

$$\beta_{13} = 0.13155, \quad \beta_{23} = 0.1313, \quad \beta_{33} = 0.13 \text{ and} \quad \mathscr{R}^3_0 = 1.006375449$$

and $\alpha = 0.025$ $\quad \gamma = 0.05$ $\quad$ and $\theta = 0.001$ are modified

The following results are shown in the figure 5.5
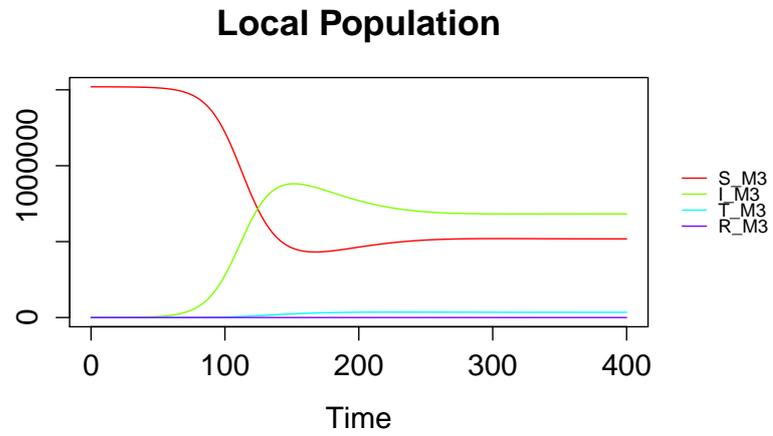
(a) Local population
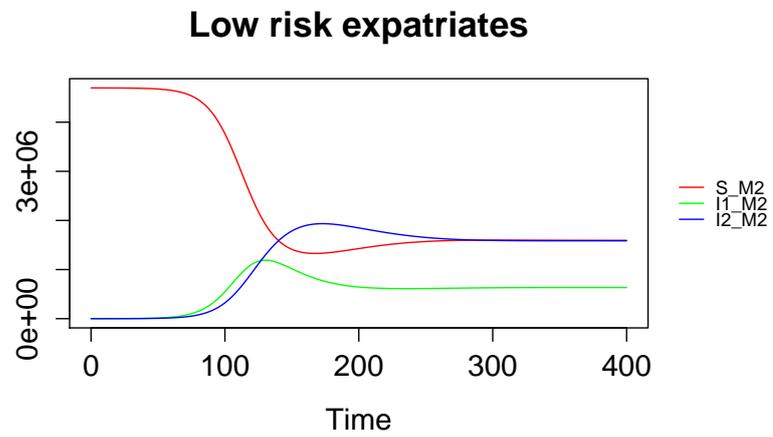
(b) low-risk population
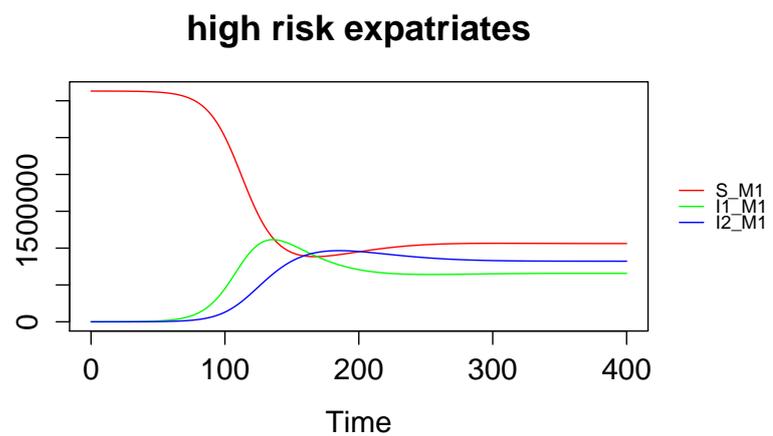
(c) high-risk population

Figure 5.5: The time series of the model compartments in case $\mathscr{R}_0^1 < 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$

What can be noticed in the time series is that, the susceptible local population has the largest drop with 34.1% and lowest fatality of 18.8% compared to other subpopulation. Also, the infected local represents a big drop of 44.8% and 2.2% for the treated local.

Once more for the general expatriate population, the fatality is 18.9%, and the susceptible compartment is reduced to 34%. That means the case of the infected unaware is more than the aware by 20.2% .

Among the high risk population,the susceptible partition is reduced to 33.9% and the fatality represents 18.9%. Furthermore, the aware infected is much higher than unaware population, the values of 26.2% and 21%, respectively.

Despite the fact that disease is endemic in local population the fatality percentage is almost the same for the three subpopulation at 18.8% to 18.9%. In addition to that the susceptible and infected for local population denoted the largest drop compared to other endemic subpopulation.

The sensitivity analysis of all the parameters, in this case, for $I^1_{M_3}$, $I^2_{M_2}$ and $I^2_{M_1}$ are given in the following bar charts 5.6.

Sensitivity of infected locals
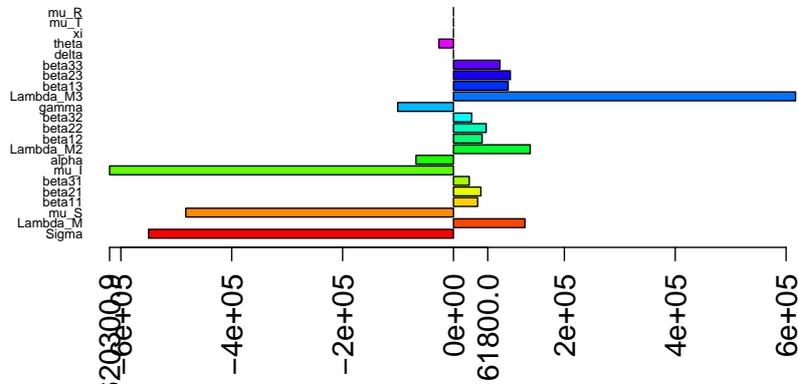
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
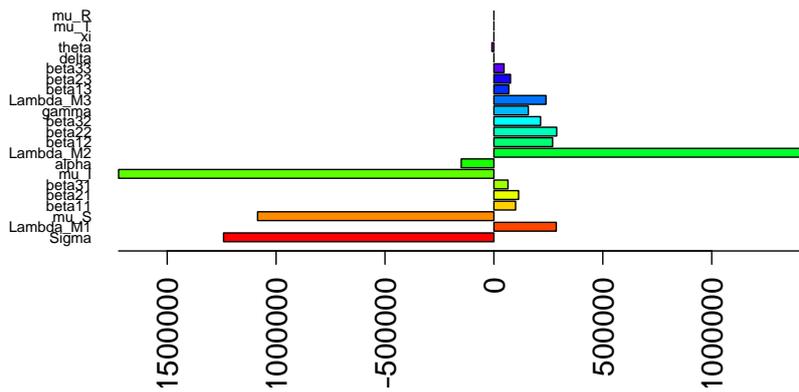
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
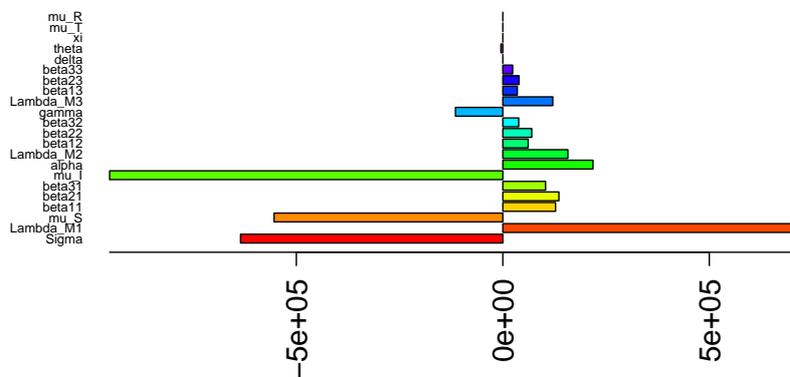
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.6: Sensitivity of the infected populations with respect to all parameters

The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3)are making the variable sensitive in positive direction. Some parameters are making the variable sensitive to negative direction for example μ_s, μ_i, θ and Σ. The γ and α can change the sensitivity of the parameter in both direction. For example $I_{M_1}^2$ and $I_{M_3}^1$ are negatively sensitive to γ but $I_{M_2}^2$ is positively sensitive to γ. Also $I_{M_2}^2$ and $I_{M_3}^1$ are negatively sensitive to α but $I_{M_1}^2$ is positively sensitive to α.

5.4.4 Endemic Case 4

Considering the case where $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 < 1$. First, the time series of this case with parameter is given as follow

$$\beta_{1\,1} = 0.115, \quad \beta_{2\,1} = 0.111, \quad \beta_{3\,1} = 0.105 \text{ and } \quad \mathscr{R}_0^1 = 1.00619547$$

$$\beta_{1\,2} = 0.1025, \quad \beta_{2\,2} = 0.1019, \quad \beta_{3\,2} = 0.1015 \text{ and } \quad \mathscr{R}_0^2 = 1.0242$$

$$\beta_{1\,3} = 0.1013, \quad \beta_{2\,3} = 0.1011, \quad \beta_{3\,3} = 0.10 \text{ and } \quad \mathscr{R}_0^3 = 0.541894473$$

The following results are shown in the figure 5.7
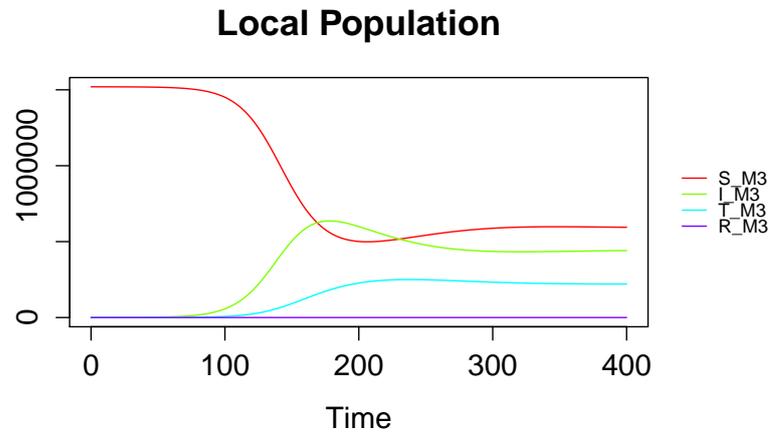
(a) Local population
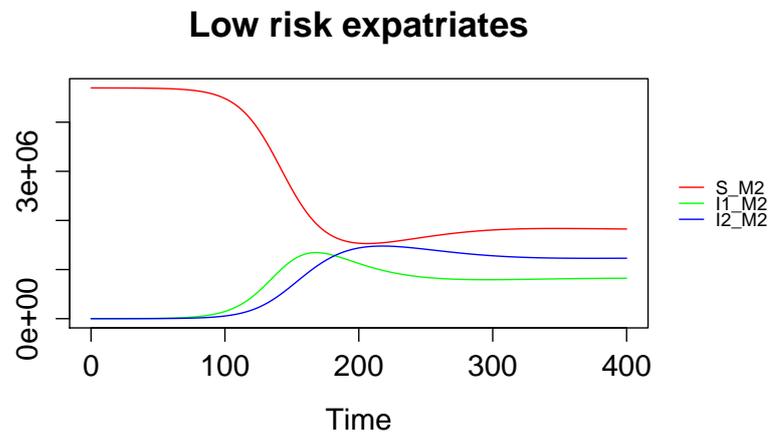
(b) low-risk population
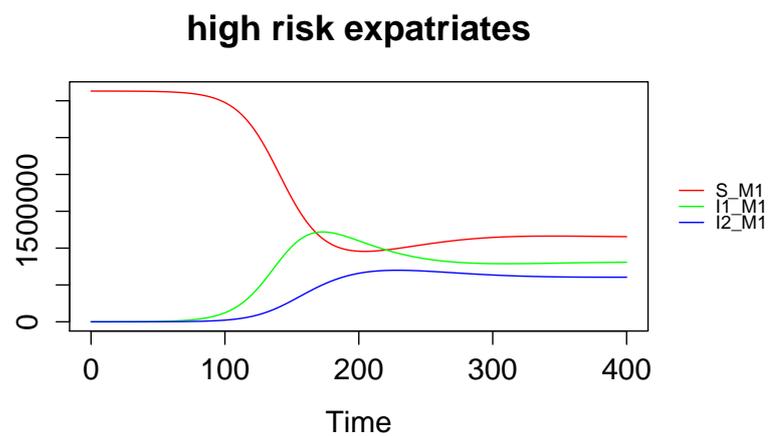
(c) high-risk population

Figure 5.7: The time series of the model compartments in case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 < 1$

It has been noticed that the disease is endemic in the high risk, $\mathscr{R}_0^1 > 1$, and low risk, $\mathscr{R}_0^2 > 1$, expatriate population steady but significant rise can be seen in the percentage of the fatality starting with 17.4% in M_3 followed by 17.5% and 18% in M_2 and M_1, respectively. Conversely, the susceptible for M_1 dropped to 36.8% followed by 38.7% in M_2 and 39% in M_3. The infected local represents 29.1% and 14.5%.

For the high risk population, the infected unaware is less than the aware with values 19.4% and 25.8%, correspondingly.

For the general expatriate population with $\mathscr{R}_0^2 > 1$, the burden of disease reduces the susceptible to the second value, 38.7%. The unaware infected represents 26.3% much higher than aware population with 8.8% difference.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.8.

Sensitivity of infected locals
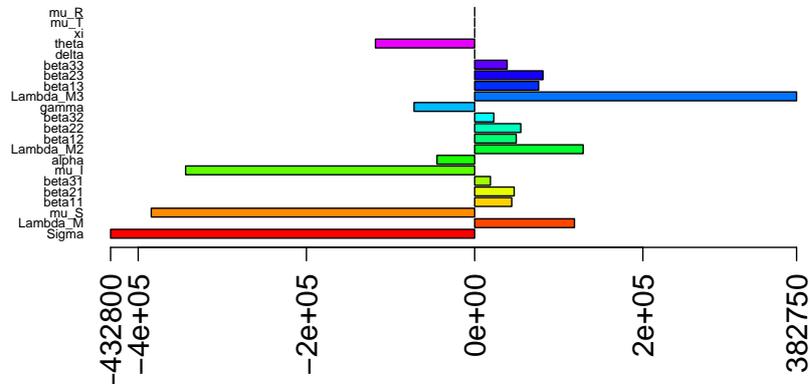
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
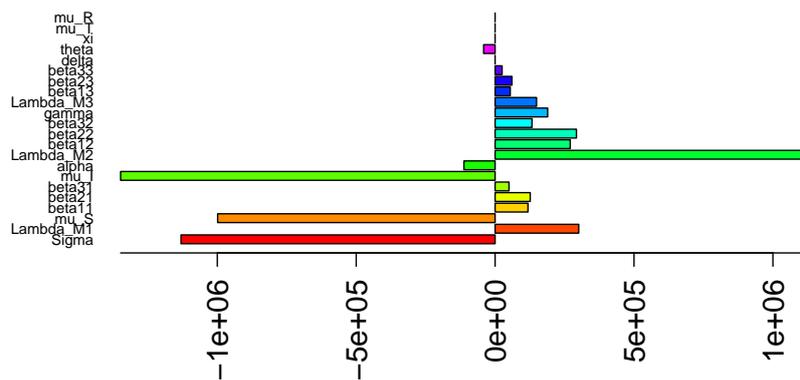
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
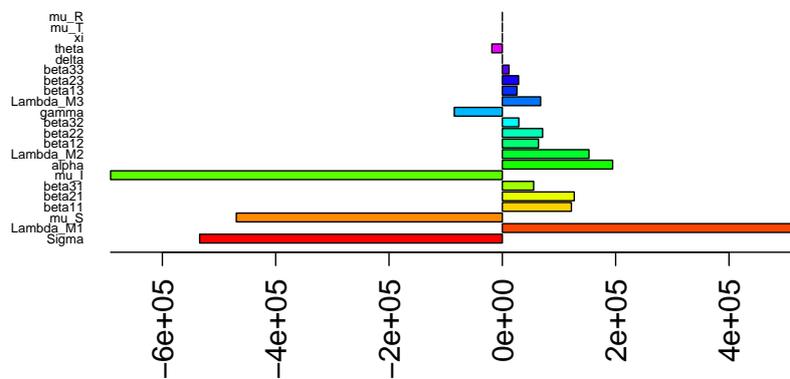
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.8: Sensitivity of the infected populations with respect to all parameters

The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3)are making the variable sensitive in a positive direction. Some parameters such as μ_s, μ_i, θ and Σ are making the variable sensitive to a negative direction. The γ and α can change the sensitivity of the parameter in both direction. For example, $I_{M_1}^2$ and $I_{M_3}^1$ are negatively sensitive to γ, but $I_{M_2}^2$ is positively sensitive to γ. Also, $I_{M_2}^2$ and $I_{M_3}^1$ are negatively sensitive to α but $I_{M_1}^2$ is positively sensitive to α.

5.4.5 Endemic Case 5

In relation to the case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$. First, the time series of this case with parameter is presented

$$\beta_{1\,1} = 0.14, \quad \beta_{2\,1} = 0.1399, \quad \beta_{3\,1} = 0.1385 \text{ and } \quad \mathscr{R}_0^1 = 1.34$$

$$\beta_{1\,2} = 0.1365, \quad \beta_{2\,2} = 0.1360, \quad \beta_{3\,2} = 0.1359 \text{ and } \quad \mathscr{R}_0^2 = 0.976$$

$$\beta_{1\,3} = 0.1357, \quad \beta_{2\,3} = 0.1352, \quad \beta_{3\,3} = 0.135 \text{ and } \quad \mathscr{R}_0^3 = 1.045$$

and $\gamma = 0.05$ and $\theta = 0.001$ are modified

The following results are shown below in figure 5.9

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
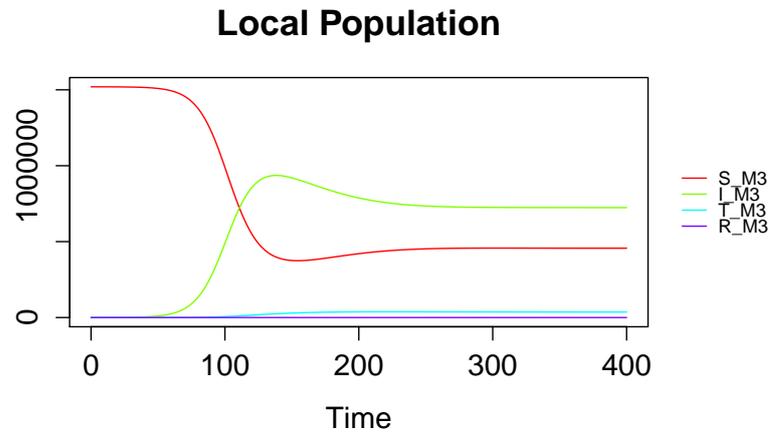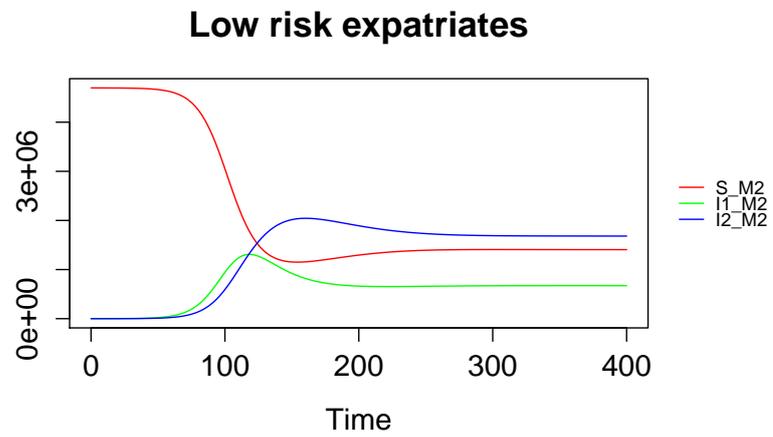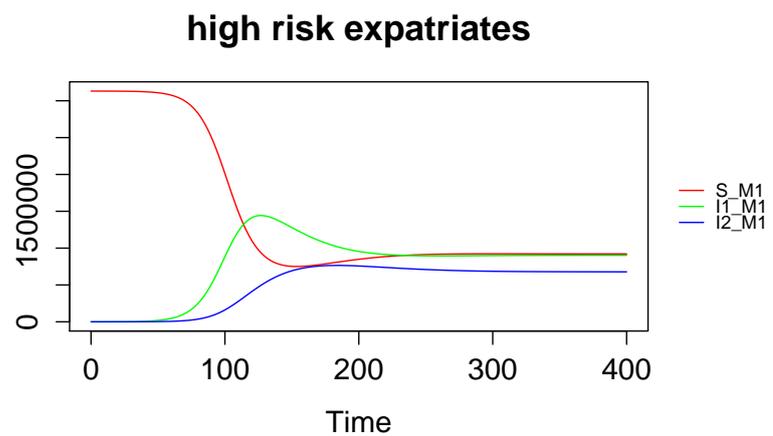
(c) high-risk population

Figure 5.9: The time series of the model compartments in case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$

The time series of this model in case 5 showed that, for the local population where the disease is endemic with $\mathscr{R}_0^3 > 1$, the susceptible local population dropped to 30.1% with fatality 20%, due to death of the disease. The infected, treated local are represented by 47.6% and 2.4% respectively.

The high risk population, with $\mathscr{R}_0^1 > 1$, the fatality is higher, compared to all sub-population, and it is 20.1%. In this subgroup, the susceptible compartment is reduced to 29.4% and infected unaware is lower than the aware with values of 21.6% and 28.8%, correspondingly.

For the general expatriate population, the fatality is 20% which is the same as local population. The burden of the disease reduces the susceptible to 29.9%. Also it can be noted that, the aware infected is much higher than unaware population, with values of 35.8% and 14.3%, respectively.

Although the disease is endemic in the high risk expatriate population, $\mathscr{R}_0^1 > 1$, and in the local population, $\mathscr{R}_0^3 > 1$, the disease is almost identical to all subpopulation in fatality with 20% to 20.1%.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.10.

Sensitivity of infected locals
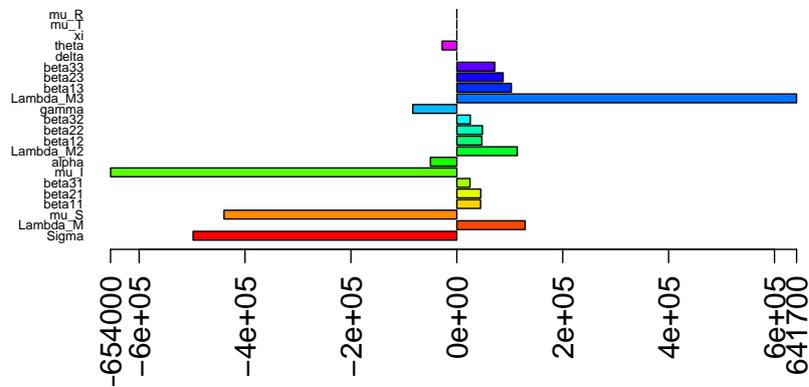
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
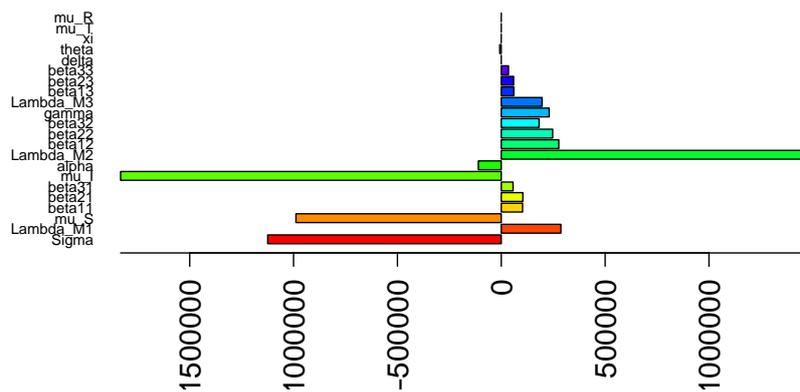
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
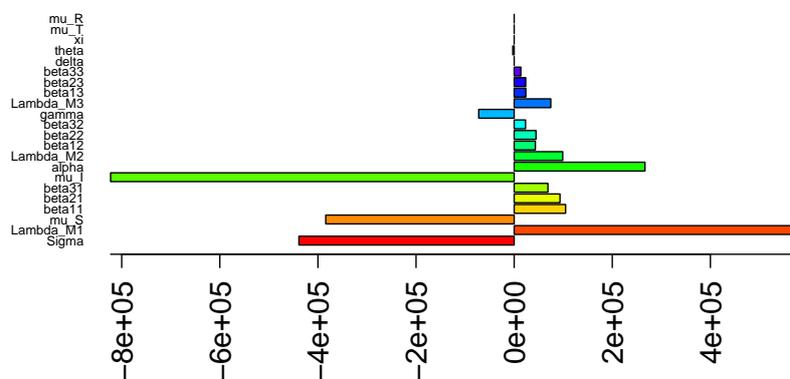
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.10: Sensitivity of the infected populations with respect to all parameters

The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3) make the variable sensitive in a positive direction. Some parameters such as μ_s, μ_i, θ and Σ make the variable sensitive to negative direction. The γ and α can change the sensitivity of the parameter in both directions. For example, $I_{M_1}^2$ and $I_{M_3}^1$ are negatively sensitive to γ, but $I_{M_2}^2$ is positively sensitive to γ. Also, $I_{M_2}^2$ and $I_{M_3}^1$ are negatively sensitive to α, but $I_{M_1}^2$ is positively sensitive to α.

5.4.6 Endemic Case 6

In case $\mathscr{R}_0^1 < 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$.The time series of this case with parameters is given as follows

$$\beta_{1\,1} = 0.132, \quad \beta_{2\,1} = 0.13188, \quad \beta_{3\,1} = 0.13121 \text{ and } \quad \mathscr{R}_0^1 = 0.982582088$$

$$\beta_{1\,2} = 0.131, \quad \beta_{2\,2} = 0.1305, \quad \beta_{3\,2} = 0.13045 \text{ and } \quad \mathscr{R}_0^2 = 1.31$$

$$\beta_{1\,3} = 0.1302, \quad \beta_{2\,3} = 0.1301, \quad \beta_{3\,3} = 0.130099 \text{ and } \quad \mathscr{R}_0^3 = 1.007$$

and $\alpha = 0.025$ and $\theta = 0.001$ are modified

The following results in figure 5.11 are detected

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
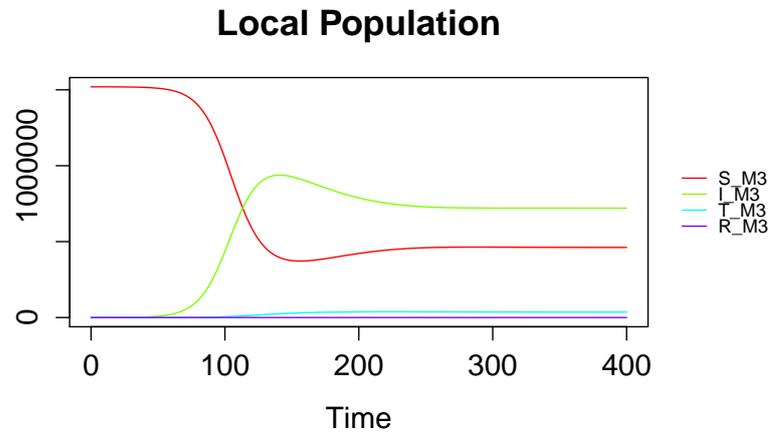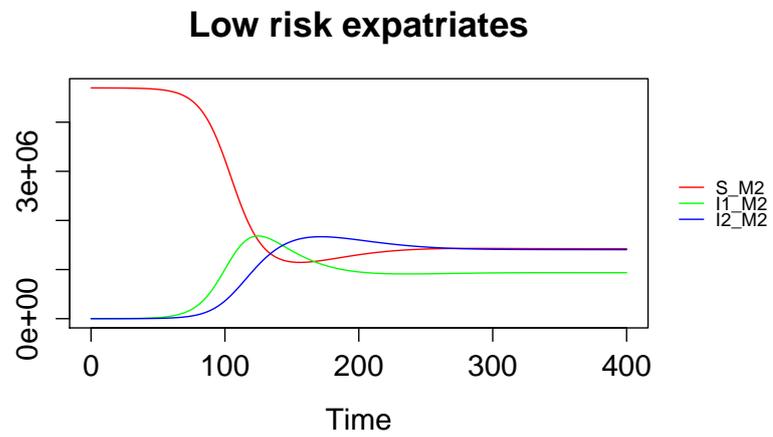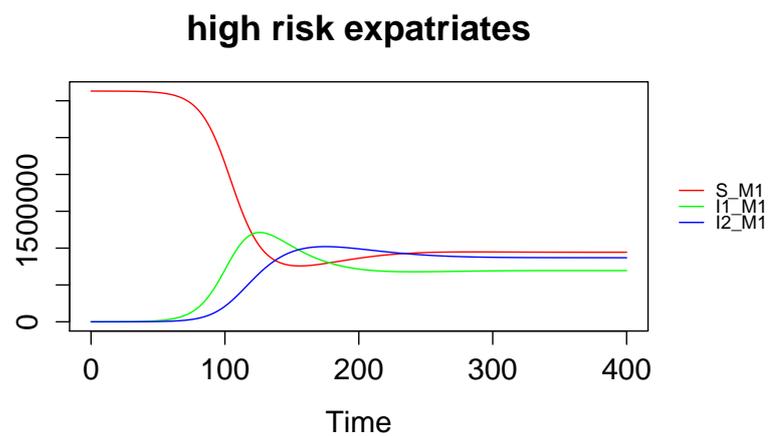
(c) high-risk population

Figure 5.11: The time series of the model compartments in case $\mathscr{R}_0^1 < 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$

From the time series in case 6, it can be seen that, for the local population where the disease is endemic with $\mathscr{R}_0^3 > 1$, the susceptible local population drops to 30.4% with a fatality 19.9%, in the result of the death of the disease. The infected and treated local are represented by rate of 47.4% and 2.4% respectively.

For the general expatriate population, with $\mathscr{R}_0^2 > 1$, the fatality is the same, compare to the all subpopulation, and it is 19.9%. In this subgroup, the susceptible compartment is reduced to 30.3% and infected unaware is lower than the aware with values 19.9% and 29.9%, correspondingly.

The high risk population, the fatality is 19.9% which is the same as local population. The burden of the disease reduces the susceptible to 30.2%. Also it can be noted that, the unaware infected are lower than aware population, values of 22.2% and 27.7% respectively.

In this situation it can be seen that, although the disease is endemic in the low risk expatriate population, $\mathscr{R}_0^2 > 1$, and in the local population, $\mathscr{R}_0^3 > 1$, the disease is identical in fatality to all subpopulation with 19.9%.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.12.

Sensitivity of infected locals
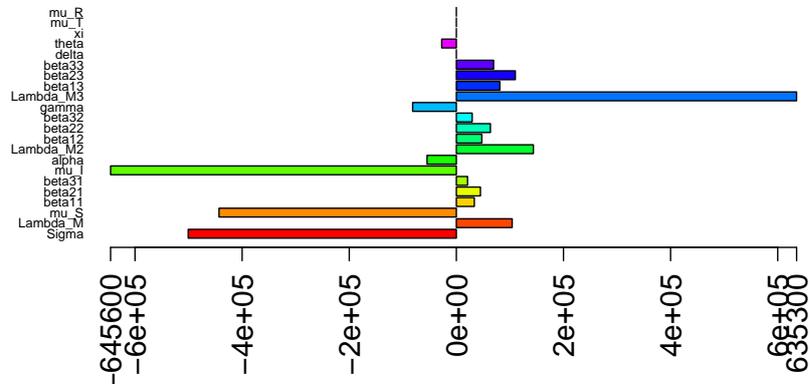
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters to the model

Sensitivity of infected low−risk expatriates
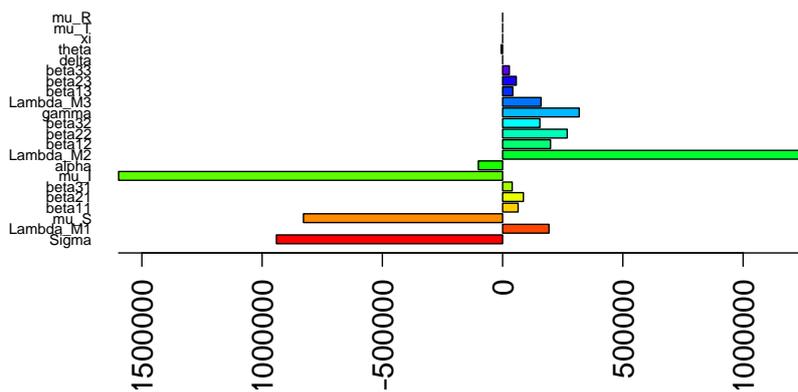
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
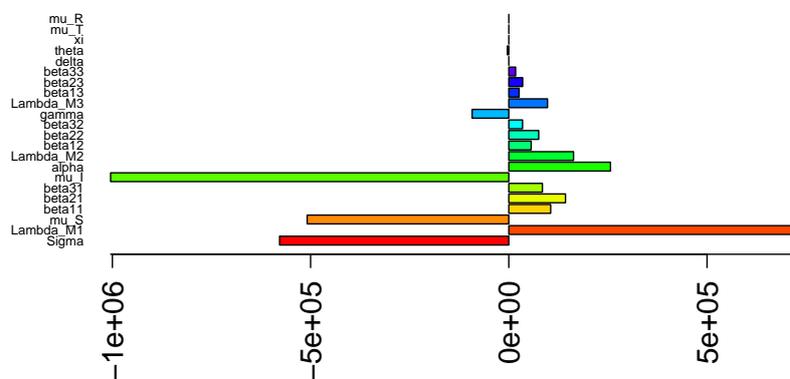
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.12: Sensitivity of the infected populations with respect to all parameters

The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3) make the variable sensitive in a positive direction. Some parameters such as μ_s, μ_i, θ and Σ make the variable sensitive to a negative direction. The γ and α can change the sensitivity of the parameter in both directions. For example, $I^2_{M_1}$ and $I^1_{M_3}$ are negatively sensitive to γ, but $I^2_{M_2}$ is positively sensitive to γ. Also, $I^2_{M_2}$ and $I^1_{M_3}$ are negatively sensitive to α, but $I^2_{M_1}$ is positively sensitive to α.

5.4.7 Endemic Case 7

The last case where $\mathscr{R}^1_0 > 1, \mathscr{R}^2_0 > 1$ and $\mathscr{R}^3_0 > 1$. The time series of this case with parameters is shown in this way

$$\beta_{1\,1} = 0.195, \quad \beta_{2\,1} = 0.192, \quad \beta_{3\,1} = 0.190 \text{ and } \quad \mathscr{R}^1_0 = 1.866$$

$$\beta_{1\,2} = 0.1895, \quad \beta_{2\,2} = 0.1880, \quad \beta_{3\,2} = 0.1875 \text{ and } \quad \mathscr{R}^2_0 = 1.889$$

$$\beta_{1\,3} = 0.187, \quad \beta_{2\,3} = 0.1868, \quad \beta_{3\,3} = 0.186 \text{ and } \quad \mathscr{R}^3_0 = 1.0079$$

The following results are shown in the figure 5.13

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
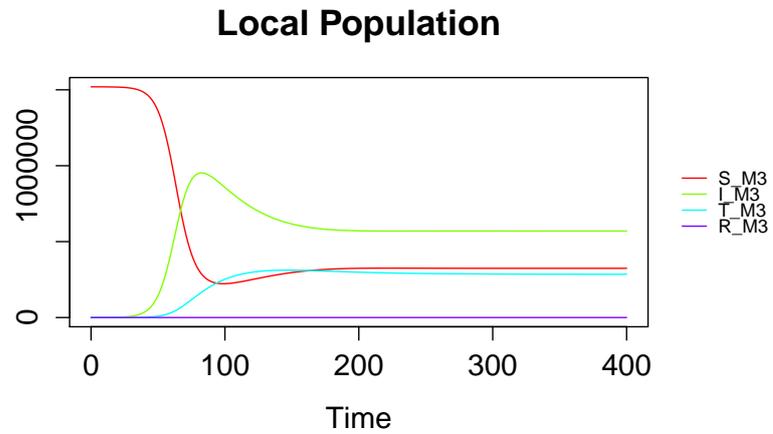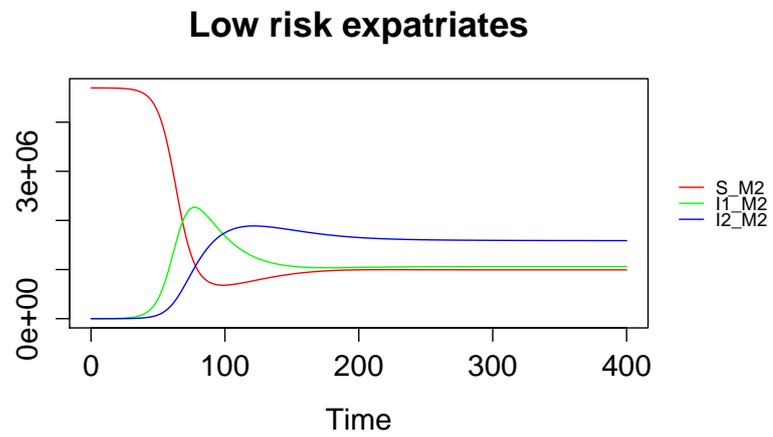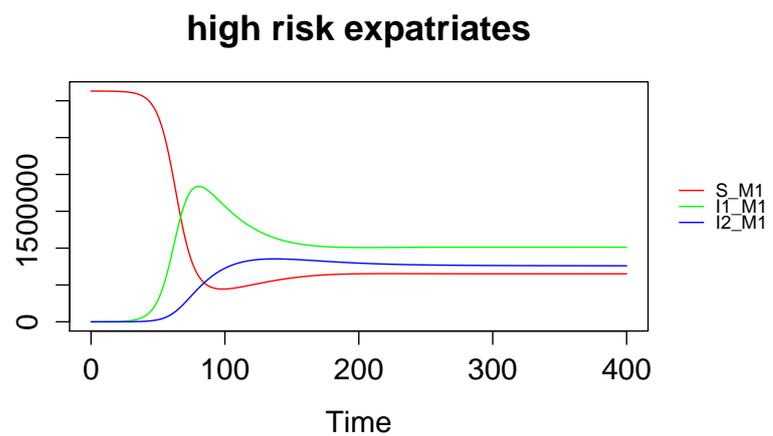
(c) high-risk population

Figure 5.13: The time series of the model compartments in case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$

Not surprisingly the disease is endemic in all subgroups. It can be seen that, for the local population, the susceptible local population drops to 21.3% with the fatality of 22.5%, due to the death of the disease. The infected and treated local are represented by 37.5% and 18.7%, respectively.

For the general expatriate population, the fatality is 22.5%. In this subgroup, the susceptible compartment is reduced to 21.2% and infected unaware is lower than the aware with values 22.5% and 33.8%, correspondingly.

In the high risk population, the fatality is 22.6%, a little more than other subgroups. The burden of the disease reduces the susceptible to 20.8%. It can be seen that, the unaware infected is higher than the aware population, the values 32.3% and 24.3%, respectively.

In this situation it can be seen that, the fatality percentage is almost the same for all endemic subpopulation with 22.5% and 22.6%. The infected local population denoted the highest drop with 37.5%

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 5.14.

Sensitivity of infected locals
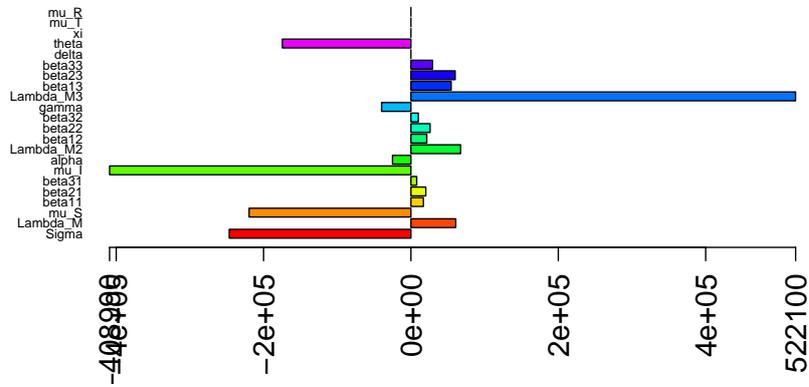
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
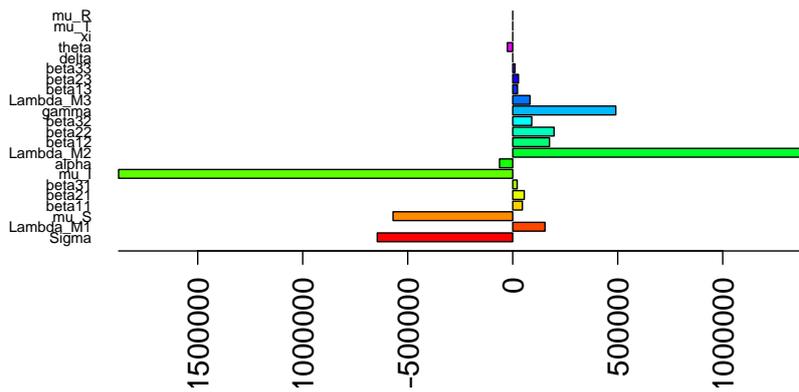
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
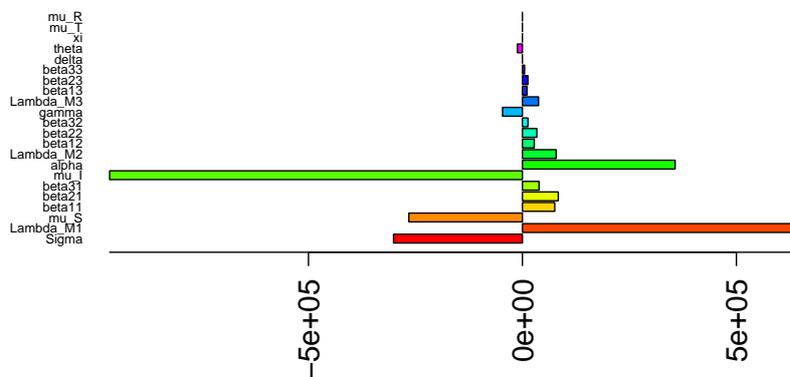
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 5.14: Sensitivity of the infected populations with respect to all parameters

The parameters $\beta_{i\ j}$ and Λ_{Mi}, (i,j=1,2,3) make the variable sensitive in a positive direction. Some parameters such as μ_s, μ_i, θ and Σ make the variable sensitive to a negative direction. The γ and α can change the sensitivity of the parameter in both directions. For example, $I^2_{M_1}$ and $I^1_{M_3}$ are negatively sensitive to γ, but $I^2_{M_2}$ is positively sensitive to γ. Also, $I^2_{M_2}$ and $I^1_{M_3}$ are negatively sensitive to α, but $I^2_{M_1}$ is positively sensitive to α.

Chapter 6: Numerical Simulation and Sensitivity Analysis of the Model for TB

6.1 Estimation of Parameters for TB

The TB cases are simulated with the parameter values listed in the next table 6.1. The estimation of unknown parameters according to the data and demographic of TB in the UAE are given. All the parameters are usually measured experimentally but it is difficult to estimate the transmission rate $\bar{\beta}_{i\,j}(i,j=1,2,3)$ of tuberculosis so the values was calculated indirectly from the infected contact rate [14]. The other parameters, where chosen from the cited references in Table 6.1

Notation	Parameter description	Range	Source
Λ_{M_1}	Constant rate by birth or immigration of M_1	0.00158	[13],estimated
Λ_{M_2}	Constant rate by birth or immigration of M_2	0.012	[13],estimated
$\bar{\beta}_{i\,j}(i,j=1,2,3)$	matrix of transmission rates between		
	susceptible and infectious individuals in(M_1, M_2, M_3)	$[6.05681 \times 10^{-6} - 4]$	[9]
Λ_{M_3}	Birth of M_3	0.01554	[13]
μ_S	The death rate susceptible population	0.01429	[13]
μ_I	The death rate of the infected population	$[5 \times 10^{-4} - 1.4 \times 10^{-2}]$	[32], estimated
μ_T	The death rate of the treated population	$[0.73 - 1.2]$	[32]
μ_R	The death rate of the recovered population	0.09	[9], estimated
α	The rate of unaware infected become aware infected in M_1	0.828248	[9], estimated
γ	The rate of unaware infected become aware infected in M_2	0.828248	[9], estimated
θ	The rate of treatment of local infected in M_3	$[1 - 2]$	[7]
ξ	The rate of recovery of successful treat of TB	$[0.7311 - 1.2]$	[32], [5]
δ	The rate of losing immunity	2.72×10^{-5}	[28] ,No recovery out of TB

Table 6.1: Parameters values for TB

6.2 Numerical Simulations of the Different Endemic Cases

6.2.1 Endemic Case 1

In case where $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 < 1$. The time series of this case with parameters is shown as follow

$\beta_{1\,1} = 2.9$, $\beta_{2\,1} = 2.8$, $\beta_{3\,1} = 2.7$ and $\mathscr{R}_0^1 = 1.15336146$

$\beta_{1\,2} = 2$, $\beta_{2\,2} = 1.505$, $\beta_{3\,2} = 1.5$ and $\mathscr{R}_0^2 = 0.89783224$

$\beta_{1\,3} = 1.43$, $\beta_{2\,3} = 1.42$, $\beta_{3\,3} = 1.4$ and $\mathscr{R}_0^3 = 0.30590817$

The following results are shown in the Figure 6.1.

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
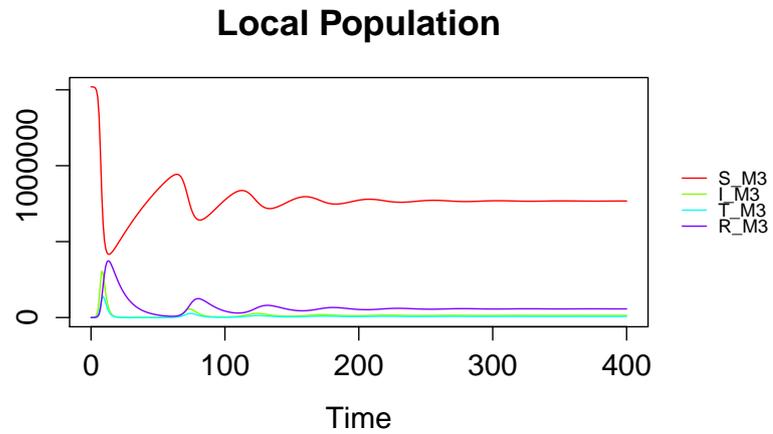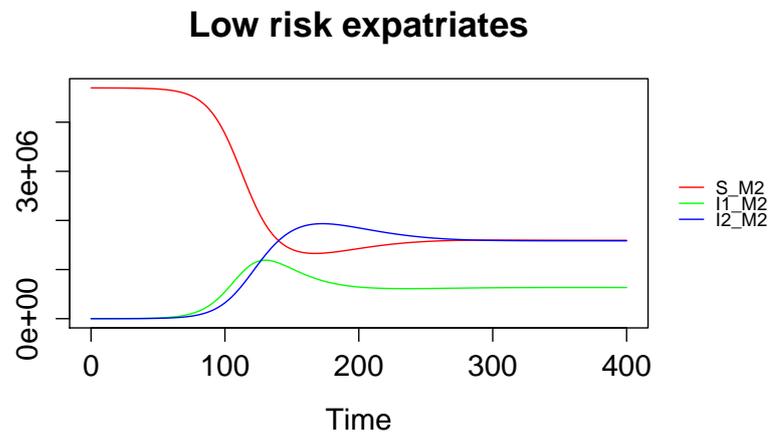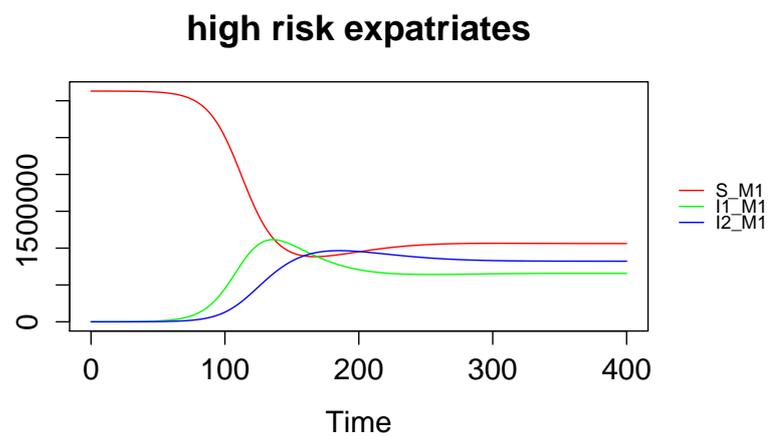
(c) high-risk population

Figure 6.1: The time series of the model compartments in case $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 < 1$

These simulation show the dynamic of the different compartment. One comment observation is that all the variables oscillate before they converge to an equilibrium points.

In the dynamic of local compartment, it is noticed that the susceptible subpopulation is reduced to 50.47% of it's size. The same figure 6.1a shows that the infected subpopulation is very low with 0.95% , the treated population is 0.45% and recovery 3.76%. This show very low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation with 44.35%

For the high risk population, as it is shown in figure 6.1c, this subpopulation has the lowest drop of susceptible to 33.78% and aware infected subpopulation with 66.46% . Hence has high isolation and deportation. It can be seen also, very low number of the unaware infected people with 1.1% and no fatality due the high level of aware infected.

Finally, in the figure 6.1b, it is shown that this case similar high risk subpopulation. In fact, there is a drop in the low risk susceptible subpopulation to 46%. The aware infected subpopulation is around 54% and no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.1.

Sensitivity of infected locals
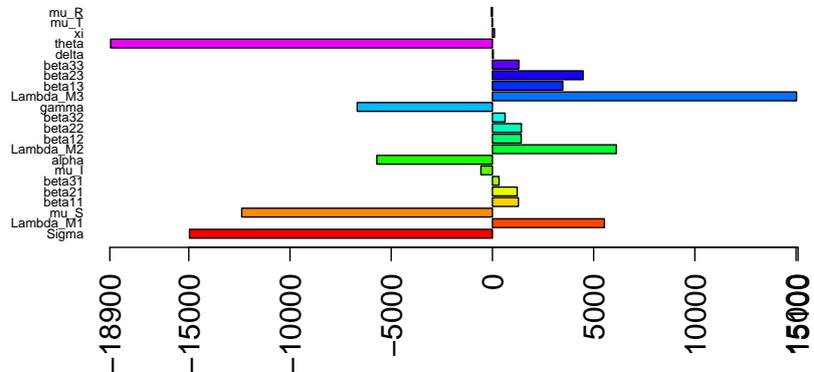
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
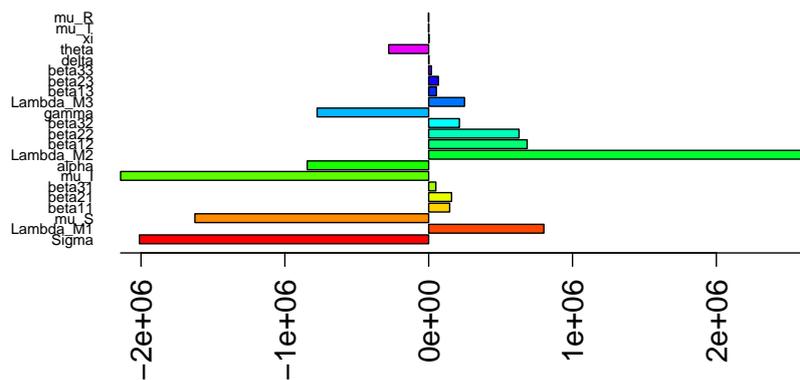
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
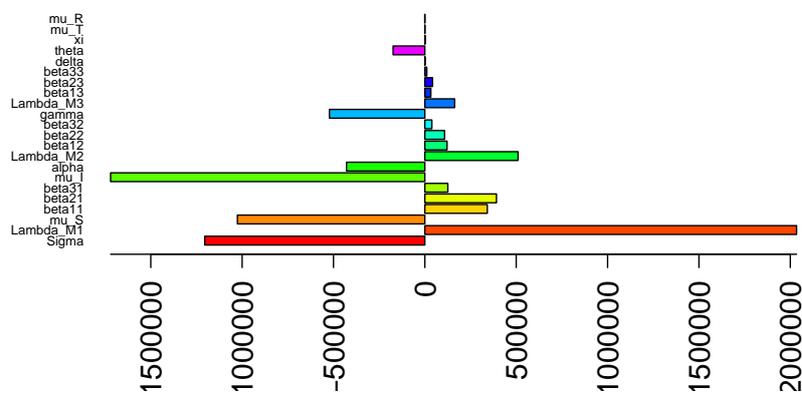
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.2: Sensitivity of the infected populations with respect to all parameters

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the bar charts 6.2. This sensitivity analysis does not show any parameter that change the sign of with respect of the three variables $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$. But, some parameters are not sensitive to the variables. On the other hand, Λ_{M_i} has a high sensitivity to I_{M_i} all with the same sign. For the infected local subpopulation, θ is highly sensitive parameter . That shows the impact of the treatment rate parameter in the dynamic of this model in this case of high infectious due to high risk subpopulation. The other high sensitive parameters are μ_I, μ_S and Σ.

6.2.2 Endemic Case 2

It is referred to the case where $\mathscr{R}_0^1 < 1$, $\mathscr{R}_0^2 > 1$ and $R_0^3 < 1$. First, the time series of this case with parameters is presented as follow

$\beta_{1\,1} = 2.25$, $\beta_{2\,1} = 2.24$, $\beta_{3\,1} = 2.2$ and $\mathscr{R}_0^1 = 0.894849408$

$\beta_{1\,2} = 2$, $\beta_{2\,2} = 1.9$, $\beta_{3\,2} = 1.85$ and $\mathscr{R}_0^2 = 1.133475917$

$\beta_{1\,3} = 1.65$, $\beta_{2\,3} = 1.62$, $\beta_{3\,3} = 1.6$ and $\mathscr{R}_0^3 = 0.349609337$

The following plots in the Figure 6.4 represented the results.

Local Population
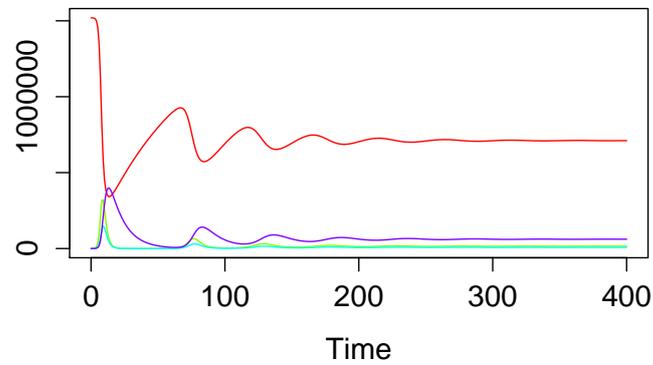
(a) Local population

Low risk expatriates
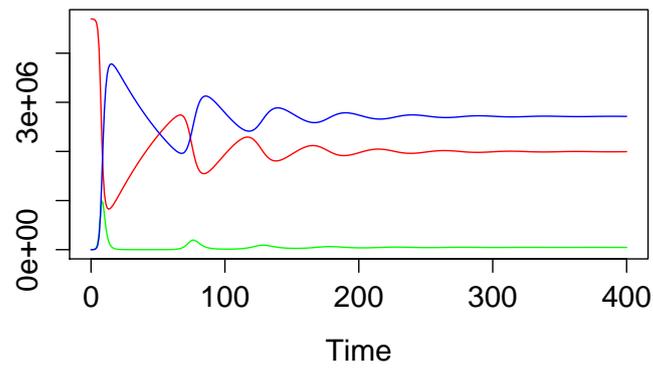
(b) low-risk population

high risk expatriates
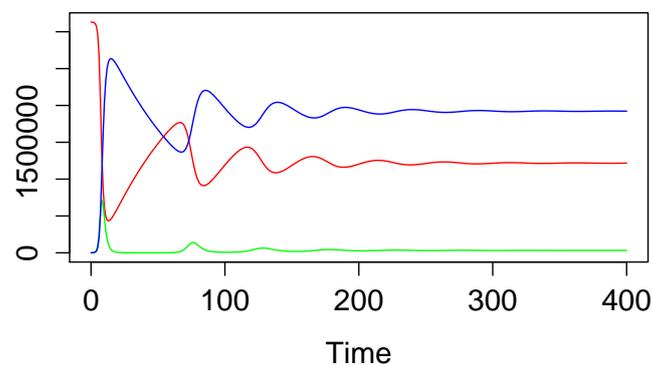
(c) high-risk population

Figure 6.3: The model compartments at $\mathscr{R}_0^1 = 0.895$, $\mathscr{R}_0^2 = 1.133$, $\mathscr{R}_0^3 = 0.349$

These simulation show the dynamic of the different compartment in this case. It can be observed that, all the variable oscillate before they converge to an equilibrium points.

Through the dynamic of local population, It is noticed that the susceptible subpopulation is reduced to 46.8% of it's size. The same figure 6.3a shows that the infected subpopulation is very low with 1% , the treated population is 0.5% and recovery 4%. This show very low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 47.7%

For the high risk population, as it is shown in figure 6.3c, the subpopulation has the lowest drop for susceptible with 38.9% and aware infected subpopulation of 61.4%. However, a very low number of the unaware infected people and no fatality due the high level of aware infected.

Finally, in the figure 6.3b, it can be seen that this case is similar to the high risk subpopulation. In fact, the drop of the low risk susceptible subpopulation is almost 42.5%. The aware infected subpopulation is around 57.8% and no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.4.

Sensitivity of infected locals
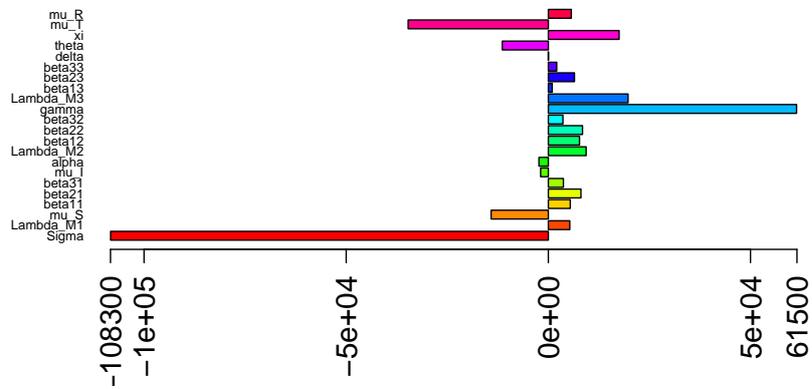
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
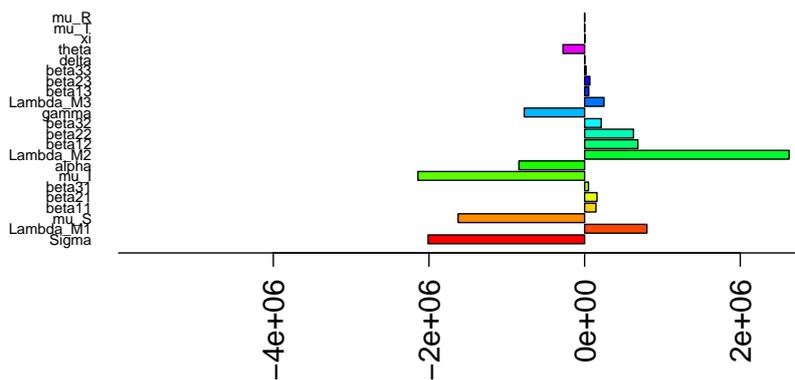
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
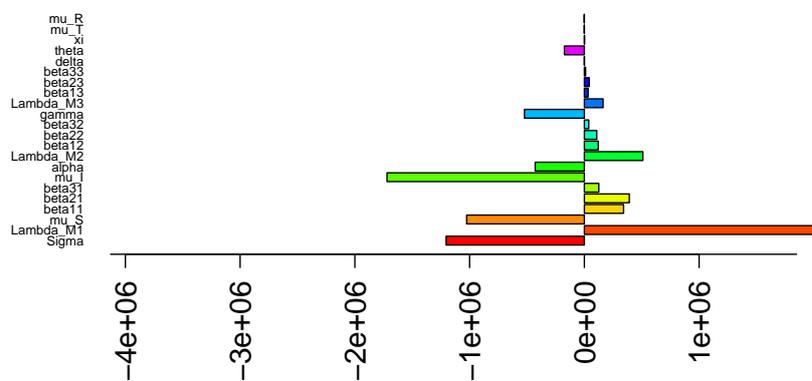
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.4: Sensitivity of the infected populations with respect to all parameters

The sensitivity analysis of all the parameters in case 2, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the bar charts 6.4. The parameters γ, μ_R and ξ change their sensitivity of the parameter in both directions. The variable $I_{M_1}^2$ and $I_{M_2}^2$ are negatively sensitive to γ, μ_R and ξ, but $I_{M_3}^1$ is positively sensitive to all of them. But, we can clearly see that all other parameters do not change the sensitivity directions . On the other hand, Σ have high sensitivity for all parameter and Λ_{M_1} for $I_{M_1}^2$ in the negative direction. For the infected local subpopulation, μ_I and α are low sensitive parameter with respect to the other subgroups. It can be seen also that, μ_R and Σ are the most infective parameters for all subpopulation.

6.2.3 Endemic Case 3

considering the case where $\mathscr{R}_0^1 < 1$ and $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$. First, the time series of this case with parameters is given as follow

$$\beta_{1\,1} = 2.42, \quad \beta_{2\,1} = 2.415, \quad \beta_{3\,1} = 2.41 \text{ and } \quad \mathscr{R}_0^1 = 0.962460252$$

$$\beta_{1\,2} = 2.405, \quad \beta_{2\,2} = 2.4, \quad \beta_{3\,2} = 2.34 \text{ and } \quad \mathscr{R}_0^2 = 0.993324711$$

$$\beta_{1\,3} = 2.36, \quad \beta_{2\,3} = 2.35, \quad \beta_{3\,3} = 2.3 \text{ and } \quad \mathscr{R}_0^3 = 1.027217544$$

and $\quad \gamma = 1.2 \quad$ and $\theta = 0.35$ are modified

The following results are given in the Figure 6.6.
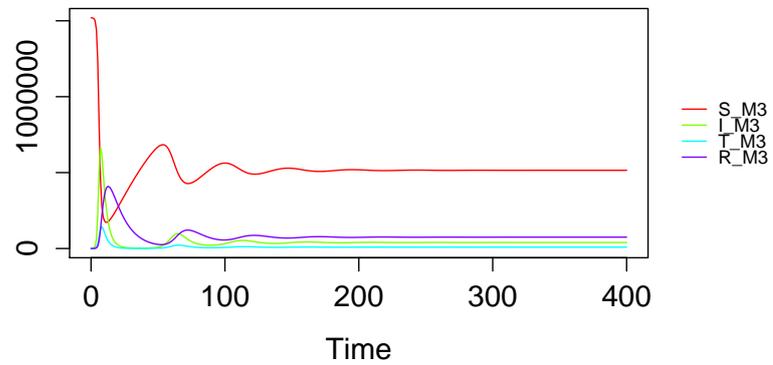
(a) Local population
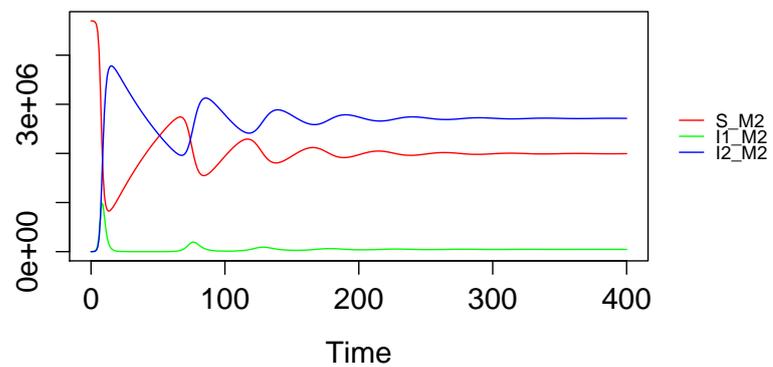
(b) low-risk population
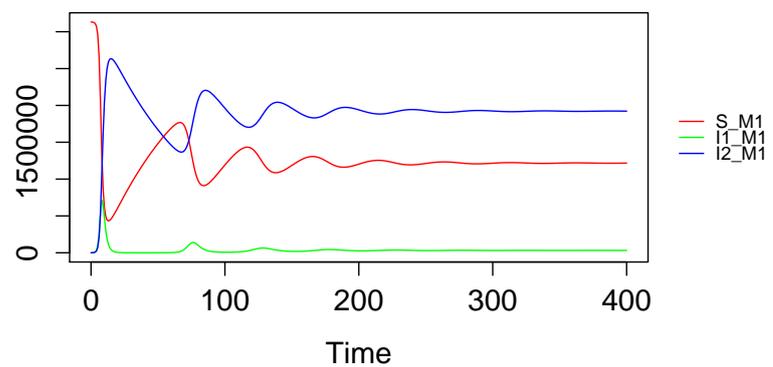
(c) high-risk population

Figure 6.5: The time series of the model compartments in case $\mathscr{R}_0^1 < 1$ and $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$

These simulation show the dynamic of the different compartment in this case. It can be observed that, all the variable oscillate before they converge to an equilibrium points.

Through the dynamic of local population, the susceptible subpopulation has the lowest drop 33.9% with respect to the other subgroup. The same figure 6.5a shows that the infected subpopulation is very low to 2.6% , the treated population is 0.6% and recovery 4.9%. This show very low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 58%

For the high risk population, as it is shown in figure 6.5c, the susceptible subpopulation dropped to 33% of it's size. The aware infected subpopulation of 67.2% , which mean high isolation and deportation. There is also very low number of the unaware infected people with 1.1% and no fatality due the high level of aware infected.

Finally, in the figure 6.5b, it can be observed that, this case is similar to the high risk subpopulation. In fact, we notice that there is a drop of the low risk susceptible subpopulation to almost 33.2%. The aware infected subpopulation is around 67.4% and no fatality.

The sensitivity analysis of all parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.6.

Sensitivity of infected locals
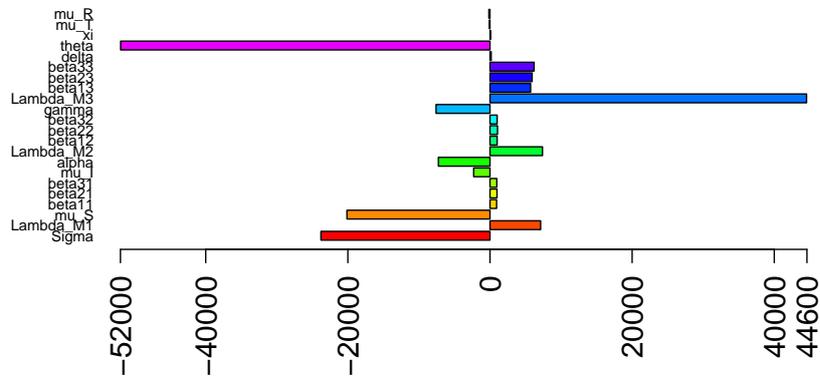
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
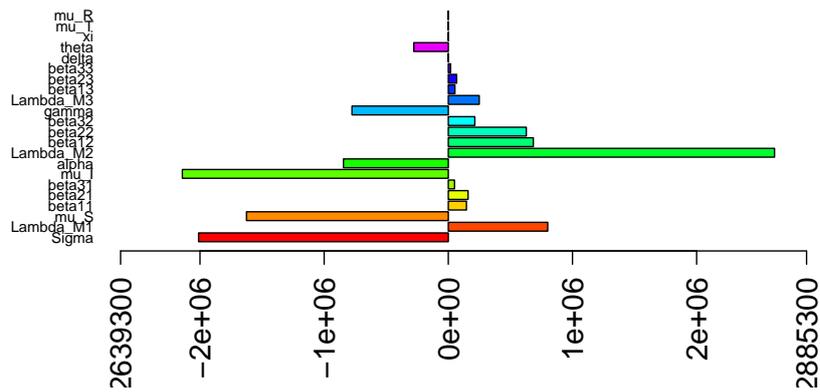
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
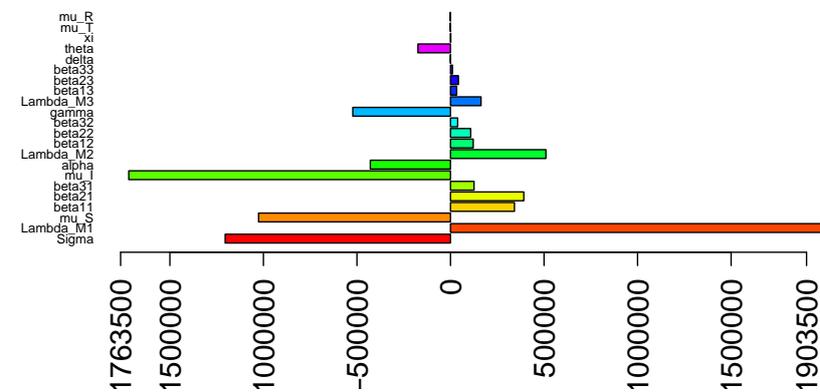
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.6: Sensitivity of the infected populations with respect to all parameters

The sensitivity analysis show that, there are no parameter that change the sign of with respect of the three variables $I^1_{M_3}$, $I^2_{M_2}$ and $I^1_{M_1}$. The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3)are making the variable sensitive in positive direction, but $\beta_{i\,j}$ have a little sensitivity in all subpopulation and Λ_{M1} is making the variable more sensitive in $I^2_{M_1}$. Some parameters are making the variable sensitive to negative direction for example θ, μ_s, μ_i, α and Σ , it can be seen that, μ_i is less sensitive in $I^1_{M_3}$ and θ. Also, $\beta_{i\,2}$ are less sensitive for all subgroups with respect to $\beta_{i\,1}$ and $\beta_{i\,3}$.

6.2.4   Endemic Case 4

In case $\mathscr{R}^1_0 > 1, \mathscr{R}^2_0 > 1$ and $\mathscr{R}^3_0 < 1$.the time series of this case with parameters is given as follows

$$\beta_{1\,1} = 2.9, \quad \beta_{2\,1} = 2.8, \quad \beta_{3\,1} = 2.7 \text{ and } \quad \mathscr{R}^1_0 = 1.15336146$$

$$\beta_{1\,2} = 2.05, \quad \beta_{2\,2} = 2, \quad \beta_{3\,2} = 1.98 \text{ and } \quad \mathscr{R}^2_0 = 1.193132544$$

$$\beta_{1\,3} = 1.97, \quad \beta_{2\,3} = 1.965, \quad \beta_{3\,3} = 1.96 \text{ and } \quad \mathscr{R}^3_0 = 0.428271438$$

The following plots in the Figure 6.8 represent the results.

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
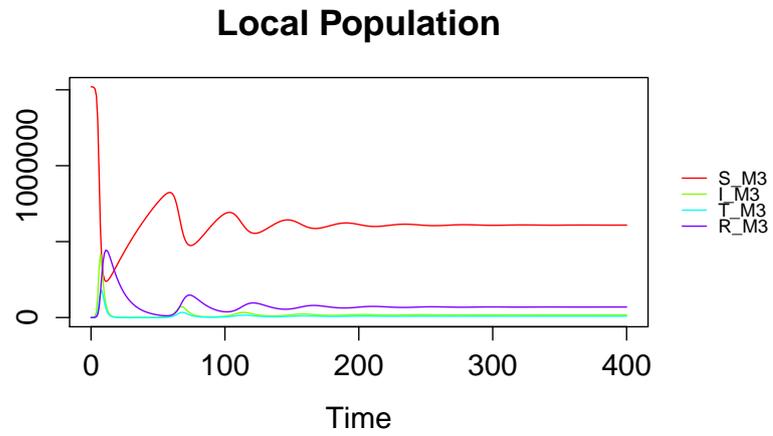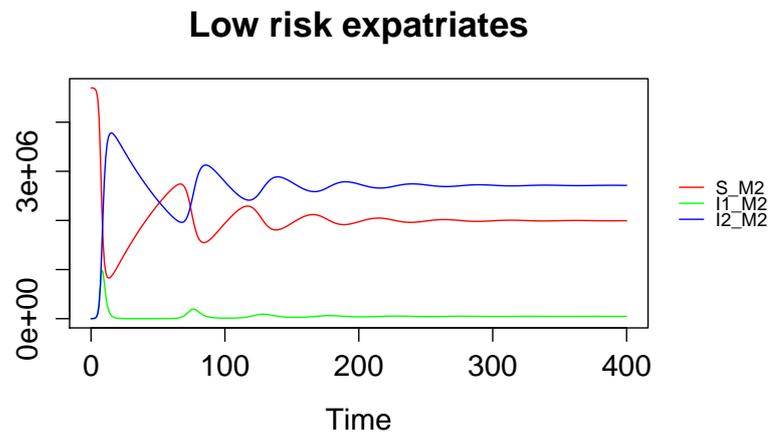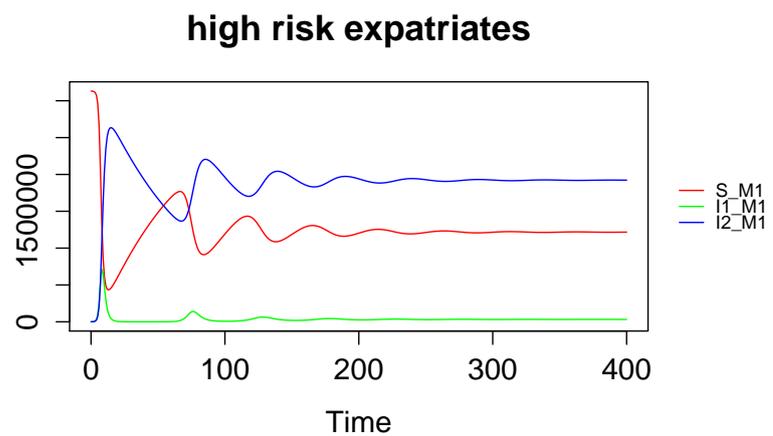
(c) high-risk population

Figure 6.7: The time series of the model compartments in case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 < 1$

For the dynamic of local compartment, It can be seen that, the susceptible subpopulation is reduced to 40.1% of it's size. The same figure 6.7a shows that the infected subpopulation low to 1.2% , the treated population is 0.6% and recovered 4.6%. This show low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 53.7%

For the high risk population, as it is shown in figure 6.7c, this subpopulation lowest drop of susceptible to 31.7% and aware infected subpopulation of 68.6% . Hence the biggest possible isolation and deportation. It can be seen also, very low number of the unaware infected people with 1.2% and no fatality due the high level of aware infected.

Finally, in the figure 6.7b, it is shown that this case is similar to the high risk subpopulation. In fact, there is a drop of the low risk susceptible subpopulation to almost 39.4%. The aware and unaware infected subpopulation are 60.9% and 1.2%, respectively with no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.8.

Sensitivity of infected locals
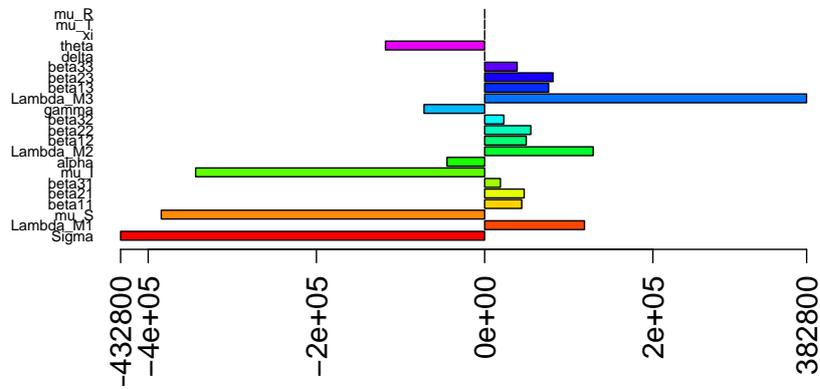
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
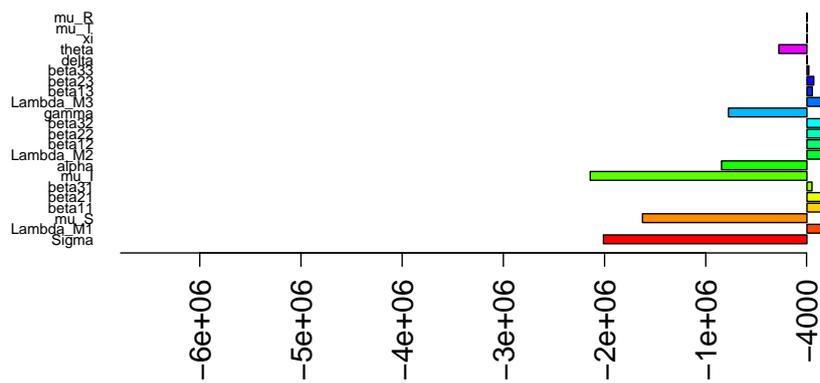
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
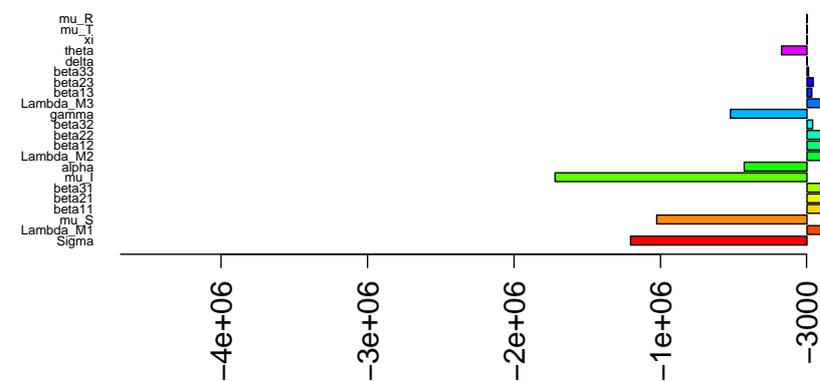
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.8: Sensitivity of the infected populations with respect to all parameters

The sensitivity analysis of case 4 for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the bar charts 6.8. It can be seen that, the type local sensitivity of $I_{M_1}^2$, $I_{M_2}^2$ with respect to parameters is similar but for $I_{M_3}^1$ is quite different. On the other hand , some parameters are not sensitive in these three variables for example, the parameters that are not sensitive for the variable $I_{M_3}^1$ are μ_R, μ_T and ξ , the parameter δ is not sensitive for all three variables. The parameters β_{ij} and Λ_{Mi}, (i,j=1,2,3)are making the variable sensitive in positive direction except β_{23} which is making the variable sensitive in positive direction in $I_{M_2}^2$. The parameters γ ,α , Σ and μ_I change the sensitivity of the parameter with respect to $I_{M_1}^2$ and $I_{M_2}^2$ compare to $I_{M_3}^1$. In fact $I_{M_1}^2$, $I_{M_2}^2$ are positively sensitive to γ, α and Σ, but $I_{M_3}^1$ is negatively sensitive to all of them.

6.2.5 Endemic Case 5

In case $\mathcal{R}_0^1 > 1$, $\mathcal{R}_0^2 < 1$ and $\mathcal{R}_0^3 > 1$. The time series of this case with parameters is given as follow

$$\beta_{11} = 2.7, \quad \beta_{21} = 2.6, \quad \beta_{31} = 2.5 \text{ and } \quad \mathcal{R}_0^1 = 1.07381929$$

$$\beta_{12} = 2.4, \quad \beta_{22} = 2.35, \quad \beta_{32} = 2.33 \text{ and } \quad \mathcal{R}_0^2 = 0.972630446$$

$$\beta_{13} = 2.32, \quad \beta_{23} = 2.31, \quad \beta_{33} = 2.3 \text{ and } \quad \mathcal{R}_0^3 = 1.027217544$$

and $\gamma = 1.2$ and $\theta = 0.35$ are modified

The plots of case 5 is presented in the Figure 6.10.

Local Population
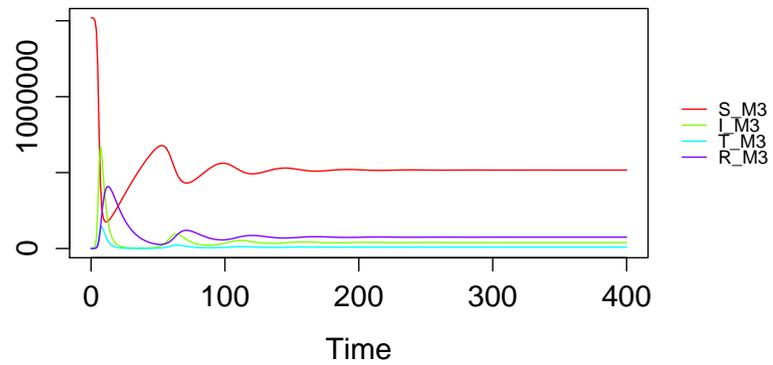
(a) Local population

Low risk expatriates
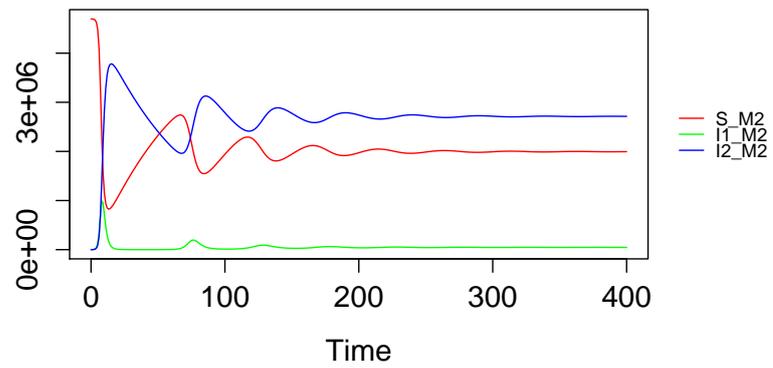
(b) low-risk population

high risk expatriates
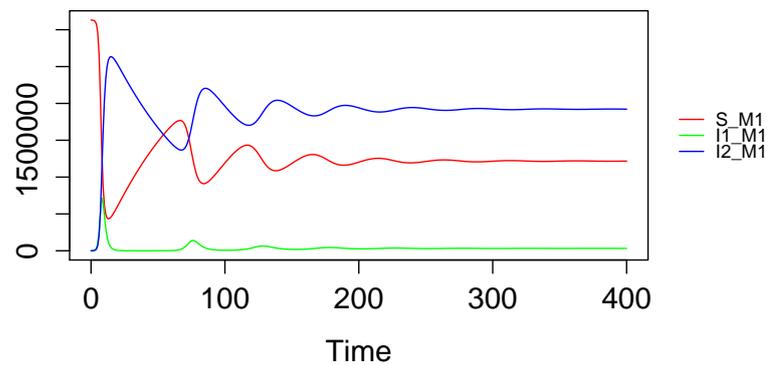
(c) high-risk population

Figure 6.9: The time series of the model compartments in case $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 > 1$

By analysing these plats, It is noticed that the susceptible subpopulation is reduced to 34% of it's size. The same figure 6.9a shows that the infected subpopulation is 2.6% , the treated population is 0.6% and recovery 4.9%. This show low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 57.9%

For the high risk population, as it is shown in figure 6.9c, This subpopulation the susceptible dropped to 31.3% and aware infected subpopulation to 69% . Hence the biggest possible isolation and deportation. It can be seen also, very low number of the unaware infected people and no fatality due the high level of aware infected.

Finally, in the figure 6.9b, it is shown that this case similar high risk subpopulation. In fact, we notice that there drop of the low risk susceptible subpopulation to almost 33.4%. The aware and unaware infected subpopulation are 67.2% and 8.8%, respectively with no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.10.

Sensitivity of infected locals
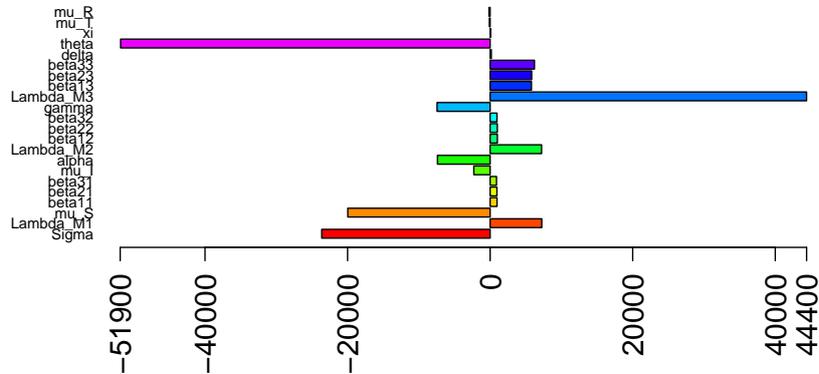
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters of the model

Sensitivity of infected low−risk expatriates
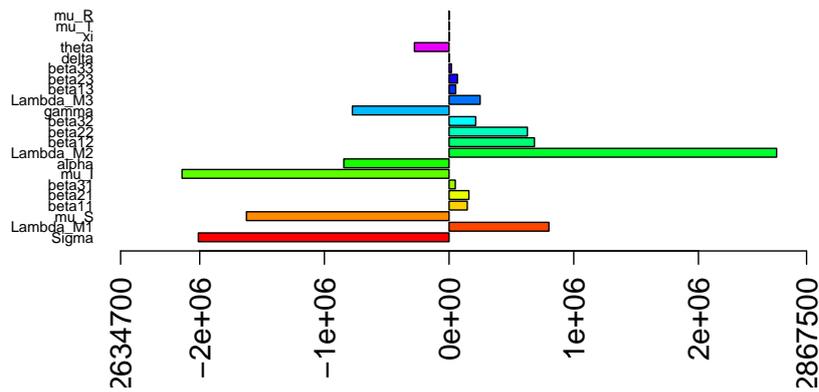
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
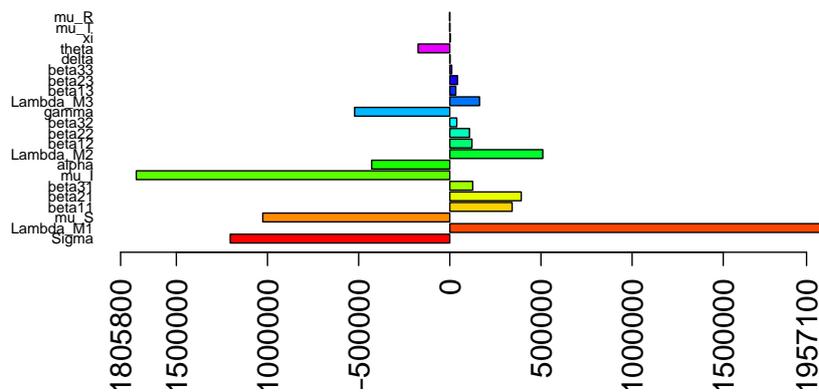
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.10: Sensitivity of the infected populations with respect to all parameters

The sensitivity analysis of this case show that, one can see from the local sensitivity of $I_{M_1}^2, I_{M_2}^2$ and $I_{M_3}^1$ that some parameters are always very sensitive for the three variables. On the other hand , some parameters are not sensitive in these three variables. The parameters that are not sensitive in these three plots are μ_R, μ_T, ξ and δ. This means μ_R and μ_T are of concern in this analysis. The parameters $\beta_{i\,j}$ and Λ_{Mi}, (i,j=1,2,3)are making the variable sensitive in positive direction. Λ_{M3} is very sensitive in $I_{M_3}^1$, Λ_{M2} is making the variable more sensitive in $I_{M_2}^2$ and Λ_{M1} is making the variable more sensitive in $I_{M_1}^2$. Also, there is no parameter change the sensitivity of the parameter in both direction.

6.2.6 Endemic Case 6

In relation to the case where $\mathscr{R}_0^1 < 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$. The time series of this case with parameters is presented.

$$\beta_{1\,1} = 3.55, \quad \beta_{2\,1} = 3.54, \quad \beta_{3\,1} = 3.535 \text{ and } \quad \mathscr{R}_0^1 = 0.98$$

$$\beta_{1\,2} = 3.53, \quad \beta_{2\,2} = 3.52, \quad \beta_{3\,2} = 3.515 \text{ and } \quad \mathscr{R}_0^2 = 2.099913278$$

$$\beta_{1\,3} = 3.51, \quad \beta_{2\,3} = 3.505, \quad \beta_{3\,3} = 3.5 \text{ and } \quad \mathscr{R}_0^3 = 1.008846093$$

and $\alpha = 1.2$ and $\theta = 0.55$ are modified.

The results shown in following plots in the Figure 6.12.

Local Population

(a) Local population

Low risk expatriates

(b) low-risk population

high risk expatriates
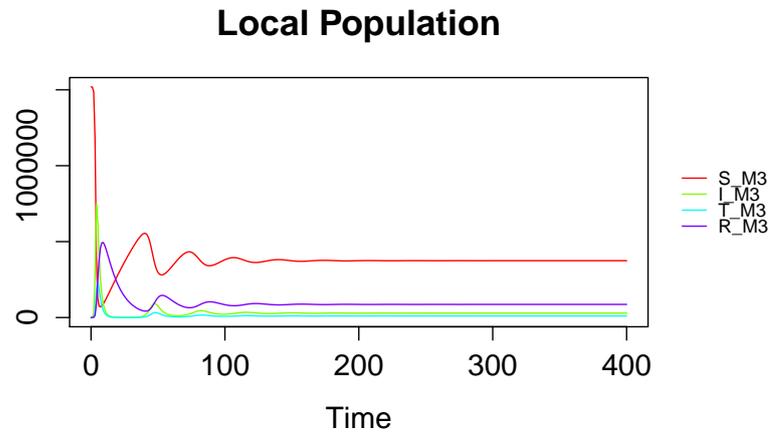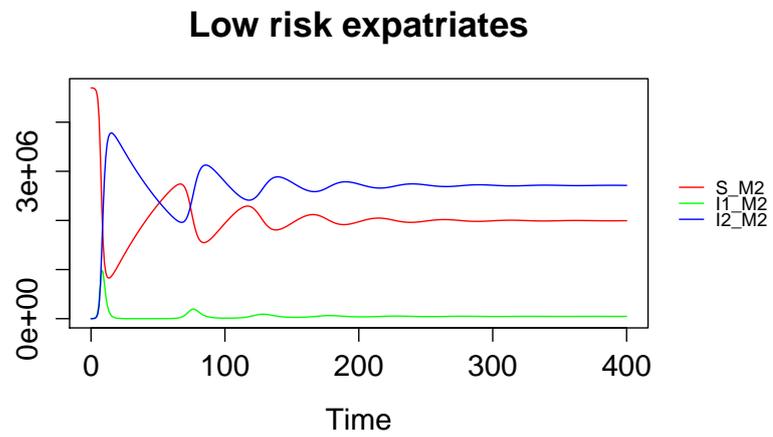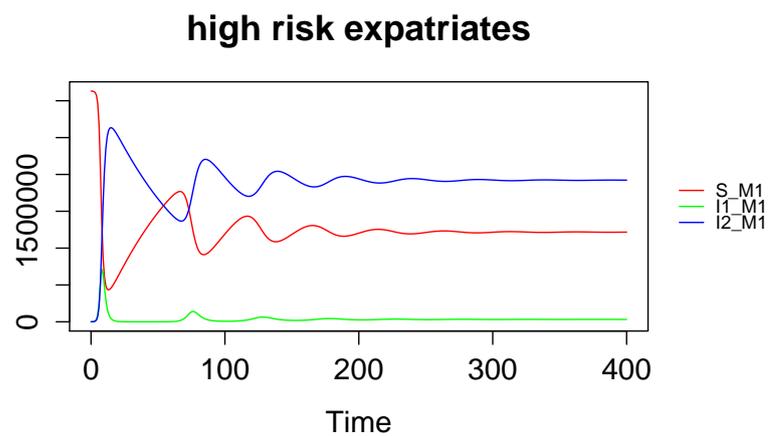
(c) high-risk population

Figure 6.11: The time series of the model compartments in case $\mathscr{R}_0^1 < 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$

These simulation show the dynamic of the different compartment in this case. One comment observation is that all the variable oscillate before they converge to an equilibrium points.

By look at the dynamic of local compartment, It is observed that, the susceptible subpopulation is reduced to 24.6% of it's size. The same figure 6.11a shows that the infected subpopulation is 1.9% , the treated population is 0.7% and recovery 5.7%. This show low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 67.1%

For the high risk population, as it is shown in figure 6.11c, the susceptible in this subpopulation dropped to 24.3% and aware infected subpopulation becomes 76.4% . Which will be isolated or deported. Also , there are very low number of the unaware infected people and no fatality due the high level of aware infected.

Finally, in the figure 6.11b, it is shown that this case is similar to high risk subpopulation by the drop of susceptible subpopulation to 24.4%. The aware infected subpopulation is also similar to the previous one with 75.8% and no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.12.

Sensitivity of infected locals
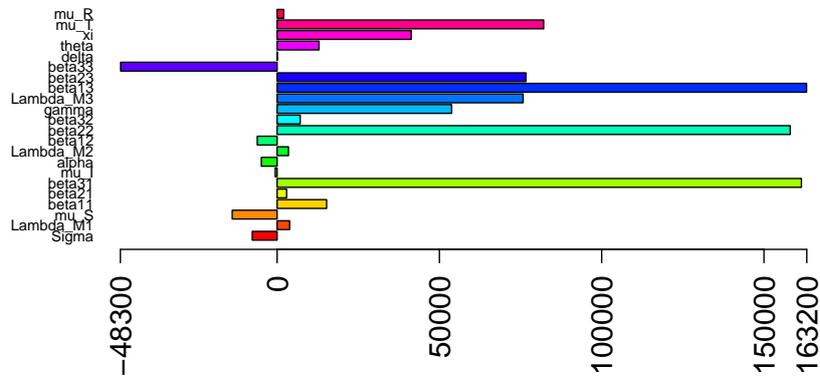
(a) Sensitivity of $I_{M_3}^1$ with respect to all parameters to the model

Sensitivity of infected low−risk expatriates
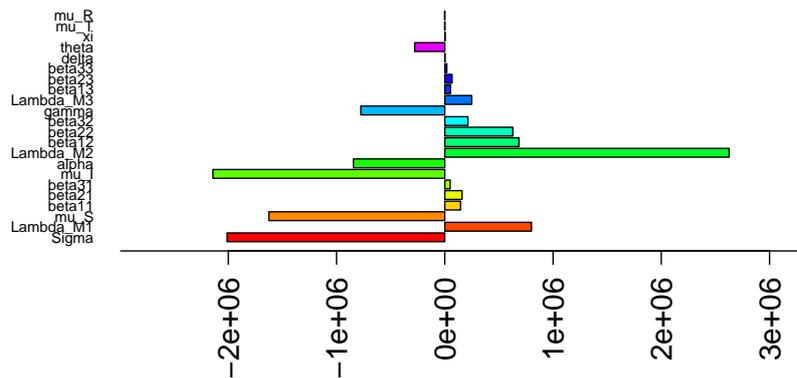
(b) Sensitivity of $I_{M_2}^2$ with respect to all parameters of the model

Sensitivity of infected high−risk expatriates
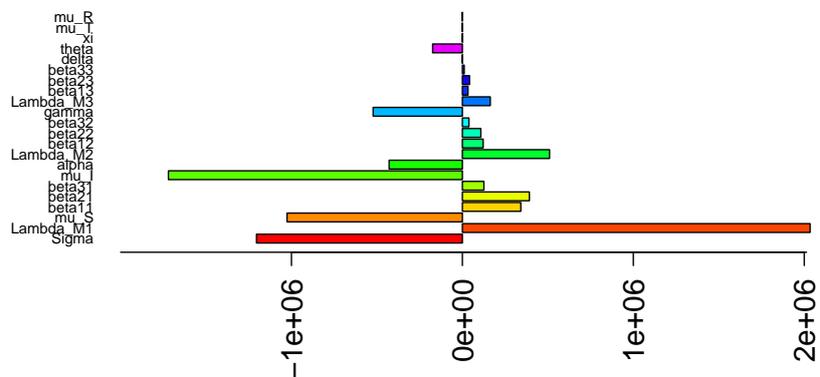
(c) Sensitivity of $I_{M_1}^2$ with respect to all parameters of the model

Figure 6.12: Sensitivity of the infected populations with respect to all parameters

The bar charts 6.12 shows the sensitivity analysis of $I^1_{M_3}$, $I^2_{M_2}$,$I^2_{M_1}$ with respect to all variable. It can be seen that, μ_T , Σ and $\beta_{1\,1}$ can change the sensitivity in both direction. For example, $I^2_{M_2}$ and $I^2_{M_1}$ are negatively sensitive to μ_T but $I^1_{M_3}$ is positively sensitive to it. On the other hand , $I^2_{M_2}$ and $I^2_{M_1}$ are positively sensitive to Σ but $I^1_{M_3}$ is negatively sensitive to this parameter. δ is not sensitive in all three variables so it is not concern in this analysis. Also, it can be observed that, the most sensitive parameters to local population are $\beta_{1\,3}$, $\beta_{2\,2}$ and $\beta_{2\,1}$ in the positive direction.

6.2.7   Endemic Case 7

The last case where $\mathscr{R}^1_0 > 1, \mathscr{R}^2_0 > 1$ and $\mathscr{R}^3_0 > 1$. The time series of this case with parameters is shown in this way

$$\beta_{1\,1} = 4.65, \quad \beta_{2\,1} = 4.64, \quad \beta_{3\,1} = 4.63 \text{ and } \quad \mathscr{R}^1_0 = 1.85$$

$$\beta_{1\,2} = 4.61, \quad \beta_{2\,2} = 4.605, \quad \beta_{3\,2} = 4.603 \text{ and } \quad \mathscr{R}^2_0 = 2.75$$

$$\beta_{1\,3} = 4.602, \quad \beta_{2\,3} = 4.6, \quad \beta_{3\,3} = 4.599 \text{ and } \quad \mathscr{R}^3_0 = 1.005$$

The results represented in following plots in the Figure 6.14.

Local Population

(a) Local population

Low risk immigrants

(b) low-risk population

high risk immigrants
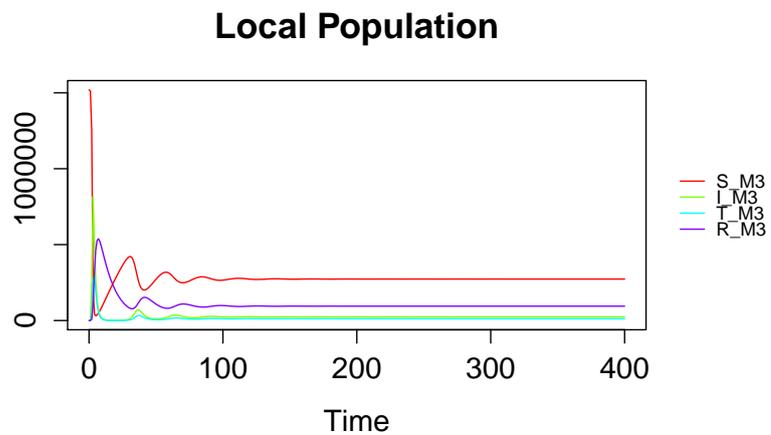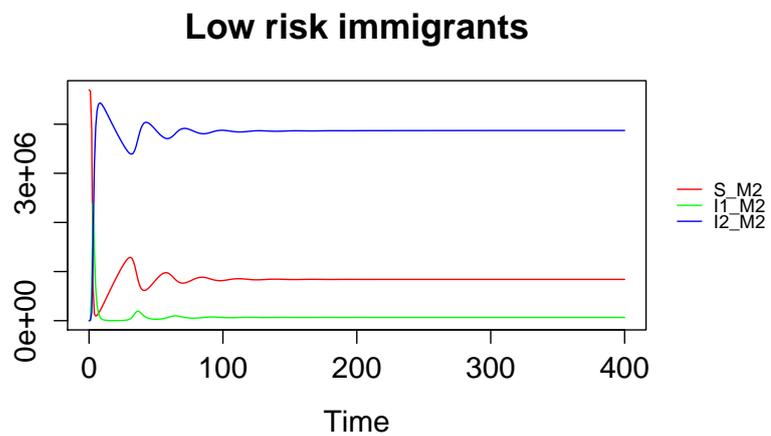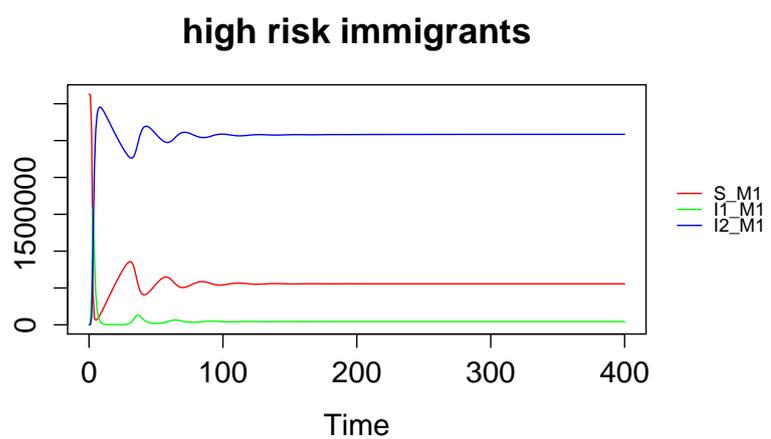
(c) high-risk population

Figure 6.13: The time series of the model compartments in case $\mathscr{R}_0^1 > 1, \mathscr{R}_0^2 > 1$ and $\mathscr{R}_0^3 > 1$

By looking at the dynamic of local compartment, It is noticed that, the susceptible subpopulation is reduced to 18% of it's size. The same figure 6.13a shows that the infected subpopulation is 1.6% , the treated population is 0.8% and recovery 6.2%. This show low persistence of the disease in local subpopulation, but with very high fatality in the local subpopulation of 73.5%

For the high risk population, as it is shown in figure 6.13c, the susceptible in subpopulation dropped to 17.8% and aware infected subpopulation becomes 82.6% . Which will be isolated or deported. Also , a very low number of the unaware infected people with 1.4% and no fatality due the high level of aware infected.

Finally, in the figure 6.13b, it is shown that this case similar to high risk subpopulation by the drop of susceptible subpopulation to similar number 17.9%. The aware infected subpopulation is also similar to the previous one with 82.4% and no fatality.

The sensitivity analysis of all the parameters, in this case, for $I_{M_3}^1$, $I_{M_2}^2$ and $I_{M_1}^2$ are given in the following bar charts 6.14.
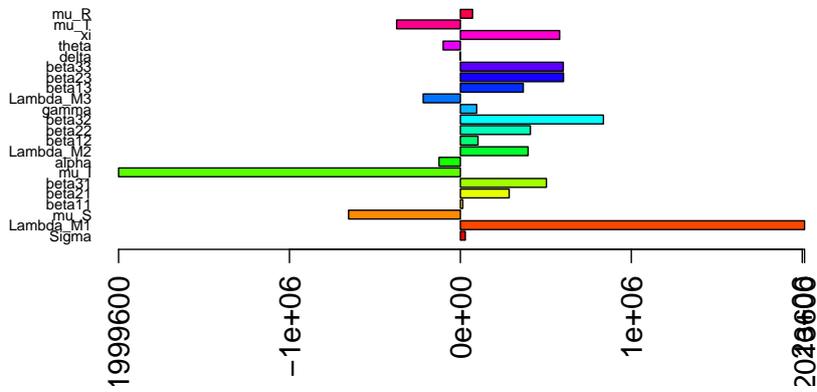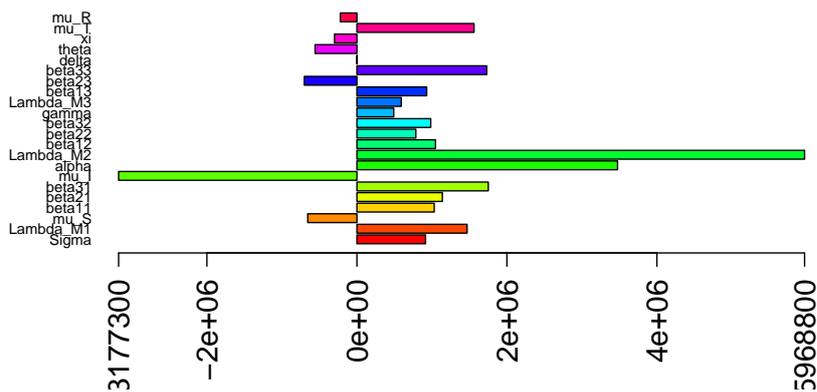
(a) Sensitivity of $I^1_{M_3}$ with respect to all parameters of the model

(b) Sensitivity of $I^2_{M_2}$ with respect to all parameters of the model
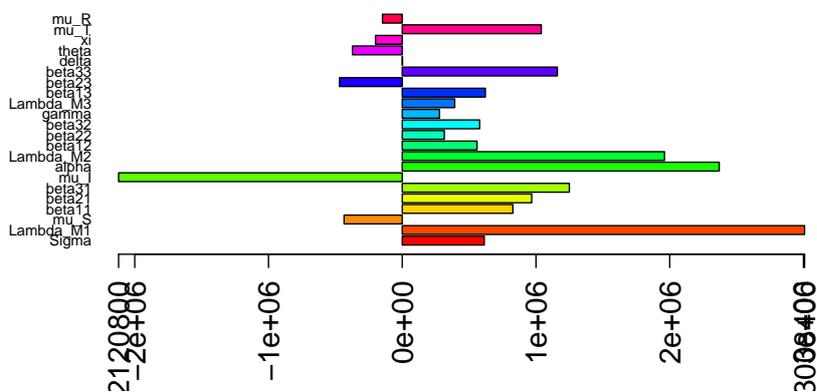
(c) Sensitivity of $I^2_{M_1}$ with respect to all parameters of the model

Figure 6.14: Sensitivity of the infected populations with respect to all parameters

In the sensitivity analysis of case 7 as shown in bar charts 6.14 it can be seen that, μ_T , μ_R and ξ can change the sensitivity in both direction. For example, $I_{M_2}^2$ and $I_{M_1}^2$ are negatively sensitive to μ_R and ξ but $I_{M_3}^1$ is positively sensitive to both of them. On the other hand, $I_{M_2}^2$ and $I_{M_1}^2$ are positively sensitive to μ_T but $I_{M_3}^1$ is negatively sensitive to this parameter. Some parameters are making the variable sensitive in a positive direction for example Σ and Λ_{Mi}, (i,j=1,2,3). Also, it can be noted that, μ_T is a sensitive parameters to all subpopulation to the negative direction and Λ_{M_1} is more sensitive in local population and high risk expatriates.

Chapter 7: Conclusion and Future Expectations

This study aims at investigating the impact of screening and measuring control in reducing the burden of diseases in the UAE. In order to do that, these policies and procedures of screening and measuring are contextualised. It is evident that UAE economy is one of the fastest growing economies in the world which relies mainly on expatriates. The vast majority of these expats are from regions that are considered *endemic*. To deal with this fact and the possibility of having imported communicable diseases in the UAE, the health authorities have implemented disease screening policies that require each person to be screened for specific communicable diseases before getting the residence. The screening must be repeated every three years to renew the residency, and in certain cases less than that.

Despite these policies, the UAE has faced cases of the spread of communicable diseases from expatriates of endemic regions. Chapter 2, has already shown data of disease prevalence (HIV and TB) cases of expatriates that were identified in the screening prior to coming to the UAE as well as cases of those who had passed the home screening but were found to be infected in the second screening after entering the UAE (see figures (2.4 and 2.5). Moreover, in the same chapter cases of HIV and TB from expatriates were detected at the tie of visa renewal though they had passed through at the time of first entry. That is revealed (see figures (2.7 and 2.8). In addition, in recent years the cases of imported malaria, cholera, and TB among expatriates from endemic regions are evident hand in hand with the pre-mentioned cases.

Facts regarding these cases invite and emergent reviser of the existing policies in the UAE. That is to say screening procedures need a certain kind of enhance-

ment. This justifies the importance of investigating these polices via mathematical modeling to arrive at possible solutions.

There is a wealth of mathematical models that have studied the spread of communicable diseases in different populations. These studies have focused either on the dynamics of diseases or the impact of some control measures on protecting the population. However, the studies on the use of the screening as a measure to protect the public from the spread of the communicable diseases are few. chapter 3, presents the important of the existing models that have focused on screening. The papers that studied HIV, other STD's, and TB are selected carefully to serve the purpose of this research. These papers are models defined for a specific country (Canada, Cameron, India) and therefore are not applicable to the UAE due to the demography of the population (as explained earlier) and the nature of health policies of this country.

Therefore, in Chapter 4, a core model that takes into consideration the demography and health policy specific to the UAE is suggested.

The model is composed of a set of differential equations that considers three types of subpopulation which are: Local subpopulation (UAE citizens), High-risk subpopulation (expatriates from endemic regions) , and Low-risk subpopulation (expatriates not from endemic regions). In order to examine these cases in line with the health policy of the country, the expatriates populations have been subdivided into susceptible, unaware and aware infected. The aware are those who are screened and detected, hence deported or, as in few cases, isolated. The local population is subdivided into susceptible,infected, treated and recovered.

First, the basic properties of the model such as positivity of the variables and the boundedness is presented. Next, using the basic reproduction number $\mathscr{R}_0$ stud-

ied the conditions under which the disease can not be established and if $\mathscr{R}_0 < 1$ then there is only the disease free equilibrium and it is locally asymptotically stable. On the other hand, it is noticed that if $\mathscr{R}_0 > 1$ the disease free equilibrium is unstable. In this case, a result of possible existence of the endemic equilibrium when $\mathscr{R}_0 > 1$ is given. This finding led to consider all the possible cases in which $\mathscr{R}_0 > 1$. By using the basic reproduction number related to each subpopulation, seven possible scenarios that give this case are revealed.

In order to illustrate these findings a model for two types of diseases that were discussed in chapter 2: HIV and TB ares considered. These two diseases are selected just to set and example, however; the model may be generalised to include more diseases.

In chapter 5, the demographic data of the UAE is used to estimate the parameters of HIV from the literature in order perform the time series simulations. These simulations showed the outcome of HIV if it has spreaded at the endemic level. Therefore, the simulation covers the seven possible scenarios depending on the level of endemicity.

By calculating the fatality of each case, it is found that the fatality does not depend on the endemicity per subpopulation. In fact, if basic reproduction number per each subpopulation is above one, the fatality of subpopulation are comparable. More precisely, the fatality for each subpopulations varies as follows:

- For the local subpopulations between 15.8% and 22.5%.

- For the high-risk subpopulations between 17.1% and 22.6%.

- For the low-risk subpopulations between 17% and 23.2%.

The local sensitivity analysis of all seven scenarios for the infected compart-

ments of each subpopulation, showed that if $\mathscr{R}_0^i > 1$ for $i = 1, 2, 3$ then screening parameters for each subpopulation γ, θ and α make the infected compartment very sensitive. It is also observed that the number of birth per endemic subpopulation makes the infected compartment correspondent very sensitive.

Finally, in chapter 6, the estimated data from the UAE and from the literature in order to perform the time series simulation for the seven cases of the endemicity is used. The fatality calculation showed that the local subpopulation is the one that will dramatically suffer from TB in the UAE. Although the disease will persist in very low levels in this subpopulation, the screening as a measure to protect the public health, may not be useful in relation to the local population due to the high fatality among this population (44.3% when only $\mathscr{R}_0^1 > 1$ and increases to 73.5% when $\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 > 1$, $\mathscr{R}_0^3 > 1$.

This discussion and analysis concluded to the fact that the screening measure might work for HIV, but for TB it should be enhanced either by constant screening of the high risk population or at least by screening them in the UAE after each visit to their respective countries. The arrival from specific areas, with high rate of infection, screening should be different than with those coming from low rate infection. For example arrival from marked areas their screening should be done twice: by their arrival and then after three months (incubation is usually 3 months).

As a continuation of this study, the condition under which an endemic equilibrium exists needs further study. This might give the conditions with respect to $\mathscr{R}_0^i$. Also the local and global stability of the equilibrium points need to be sorted out.

Furthermore, the structure of the model should be extended to more com-

partments in order to include other diseases that the UAE might face. By using real data, if available, this step can be augmented.

Bibliography

[1] *Global health observatory.*

[2] R. N. A, A. TRIPATHI, AND D. SHARMA, *A nonlinear aids epidemic model with screening and time delay*, Scientific Research and Essays, 217 (2011), pp. 4416–4426.

[3] D. H. AUTHORITY, *Prevention of communicable diseases law no 27-1981*, 2014.

[4] M.-C. BOILY, C. LOWNDES, AND M. ALARY, *The impact of hiv epidemic phases on the effectiveness of core group interventions: insights from mathematical models*, Sexually Transmitted Infections, 78 (2002), pp. 78–90.

[5] S. BOWONG AND J. KURTHS, *Parameter estimation based synchronization for an efidemic model with application to tuberculosis in cameroon*, ELSEVIER, 374 (2010), pp. 4496–4505.

[6] R. BROOKMEYER, *Incubation period of infectious diseases. in encyclopedia of biostatistics*, Edited by Armitage P, Colton T. New York: Wiley, 2 (1998), pp. 2011–2016.

[7] C.CASTILLO-CHAVEZ AND Z. FENG, *Mathematical models for the disease dynamics of tuberculosis*, Cornell University, Ithaca, NY, 14853, (1996), p. .

[8] H. A. A. DHABI, *Communicable diseases bulletin, first quarter*, 2012.

[9] M.-N. DP, R. S, E. R, AND D. P, *Parameter identification in a tuberculosis model for cameroon.*, Plos One [PLoS One], 10 (2015), p. e0120607.

[10] P. V. D. DRIESSCHE AND J. WATMOUGH, *Reproduction numbers and subthreshold endemic equilibria for compartmental models of disease transmission*, Math. Biosci, 180 (2002), pp. 221–236.

[11] P. V. D. DRIESSCHE AND J. WATMOUGH, *Further notes on the basic reproduction number*, Mathematical Epidemiology, 1945 (2008), pp. 159–178.

[12] K. EXECUTIVE BOARD OF THE HEALTH MINISTER'S COUNCIL FOR G.C.C STATES, *Expatriate workers - check-up project*, 2010.

[13] C. W. FACTBOOK, *Cia world factbook*, 2014.

[14] FOLEY AND JANET, *Mathematical models in population biology and epidemiology*, Ecology, 83 (2002), p. 294.

[15] I. H, DE BOER RJ, S. K, M. S, M. N, K. Y, A. K, AND I. S, *Improving the estimation of the death rate of infected cells from time course data during the acute phase of virus infections: application to acute hiv-1 infection in*

a humanized mouse model, Theoretical Biology and Medical Modelling, 11 (2014), p. 22.

[16] S. HOVE-MUSEKWA AND F. NYABADZA, *The dynamics of an hiv/aids model with screened disease carriers*, Computational and Mathematical Methods in Medicine, 10 (2009), pp. 287–305.

[17] W. J, D. R, G. M, AND R. JV, *Sensitivity analysis of infectious disease models: methods, advances and their application*, Journal Of The Royal Society, Interface / The Royal Society, pp. 2012–1018.

[18] Z. JIA, S. CHENG, AND X. JIA, *A mathematical model for evaluating tuberculosis screening strategies*, Journal Of Evidence-Based Medicine, 4 (2011), pp. 48–52.

[19] D. KIRSCHNER AND A. PERELSON, *A model for the immune system response to hiv: Azt treatment studies*, Mathematical Population Dynamics, 1 (1993), pp. 295–310.

[20] S. KURLANDER, A. ANANI, A. R. QURESHI, M. MAKARY, AND C. FULLER, *Healthcare regulation in the united arab emirates, 2013 update*, 2013.

[21] H. NISHIURA, *Early efforts in modeling the incubation period of infectious diseases with an acute course of illness*, Emerging Themes in Epidemiology, 4 (2007), p. 2.

[22] W. H. ORGANIZATION, *Global health observatory (gho) data, hiv/aids*, 2014.

[23] R. SAFIEL, E. S. MASSAWE, AND D. O. MAKINDE, *Modelling the effect of screening and treatment on transmission of hiv/aids infection in a population*, American Journal of Mathematics and Statistics, 2 (2012), pp. 75–88.

[24] R. SAFIEL, E. S. MASSAWE, AND D. O. MAKINDE, *Modelling the effect of screening and treatment on transmission of hiv/aids infection in a population*, American Journal of Mathematics and Statistics, 2 (2012), pp. 75–88.

[25] I. SHABANI, E. S. MASSAWE, AND O. D. MAKINDE, *Modelling the effect of screening on the spread of hiv infection in a population with variable inflow of infective immigrants*, Scientific Research and Essays, 6 (2011), pp. 4397–4405.

[26] I. SHABANI, E. S. MASSAWE, AND O. D. MAKINDE, *Modelling the effect of screening and treatment on transmission of hiv/aids infection in a population with variable inflow of infective immigrants*, American Journal of Mathematics and Statistics, 2 (2012), pp. 75–88.

[27] SHAROMI, O. PODDER, C. N. GUMEL, A. B., AND SONG, *Mathematical analysis of the transmission dynamics of hiv/tb coinfection in the presence of treatment*, Mathematical biosciences and engineering, pp. 145–174.

[28] P. TC AND B. SM, *Quantifying the intrinsic transmission dynamics of tuberculosis.*, Theoretical Population Biology, 54 (1998), pp. 117–32.

[29] H. R. THIEME, *Modelling in population biology*, Pronceton University Press, Princeton, (2003), p. .

[30] A. TRPATHI, R. NARESH, AND D. SHARMA, *Modelling the effect of screening of unaware infectives on the spread of hiv infection*, Scientific Research and Essays, 184 (2007), pp. 1053–1068.

[31] A. R. TUITE, D. FISMAN, AND S. MISHRA, *Screen more or screen more often? using mathematical models to inform syphilis control strategies*, BMC Public Health, 13 (2013).

[32] M. B. VARUGHESE, D. LANGLOIS-KLASSEN, R. LONG, AND M. LI, *Preventing tuberculosis in the foreign-born population of canada: a mathematical modelling study*, International Journal of Tuberculosis and Lung Disease, 18 (2013), pp. 405–412.

[33] WHO-EM/PME/002/E, *Country cooperation strategy for who and the united arab emirates 2012âĂŞ2017*, 2012.

[34] M. XIRIDOU, H. J. VRIEND, A. K. LUGNER, J. WALLINGA, J. S. FENNEMA, J. PRINS, S. GEERLINGS, B. RIJINDERS, M. PRINS, H. J. DE VRIES, M. J. POSTMA, M. G. POSTMA, M. G. VAN VEEN, M. F. S. VAN DER LOEFF, AND M. A. VAN DER SANDE, *Modelling the impact of chlamydia screening on the transmission of hiv among men who have sex with men*, BMC Infectious Diseases, 13 (2013).

Appendix

For the numerical simulation the R foundation for statistical computing is used , R version 3.1.1 (2014-07-10) and RStudio Version 0.98.1103 © 2009-2014 RStudio, Inc. They are practical softwares to calculate the basic reproduction number R_0 and to analyze the local sensitivity.

A Sample R Code for the HIV Simulations

In this section an example of the simulations for the case 1 for HIV ($\mathscr{R}_0^1 > 1$, $\mathscr{R}_0^2 < 1$ and $\mathscr{R}_0^3 < 1$) is represented as follows:

The simulation require the installation of the following packages

```
install.packages("FME")
library("FME")
install.packages("deSolve")
library("deSolve")
```

Time Series Simulation

```
pars1<- list(Sigma=9350000,Lambda_M1=44756, mu_S=0.01429,
beta11=0.125, beta21=0.11,beta31=0.105,mu_I= 0.02,
alpha=0.015 ,Lambda_M2=67134,   beta12=0.095, beta22=0.092,
 beta32=0.905,gamma=0.03,Lambda_M3=21721, beta13=0.09,
 beta23=0.082, beta33=0.081,  delta=0.01,
 theta=0.01 , xi=0.0 , mu_T=0.02,  mu_R=0)
"Sigma","Lambda_M1", "mu_S", "beta11","beta21","beta31",
"mu_I","alpha", "Lambda_M2", "beta12","beta22"," beta32",
"gamma","Lambda_M3", "beta13", "beta23"," beta33", "delta",
" theta" ," xi" ," mu_T","mu_R"
```

```r
HIV<-function(pars1,times=seq(0,400,by=1)){
derivs<-function(t,state,pars1){
with(as.list(c(state,pars1)),{
dS_M1 <- Lambda_M1 - mu_S*S_M1
- (beta11*S_M1*I1_M1+beta21*S_M1*I1_M2+beta31*S_M1*
I1_M3)/Sigma
 dI1_M1 <- (beta11*S_M1*I1_M1+beta21*S_M1*I1_M2+
 beta31*S_M1*I1_M3)/Sigma - (mu_I+alpha)*I1_M1
 dI2_M1<- alpha*I1_M1 -mu_I*I2_M1
 dS_M2 <- Lambda_M2 - mu_S*S_M2
- (beta12*S_M2*I1_M1+beta22*S_M2*I1_M2+beta32*S_M2*
I1_M3)/Sigma
 dI1_M2 <- (beta12*S_M2*I1_M1+beta22*S_M2*I1_M2+
 beta32*S_M2*I1_M3)/Sigma - (mu_I+gamma)*I1_M2
dI2_M2<- gamma*I1_M2 -mu_I*I2_M2
dS_M3 <- Lambda_M3 - mu_S*S_M3
- (beta13*S_M3*I1_M1+beta23*S_M3*I1_M2+beta33*S_M3*
I1_M3)/Sigma + delta*R_M3
 dI1_M3 <- (beta13*S_M3*I1_M1+beta23*S_M3*I1_M2+
 beta33*S_M3*I1_M3)/Sigma  - (mu_I+theta)*I1_M3
dT_M3<- theta*I1_M3 -(xi+mu_T)*T_M3
 dR_M3<- xi*T_M3-(delta+mu_R)*R_M3
return(list(c(dS_M1, dI1_M1,dI2_M1,dS_M2, dI1_M2,dI2_M2,dS_M3,
 dI1_M3,dT_M3, dR_M3)))    })  }
state<-c(S_M1=3131600, I1_M1=300,I2_M1=100,S_M2=4697700,
I1_M2=210,I2_M2=90,S_M3=1519900, I1_M3=70,T_M3=30, R_M3=0)
return(ode(y=state,times=times,func=derivs,parms=pars1))   }
out<-HIV(pars1)
out
par(mfrow=c(2,2))
```

```
plot(out[,1],out[,2],main="Susceptible high-risk expatriate",
 ylab="S_M1", xlab="time",type="l",col="green")
 plot(out[,1],out[,3],main="Infected Unawared
 high-risk expatriate",
  ylab="I1_M1", xlab="time",type="l",col="blue")
 plot(out[,1],out[,4],main="Infected Awared
 high-risk expatriate",
 ylab="I2_M2", xlab="time",type="l",col="red")
plot(out[,1],out[,2]+out[,3]+out[,4],main="total
 high-risk expatriate",
ylab="M2", xlab="time",type="l",col="black")
 par(mfrow=c(2,2))
plot(out[,1],out[,5],main="Susceptible low-risk
 expatriate",
 ylab="S_M2", xlab="time",type="l",col="green")
 plot(out[,1],out[,6],main="Infected Unawared
 low-risk expatriate",
  ylab="I1_M_2", xlab="time",type="l",col="blue")
 plot(out[,1],out[,7],main="Infected Awared
 low-risk expatriate",
  ylab="I2_M_2",xlab="time",type="l",col="red")
 par(mfrow=c(2,2))
 plot(out[,1],out[,8],main="Susceptible locals", ylab="S_M3",
  xlab="time",type="l",col="green")
 plot(out[,1],out[,9],main="Infected locals", ylab="I_M3",
 xlab="time",type="l",col="blue")
 plot(out[,1],out[,10],main="treated locals", ylab="T_M3",
  xlab="time",type="l",col="red")
 plot(out[,1],out[,11],main="treated locals", ylab="R_M3",
 xlab="time",type="l",col="purple")
par(mar=c(5.1, 4.1,4.1, 6.1), xpd=TRUE)
```

```r
 SHR<- out[,2]

I1HR<- out[,3]

I2HR<- out[,4]

COLORS <- rainbow(3)

HR<- data.frame(SHR=SHR,I1HR=I1HR,I2HR=I2HR)

 matplot(out[,1],HR, type = "l", xlab="Time",

          ylab="", main="high risk expatriates",

          col = COLORS,lty=1)

 legend("right",c("S_M1", "I1_M1","I2_M1"),

         bty = "n", col = COLORS,cex = 0.6,lty=1,

         inset=c(-0.2,0))

par(mar=c(5, 4, 4, 2) + 0.1)

par(mar=c(5.1, 4.1, 4.1, 6.1), xpd=TRUE)

SLR<- out[,5]

I1LR<- out[,6]

I2LR<- out[,7]

COLORS <- rainbow(3)

LR<- data.frame(SLR=SLR,I1LR=I1LR,I2LR=I2LR)

matplot(out[,1],LR, type = "l", xlab="Time",

        ylab="", main="Low risk expatriates",

        col = COLORS,lty=1)

legend("right",c("S_M2", "I1_M2","I2_M2"),

        bty = "n", col = COLORS,cex = 0.6,lty=1,

        inset=c(-0.2,0))

par(mar=c(5, 4, 4, 2) + 0.1)

par(mar=c(5.1, 4.1, 4.1, 6.1), xpd=TRUE)

SL<- out[,8]

IL<- out[,9]

TL<- out[,10]

RL<- out[,11]

COLORS <- rainbow(4)
```

```
LP<- data.frame(SL=SL,IL=IL,TL=TL, RL=RL)

matplot(out[,1],LP, type = "l", xlab="Time",

        ylab="", main="Local Population",

        col = COLORS,lty=1)

legend("right",c("S_M3", "I_M3","T_M3", "R_M3"),

       bty = "n", col = COLORS,cex = 0.6,lty=1,

       inset=c(-0.2,0))

par(mar=c(5, 4, 4, 2) + 0.1)
```

Sensitivity Simulation

```
SnsI1_M3<-sensFun(func=HIV,parms=pars1, sensvar="I1_M3",

varscale=1)

SnsI1_M3

plot(SnsI1_M3,legpos="NULL",main="Sensitivity for

Infected Locals")

legend("right",legend=c("Sigma","Lambda_M1", "mu_S",

"beta11","beta21","beta31","mu_I","alpha","Lambda_M2",

 "beta12","beta22"," beta32","gamma","Lambda_M3","beta13",

 "beta23"," beta33", "delta"," theta",

" xi" ," mu_T","mu_R"),bty = "n",col = rainbow(22),

cex = 0.55,lty=1,inset=c(-0.5,0), ncol=2)


plot(SnsI1_M3)

summary(SnsI1_M3)

IM3<- c(-341059.699, 82699.308, -307760.226, 19806.677,

28298.342,7063.359,-347375.495,-33537.529, 107819.265,

11560.403,20000.566,42250.555, -62241.749, 405815.053,

77938.867,101031.990,33108.988,0,-130582.151,0,0,0)

par(las=2)

barplot(IM3,names=c("Sigma","Lambda_M1", "mu_S", "beta11",
```

```
"beta21","beta31","mu_I","alpha","Lambda_M2", "beta12",

"beta22"," beta32","gamma","Lambda_M3","beta13",

"beta23"," beta33", "delta"," theta" ," xi" ," mu_T","mu_R"),

  col = rainbow(22), xlim=c(-347375.5,405815.1),horiz=TRUE,

  cex.names=0.45,main="Sensitivity of infected locals" )

axis(1, at=c(-347400,405900),by=100)

pairs(SnsI1_M3)

SnsI2_M2<-sensFun(func=HIV,parms=pars1, sensvar="I2_M2",

varscale=1)

SnsI2_M2

plot(SnsI2_M2,legpos="NULL",main="Sensitivity for Infected

low risk")

legend("right",legend=c("Sigma","Lambda_M1", "mu_S",

"beta11","beta21","beta31","mu_I","alpha","Lambda_M2",

 "beta12","beta22"," beta32","gamma","Lambda_M3","beta13",

 "beta23"," beta33","delta"," theta" ,

" xi" ," mu_T","mu_R"),bty = "n",col = rainbow(22),

 cex = 0.55,lty=1,inset=c(-0.5,0), ncol=2)

plot(SnsI2_M2)

summary(SnsI2_M2)

IM2<- c( -828042.49, 108214.90,  -721162.41, 43468.15,

 67934.91, 14004.09,-1817207.31, -41393.03,  1352665.96,

 73558.67,115900.18,297531.00,474558.11, 229711.32,

 74202.59,114167.05,27275.96,0,-67639.74,0,0,0)

par(las=2)

barplot(IM2,names=c("Sigma","Lambda_M1", "mu_S", "beta11",

"beta21","beta31","mu_I","alpha","Lambda_M2", "beta12",

"beta22"," beta32","gamma","Lambda_M3", "beta13", "beta23",

" beta33", "delta"," theta"," xi" ," mu_T","mu_R"),

col = rainbow(22), xlim=c(-1817207,1352666),

horiz=TRUE,cex.names=0.5,
```

```
main="Sensitivity of infected low-risk expatriates" )

axis(1, at=c(-1817207,1352666),by=100)

pairs(SnsI2_M2)

SnsI2_M1<-sensFun(func=HIV,parms=pars1, sensvar="I2_M1",

varscale=1)

SnsI2_M1

plot(SnsI2_M1,legpos="NULL", main="Sensitivity for

Infected high risk")

legend("right",legend=c("Sigma","Lambda_M1", "mu_S",

 "beta11","beta21","beta31","mu_I","alpha","Lambda_M2",

 "beta12","beta22"," beta32","gamma","Lambda_M3","beta13",

  "beta23"," beta33", "delta"," theta" ," xi" ," mu_T","mu_R"),

  bty = "n",col = rainbow(22), cex = 0.55,

lty=1,inset=c(-0.5,0), ncol=2)

plot(SnsI2_M1)

summary(SnsI2_M1)

IM1<- c(-413154.196,543186.296,-366757.785, 91337.292,

116348.268,35904.278,-796993.540,  263297.835,  122018.237,

15863.339, 27875.535,56862.891,-69679.457, 61650.944,

23511.218,37109.780,8341.627,0,-17716.370,0,0,0)

par(las=2)

barplot(IM1,names=c("Sigma","Lambda_M1", "mu_S", "beta11",

"beta21","beta31","mu_I","alpha","Lambda_M2", "beta12",

"beta22"," beta32","gamma","Lambda_M3","beta13", "beta23",

" beta33", "delta"," theta" ," xi" ,

" mu_T","mu_R"),col = rainbow(22), xlim=c(-796993.5,543186.3),

horiz=TRUE,cex.names=0.45,

main="Sensitivity of infected high-risk expatriates")

axis(1, at=c(-797000,543200),by=100)

pairs(SnsI2_M1)
```