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Modified Surrogate Cutting Plane Algorithm (MSCPA) for Integer Linear Programming Problems

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MODIFIED SURROGATE CUTTING PLANE ALGORITHM (MSCPA) FOR INTEGER LINEAR PROGRAMMING PROBLEMS

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تعديل خوارزمية قطع المستوي لحل مسائل البرمجة الخطية الصحيحة

ملخص

هذا العمل يقدم خوارزمية جديدة لحل مشاكل البرمجة الخطية الصحيحة. يمكن أن تساعد هذه الطريقة في تحسين تعقيد حل مثل هذة المشاكل ، من خلال تقليل الحساب . تتمثل مزايا الطريقة المقترحة في تقليل الوقت المخصص للحل وتقليل التعقيد الخوارزمي.تم مناقشة بعض الأمثلة العددية المحددة لتوضيح صحة وتطبيق الطريقة المقترحة. تتم مقارنة النتائج العددية مع حل مشاكل البرمجة الخطية الصحيحة باستخدام طريقة قطع المستوي (طريقة Gomory).

Abstract

This work concerned with introducing a new algorithm for solving integer linear programming problems. The improved algorithm can help by decreasing a calculation the complexity of these problems, an advantages of the proposed method are to reduce the solution time and to decrease algorithmic complexity. Some specific numerical examples are discussed to demonstrate the validity and applicability of the proposed method. The numerical results are compared with the solution of integer linear programming problems by using cutting plane method (Gomory method).

1. INTRODUCTION

It is well known that Linear Programming (LP) is a technique for the optimization (maximization or minimization) of a linear objective function subject to linear equality and inequality, the feasible region of the LP model is continuous in the sense that each variable is restricted to over a continuous interval. If variables are further restricted to integer values, it becomes an integer (ILP) model. Branch-and-bound and cutting-plane methods have been principle tools for solving ILP models for about fifty years [1].

In 1975 the Soviet Leonid Kantorovich and the Dutch American T. C. Koopmans shared the Nobel Prize in Economics for their work, at the end of the 1930's, on formulating and solving LP problems. A LP algorithm determines a point of the LP polytope, where the objective function takes its optimal value, if such a point exists. In 1947 George B. Dantzic invented the SIMPLEX algorithm that for the first time efficiently tackled the LP problem in most cases. Note that earlier, in 1941, Frank Lamen Hitchcock gave a very similar to the SIMPLEX algorithm solution for the Transportation Problem. In 1948 Dantzic, adopting a conjecture of John von Neuman, who worked on an equivalent problem in Game Theory, provided a formal proof of the theory of Duality [2].The name linear integer programming refers to the class of combinatorial constrained optimization problems with integer variables, where the objective function is a linear function and the constraints are linear inequalities. This formulation includes also equality constraints, because each equality constraint can be represented by means of two inequality constraints. A wide variety in logistics, economics, social science and politics can be formulated as linear integer optimization problems. The combinatorial problems, like the knapsack-capital budgeting problem, warehouse location problem, travelling salesman problem, decreasing costs and machinery selection problem, network and graph problems, such as maximum flow problems, set covering problems, matching problems, weighted matching problems, spanning trees problems and many scheduling problems can also be solved as linear integer optimization problems(Some optimization problems, having nonlinear objective functions and linear constraints can be transformed in ILP optimization problems by simple approximation of the corresponding nonlinear functions by piecewise linear functions . Solving integer programming optimization problems, that is, finding an optimal solution to such kind of problems, can be a difficult task. To solve a non-convex integer programming problem could be an algorithmically unsolvable task [3].

In this work, a new algorithm was presented to solve the problems of integer linear programming ILP This method can help to improve, by decreases a calculation complexity of solution of integer linear programming problem , an advantages of the proposed method are the reduced time for solution and decreased algorithmic complexity.

2. Integer Linear Programming [4,5,6,7]

The name integer linear programming (ILP) refers to the class of combinatorial constrained optimization problems with integer variables, where the objective function is a linear function and the constraints are linear inequalities .Many problems in operations research and combinatorial optimization can be formulated as ILPs. (ILP) optimization problem can be stated in the following general form:

$Max\ X_0 = \sum_{j=1}^{n} c_j\, x_j$

Subject to

$\sum_{j=1}^{n} a_{ij} x_j \leq bi \quad i = 1,2,\ldots,m \quad \ldots \ (1)$

$x_j \geq 0 \quad j = 1,2,\ldots,n$

$x_j \epsilon\, Z^n$

c_j, a_{ij}, b_i all integers

Where $a_{ij} \in R^{m\times n}, b\, \epsilon\, R^m, c\, \epsilon\, R^n$. If the problem has both integer and continuous variables, then it is a mixed integer linear programming problem, while if all variables are integer it is a pure integer linear programming problem. A wide variety of real life problems in economics, social science and politics can be formulated as linear integer optimization problems. The combinatorial problems, like the knapsack-capital budgeting problem, decreasing costs and machinery selection problem, network and graph problems, many scheduling problems can also be solved as linear integer optimization problems.

3. Definitions [8]

Definition 3.1: (**Feasible region**) is the collection of all feasible solutions.

Definition 3.2 :(**Optimal solution**) is a feasible solution that has the most favorable value of the objective function.

4. Modified Surrogate Cutting Plane Algorithm (MSCPA)

Most methods for solving integer linear programming problems depended on the non-integer solutions that result from solving the problem of linear programming problem. In this paragraph, we will introduce a new algorithm for solving ILPP that does not depend on the solution of LP, in addition to that offer simpler solutions and the minimum number of tables to resolve the problem with help of the development of simplex tableau as shown in the following examples:

Max $X_0 = C_B X_B + C_N S_N$

Subject to

$B X_B + N S_N = b$

$X_B \geq 0$ Integers, $S_N \geq 0$

Hence, we have $X_B + B^{-1} N S_N = B^{-1} b$. Therefore,

$X_0 = C_B (B^{-1} b - B^{-1} N S_N) + C_N S_N$

$X_0 = C_B B^{-1} b - C_B B^{-1} N S_N + C_N S_N$

$X_0 + (C_B B^{-1} N - C_N) S_N = C_B B^{-1} b$

At $S_N = 0$, we have $X_B = B^{-1} b$, and $X_0 = C_B B^{-1} b$.

Then the above problem can be solved by using development integer simplex as follows in the following table:

Basic	R.H.S	$-t_N$
X_0	$C_B B^{-1} b$	$C_B B^{-1} N - C_N$
X_B S_N	0 $B^{-1} b$	-I $B^{-1}N$
SC_i	$[\frac{pivot\ row}{pivot\ element}]$	

Table 4.1. Modified integer simplex Table for (MSCPA)

Then apply simplex algorithm [8].

5. Numerical Example:

To demonstrate the solution process for (ILPP) by using MSCPA let us treat the following examples.

Example 5.1:

$Max\ X_0 = x_1 + x_2 + x_3$

$Sub.To \quad -4x_1 + 5x_2 + 2x_3 \leq 4$

$-2x_1 + 5x_2 \leq 5$

$3x_1 - 2x_2 + 2x_3 \leq 6$

$2x_1 - 5x_2 \leq 1$

$x_1, x_2, x_3 \geq 0 \quad and\ integers$

Solution

Step 1. Apply simplex method to find the entering variable and leaving variable for among

S_i variables. $t1$ is an entering variable and S_4 is leaving variable.

Table (5.1.1)

	Basic solution	$t1$	$t2$	$t3$
X_0	0	-1	-1	-1
x_1	0	-1	0	0
x_2	0	0	-1	0
x_3	0	0	0	-1
S_1	4	-4	5	2
S_2	5	-2	5	0
S_3	6	3	-2	2
S_4	1	(2)	-5	0
SC1	0	((1))	-3	0

Step 2. Determine the pivot element then find the first cutting sc_1 as follows:

$sc_1 = ([\frac{1}{2}],[\frac{2}{2}],[\frac{0}{2}])$=(0,1,-3,0), where [.] Denoted the integer function.

Step 3. Apply simplex algorithm to find the new table as follows:

Table (5.1.2)

	Basic solution	SC1	$t2$	$t3$
X_0	0	1	-4	-1
x_1	0	1	-3	0
x_2	0	0	-1	0
x_3	0	0	0	-1
S_1	4	4	-7	2
S_2	5	2	-1	0
S_3	6	-3	(7)	2
S_4	1	-2	-1	0
SC2	0	-1	((1))	0

Step4. Continue with the operations of simplex method until we obtain the optimum integer solution in the 9th table as shown

Table (5.1.9)

	Basic solution	SC6	SC7	SC5
X_0	5	0	1	0
x_1	3	2	-3	3
x_2	2	-1	-1	1
x_3	0	-3	5	-4
S_1	6	9	-17	15
S_2	1	-1	-1	1
S_3	1	2	-3	1
S_4	5	1	1	-1

The optimal solution is: $X_0 = 5$,$x_1 = 3$, x_2 =2, $x_{3=}0$

Note: When solving the problem by cutting plane of Gomory, the optimal integer solution of the problem was obtained at the 21th tables.

Example (5.2)

$$Max\ X_0 = 5x_1 + 6x_2$$
$$Sub.To \quad 2x_1 + 3x_2 \leq 18$$
$$2x_1 + x_2 \leq 12$$
$$x_1 + x_2 \leq 8$$
$$x_1, x_2 \geq 0 \quad and\ integer$$

Solution:

Table (5.2.1)

	Basic solution	t_1	t_2
X_0	0	-5	-6
x_1	0	-1	0
x_2	0	0	-1
S_1	18	2	(3)
S_2	12	2	1
S_3	8	1	1
SC1	6	0	((1))

The optimal integer solution of the problem is shown below in table (5.2.5), while by Gomory cutting plane method the optimal integer solution has been obtained in 17 tables.

Table (5.2.5)

	Basic solution	t_1	t_2
X_0	39	3	1
x_1	3	3	-1
x_2	4	-2	1
S_1	0	0	-1
S_2	2	-4	1
S_3	1	-1	0

5. Conclusion.

MSCPA algorithm was introduced to find the new optimal integer solution to the problems of (ILPP). The examples shown, the technique of this algorithm demonstrated ease of computation and speed of solution by reducing the number of solution tables compared to other methods.

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