Application of the ANFIS Approach for Estimating the Mechanical Properties of Sandstones

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APPLICATION OF THE ANFIS APPROACH FOR ESTIMATING THE MECHANICAL PROPERTIES OF SANDSTONES

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(Received 1st February and Accepted 3rd June 2020)

Abstract

The reliable determination of the geomechanical parameters of rocks such as unconfined compressive strength (UCS) and elastic modulus (E) using laboratory methods is difficult and time-consuming. In this regard, the development of predictive models for the estimation of these properties seems to be essential in rock engineering projects. The main purpose of this work is to construct an ANFIS approach to estimate the UCS and E of sandstones. For this purpose, a database of laboratory tests conducted on 130 sandstone samples was prepared. The data included porosity, P-wave velocity, dry density, slake durability index, and water absorption as input variables and UCS and E as the output variables. The performance of the proposed ANFIS model is evaluated based on the criteria including coefficient of determination (R²), root mean squared error (RMSE), and variance account for (VAF). The prepared ANFIS model showed a good performance in predicting UCS with R², RMSE, and VAF values of 0.910, 0.070, and 97.00%, respectively. Likewise, the ANFIS model prepared for predicting the E showed a high performance, with its R², RMSE, and VAF being 0.866, 0.086, and 89.00%, respectively.

Keywords - Unconfined compressive strength; Elastic modulus; Physical properties; ANFIS; Sandstone
1. INTRODUCTION

In all engineering projects constructed on and inside rocks, study and precise measurement of UCS and E are especially important, mainly because of its essential role in the geotechnical problems. The direct way for measuring UCS and E in the laboratory based on the ISRM and the ASTM introduced methods is complicated and costly and involves destructive tests. Such complexity is more obvious in case of weak, extremely fissured, porous, thinly-bedded, foliated and clay-bearing rocks. Many researchers have tried using statistical techniques, ANN and ANFIS approaches were solved these problems [1-7]. For example, Verma and Singh [8] suggested an ANFIS model to forecast the P-wave velocity and observed that ANFIS has acceptable performance for nonlinear, multivariable and complex problems. Singh et al. [9] predicted the elastic modulus of different rock types using ANN and ANFIS models and observed that ANFIS has a higher prediction performance. In another study, Singh et al. [10] using an ANFIS approach estimated UCS of different rocks. To train, test and validate the constructed network, they utilized a total of 85, 10 and ten datasets, respectively. They used point load index ($I_{ps}$), dry density ($\gamma_d$), and water absorption (Ab) as the input variables. Lastly, they observed that the results achieved from E prediction are very dependable and close to the actual values. Saedi et al. [11] implemented various modeling techniques to estimate the strength parameters (UCS and E) of migmatites using the 120 datasets. Their results confirmed that the prediction accuracy of the ANN technique is higher compared to the ANFIS approach. Also, Saedi et al. [12] used a fuzzy clustering-based ANN and multivariable regression (MR) methods to predict the rock mass diggability index. Based on the results, they concluded that the estimation power of the ANFIS technique is better compared to the ANN method and MR analysis. Jahed Armaghani [13] used various approaches such as ANN, ANFIS, and MR to estimate the UCS and E of granite rocks. They used 45 granite sample sets to construct the proposed models and concluded that the ANFIS approach is more accurate than the MRA and ANN techniques. Sommez et al. [14] using a FIS approach and regression methods predicted the UCS and E of Ankara agglomerate using the results of the petrographic analysis for 164 samples. They reported that the FIS system be able to estimate the UCS with more accuracy than the regression methods; however, they showed that regression equations are more suitable for predicting the E. Yesiloglu-Gultekin et al. [15] utilized ANN and ANFIS techniques for predicting the strength of granitic samples and in order to assess the performance of mentioned models, compared their results with regression analysis. Based on the results, they reported that the ANFIS approach has a better predictive performance compared to the ANN and regression methods. In another study, Singh et al. [16] implemented the ANN and ANFIS model for estimating UCS by density, S-wave velocity, and VP of rock samples. They observed that the estimation potential of ANN approach is higher than the ANFIS method. Yilmaz and Yuksel [17] developed an ANN model and a neuro-fuzzy model for forecasting the UCS and E of gypsum samples using 121 datasets that belong to the Hafik formation in the Sivas basin. They chose the Schmidt number, water content, P-wave velocity, and point load index as input parameters. Finally, they observed that the ANN and neuro-fuzzy techniques have better performance compared to the regression analysis. Also, Sharma et al. [18] using the ANFIS and ANN approaches predicted the UCS of 13 rock types selected from various geological formations in India. Based on the results, it was observed that the ANFIS technique estimates UCS with higher accuracy compared to the ANN model. In the recent past year the application of artificial intelligence in geotechnical engineering is underlined in many studies [19-30]. Other researchers highlighted the feasibility of soft computing in solving civil Engineering problems [31-38]. Some performed works to estimate the UCS and E using soft computation techniques are listed in table 1. The goal of this work is to construct a prediction model using ANIS to estimate the UCS and E of 130 sandstone samples. The input variables selected in this research are P-wave velocity ($V_p$), dry density ($\gamma_d$), porosity (n), water absorption ($A_b$) and slake durability index ($ID_2$). The reason for selecting these properties as input parameters to develop the predictive models is that they are non-destructive, quick, economical, and feasible.
Table 1. Some performed works to estimate the UCS and E of various rocks using soft computation methods

<table>
<thead>
<tr>
<th>References</th>
<th>Technique</th>
<th>Input</th>
<th>Output</th>
<th>No. of dataset</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verma and Singh [8]</td>
<td>ANFIS</td>
<td>R, W, ρ, BTS, Vp</td>
<td>UCS</td>
<td>-</td>
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<tr>
<td>Singh et al. [9]</td>
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<td>Saedi et al. [11]</td>
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<td>CPI, Is50, BTS, BPI</td>
<td>UCS</td>
<td>120</td>
<td>0.854</td>
</tr>
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<td>CPI, Is50, BTS, BPI</td>
<td>E</td>
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<td>γd, Vp, Q, Pl</td>
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<td>0.985</td>
</tr>
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<td>BPS, Vp</td>
<td>UCS</td>
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<td>0.865</td>
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<td>Yilmaz and Yuksek [39]</td>
<td>ANFIS</td>
<td>Vp, Is(50), Rn, WC</td>
<td>E</td>
<td>121</td>
<td>0.955</td>
</tr>
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<td>0.943</td>
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<td>UCS</td>
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<td>0.978</td>
</tr>
<tr>
<td>Gokceoglu and Zorlu [40]</td>
<td>FIS</td>
<td>Is(50), BPI, Vp, BTS</td>
<td>E</td>
<td>-</td>
<td>0.79</td>
</tr>
<tr>
<td>Majidi and Rezaei [41]</td>
<td>ANN</td>
<td>R, Rn, n, ρ</td>
<td>UCS</td>
<td>93</td>
<td>0.97</td>
</tr>
<tr>
<td>Mohamad [42]</td>
<td>ANN</td>
<td>R, W, ρ, BTS, Vp, Is(50)</td>
<td>UCS</td>
<td>40</td>
<td>0.971</td>
</tr>
<tr>
<td>Sarkar et al. [43]</td>
<td>ANN</td>
<td>Vp, Is(50), Id, ρ</td>
<td>UCS</td>
<td>40</td>
<td>0.99</td>
</tr>
<tr>
<td>Torabi-Kaveh et al. [44]</td>
<td>ANN</td>
<td>ρ, n, Vp</td>
<td>UCS</td>
<td>105</td>
<td>0.95</td>
</tr>
<tr>
<td>Torabi-Kaveh et al. [44]</td>
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<td>ρ, n, Vp</td>
<td>E</td>
<td>105</td>
<td>0.76</td>
</tr>
<tr>
<td>Abdi et al. [45]</td>
<td>ANN</td>
<td>ρ, n, Vp, Ab</td>
<td>UCS</td>
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<td>0.872</td>
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<tr>
<td>Abdi et al. [45]</td>
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<td>E</td>
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<td>Yilmaz and Yuksek [17]</td>
<td>ANN</td>
<td>ne, Is(50), Rn, Id</td>
<td>E</td>
<td>121</td>
<td>0.91</td>
</tr>
<tr>
<td>Dehghan et al. [46]</td>
<td>ANN</td>
<td>Vp, Is(50), Rn, n</td>
<td>E</td>
<td>30</td>
<td>0.77</td>
</tr>
<tr>
<td>Majdi and Beiki [47]</td>
<td>GA-ANN</td>
<td>ρ, RQD, n, NJ, GSI</td>
<td>E</td>
<td>120</td>
<td>0.89</td>
</tr>
<tr>
<td>Beiki et al. [48]</td>
<td>GA</td>
<td>ρ, n, Vp</td>
<td>E</td>
<td>72</td>
<td>0.67</td>
</tr>
<tr>
<td>Bejarbaneh et al. [49]</td>
<td>ANN</td>
<td>Is(50), Rn, Vp</td>
<td>E</td>
<td>96</td>
<td>0.81</td>
</tr>
<tr>
<td>Momeni et al [50]</td>
<td>PSO-ANN</td>
<td>SRn, Vp, Is(50), ρ</td>
<td>UCS</td>
<td>66</td>
<td>0.95</td>
</tr>
<tr>
<td>JahedArmaghani et al. [51]</td>
<td>ANFIS</td>
<td>Vp, ρ, PSV</td>
<td>UCS</td>
<td>45</td>
<td>0.98</td>
</tr>
</tbody>
</table>

$BPI$ block punch index, $I_d$ slake durability index, $n_e$ effective porosity, $RQD$ rock quality designation, $NJ$ number of joints per meter, $GSI$ geological strength index, $GA$ genetic algorithm, $WC$ water content, $WA$ water absorption, $FIS$ fuzzy inference system, $ANFIS$ adaptive neuro-fuzzy inference system, $n$ porosity, $ρ$ density, $R$ lithology type, $Rn$ Schmidt hammer number, $BTS$ Brazilian tensile strength, $Vp$ P-wave velocity, $Is50$ point load index, $SDI$ slake durability index, $Q$ quartz content, $Pl$ plagioclase content, $γ_d$ dry density, $CPI$ cylindrical punch Index, $W$ weathering degree.
2. MATERIALS AND METHODS

To perform this study, ten sandstone blocks in dimension 40cm×40cm×20cm were selected from different locations in Central Iran and Sanandaj-Sirjan zones (Figure 1). These sandstones belong to the Upper Red Formation (URF) in southwest Qom and northeast Hamedan, and Jurassic sandstones in the east of Hamedan. Then, the collected blocks were cored in the laboratory to prepare core samples with NX size (54.1mm diameter) based on the ISRM [52]. To develop the ANFIS model, different physicomechanical properties of 130 sandstone samples such as UCS, E, Vp, n, γd, ID2 and Ab were determined according to the ISRM [52]. A total of 130 datasets were used of which 91 datasets were selected for the training stage, and 39 data were considered for the testing stage.

![Figure 1. Map of sampling points](image)

2.1. ADAPTIVE NEURAL FUZZY INFERENCE SYSTEM (ANFIS)

ANFIS was introduced by Jang [53] as an approximate function system. This technique is a hybrid learning method that is broadly employed in rock mechanic and engineering geology. ANFIS is one of the most common tools to predict the rock mechanical characteristics through combining the learning capability of ANN and the fuzzy inference potential [54-56]. Figures 2a and 2b present First order Sugeno fuzzy logic and equivalent ANFIS structure, respectively. As illustrated in this figure, the structure of this network consists of two input factors (x, y) and one output parameter (f). A typical ANFIS structure is involved of five parts in the inference method, each
including some nodes that are determined by the function of the node.

The outputs of the previous layer are used as input nodes for the current layer. In this model, x and y are as inputs and f is as output. Thus, two fuzzy “if-then” rules could be illustrated as follows [58, 59, 60]:

If x is A1 and y is B1 then \( f_1 = p_1x + q_1y + r_1 \); (rule 1)
If x is A2 and y is B2 then \( f_2 = p_2x + q_2y + r_2 \); (rule 2)

The membership functions of x and y inputs are A1, A2, and B1, B2, respectively. Also, \( p_1, q_1, r_1, p_2, q_2, \) and \( r_2 \) are defined as output function parameters.

Layer 1: in this layer all neurons i are adaptive nodes:
\[ O_{1;i} = \mu A_i(x) \] (1)
\[ O_{1;i} = \mu B_i(y) \] (2)
For \( i = 1, 2 \); x and y are input nodes, and A and B are considered as linguistic labels. Also, \( \mu A_i(x) \) and \( \mu B_i(y) \) are mentioned as membership functions.
Layer 2: All neurons are a fixed node labeled \( \Pi \), whose output neuron is created by all the received signals:
\[ O_{2;i} = \omega_i = \mu A_i(x)\mu B_i(y) \text{with } i = 1, 2. \] (3)

The neuron of output indicates the firing strength of a rule.
Layer 3: The ith neuron in the current layer is labeled as encircled N. The output \( (O_{3;i}) \) is calculated using the ratio of the ith rule’s firing strength to the summation of all rules’ firing strength:
\[ O_{3;i} = \omega_i/\left(\omega_1 + \omega_2\right) \text{ with } i = 1, 2 \] (4)
Layer 4: in this layer includes the node functions given as follows:
\[ O_{4;i} = \omega_i f_i = \omega_i (px + qy + ri), \] (5)
where \( p_i, q_i, \) and \( r_i \) are parameter sets and named the consequent parameters. Also, \( \omega_i \) is defined as the output of Layer3.
Layer 5: In the final step, the output value of this layer is calculated using the sum of all incoming signals:
\[ O_{5;i} = \Sigma \omega_i f_i = \omega_i \Sigma f_i; \text{ } i = 1, 2. \] (6)

Generally, a large rule number or a large parameter number equivalently, may lead to inefficiency or difficulty in system implementation and computation. A fuzzy system with many rules may be hard to design and have large storage consumption, high computation complexity and poor convergency in parameter tuning. To construct fuzzy systems using as less as possible
fuzzy rules with guaranteed performance is a meaningful problem, which has attracted much attention for a long time in the fuzzy community [62-65]. The basic of ANFIS learning rule is Back propagation (BP) gradient descent that calculates error signals recursively from the output nodes backward to the input neurons. According to network structure (Fig. 2b), the output \( f \) could be defined as a linear combination of the following factors. To implement the learning process of the fuzzy technique via differentiable functions, this model uses a hybrid learning rule owing to its simplicity. The technique applied for learning the parameters of membership functions in ANFIS approach is classical BP method. Forward and backward hybrid learning are used as different passes in the ANFIS model. In the forward pass, the algorithm utilizes the least-squares procedure to recognize the consequent factors on layer 4. Lastly, the final output could be determined as follows:
\[
f = \frac{\omega_1}{\omega_1 + \omega_2} f_1 + \frac{\omega_1}{\omega_1 + \omega_2} f_2 = \omega_1 (p_1 + q_1 + r_1) + \omega_2 (p_2 + q_2 + r_2) = (\omega_1 x)p_1 + (\omega_1 y)q_1 + \omega_1 r_1 + (\omega_2 x)p_2 + (\omega_2 y)q_2 + \omega_2 r_2
\] (7)
where \( p_1, q_1, r_1, p_2, q_2, \) and \( r_2 \) are the consequent parameters.

In the present research, the ANFIS model was applied for estimating UCS and E of sandstone samples. For this purpose, all the data set were randomly assigned to two groups of training (70%) and testing (30%) data. The number of fuzzy rules was determined by trial and error. Several fuzzy rule combinations were used to the UCS and E data points separately. Finally, it was observed that input variables with six fuzzy rules outperform the other types of ANFIS to UCS and E prediction. Afterward, ANFIS models were created for predicting the UCS and E. Figure 3 presents the proposed ANFIS model.

![ANFIS model structure](image)

**Figure 3.** ANFIS model structure for the estimation of UCS and E

### 3. RESULTS AND DISCUSSIONS

In the data analysis step, \( n, \gamma_d, V_p, \) ID, and \( A_b \) of 130 samples were used to construct the ANFIS model for forecasting the UCS and E. To make a dataset for this study, engineering characteristics of these core samples were measured in accordance to the ISRM [52]. Since the number of data is very high in this study, some of them were accidentally chosen and listed in Table 2. The results of the statistical analysis conducted for the original dataset including the minimum, maximum, average and standard deviation are listed in Table 3. Some cylindrical core samples prepared for UCS tests and failure modes after performing UCS tests are shown in Figure 4.
Table 2. The physical and mechanical characteristics for some sandstone samples under study

<table>
<thead>
<tr>
<th>Sample no</th>
<th>n (%)</th>
<th>2( gr/cm³)</th>
<th>Vp (km/sec)</th>
<th>ID2 (%)</th>
<th>Ab (%)</th>
<th>UCS (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>2.63</td>
<td>2.88</td>
<td>5.24</td>
<td>99.15</td>
<td>0.52</td>
<td>129.31</td>
<td>19.41</td>
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<tr>
<td>S2</td>
<td>5.88</td>
<td>2.65</td>
<td>4.42</td>
<td>97.13</td>
<td>0.73</td>
<td>96.21</td>
<td>17.44</td>
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<tr>
<td>S3</td>
<td>6.86</td>
<td>2.53</td>
<td>2.34</td>
<td>97.58</td>
<td>1.33</td>
<td>39.51</td>
<td>8.76</td>
</tr>
<tr>
<td>S4</td>
<td>6.62</td>
<td>2.49</td>
<td>2.61</td>
<td>97.09</td>
<td>1.47</td>
<td>67.40</td>
<td>6.65</td>
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<tr>
<td>S5</td>
<td>4.75</td>
<td>2.83</td>
<td>4.45</td>
<td>98.21</td>
<td>1.35</td>
<td>98.45</td>
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<tr>
<td>S6</td>
<td>3.15</td>
<td>2.91</td>
<td>4.67</td>
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<td>115.54</td>
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<td>S7</td>
<td>8.68</td>
<td>2.35</td>
<td>2.44</td>
<td>97.25</td>
<td>2.32</td>
<td>39.45</td>
<td>6.91</td>
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<tr>
<td>S8</td>
<td>13.43</td>
<td>2.23</td>
<td>1.14</td>
<td>96.10</td>
<td>6.76</td>
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<tr>
<td>S9</td>
<td>2.68</td>
<td>2.88</td>
<td>5.33</td>
<td>98.75</td>
<td>0.69</td>
<td>126.76</td>
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<td>S10</td>
<td>2.23</td>
<td>2.88</td>
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<td>S11</td>
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<tr>
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<td>6.29</td>
<td>2.69</td>
<td>4.35</td>
<td>97.30</td>
<td>1.76</td>
<td>55.07</td>
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<td>97.73</td>
<td>1.45</td>
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<tr>
<td>S20</td>
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<td>2.36</td>
<td>2.34</td>
<td>97.04</td>
<td>3.41</td>
<td>41.25</td>
<td>7.22</td>
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Table 3. Values of minimum, maximum, average and standard deviation for used parameters (130 samples)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Ave</th>
<th>Std. deviation</th>
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</thead>
<tbody>
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<td>n (%)</td>
<td>2.19</td>
<td>14.98</td>
<td>6.97</td>
<td>2.63</td>
</tr>
<tr>
<td>Vp (km/sec)</td>
<td>1.09</td>
<td>5.53</td>
<td>3.25</td>
<td>0.98</td>
</tr>
<tr>
<td>2(gr/cm³)</td>
<td>2.10</td>
<td>2.94</td>
<td>2.63</td>
<td>0.18</td>
</tr>
<tr>
<td>ID2 (%)</td>
<td>95.17</td>
<td>99.56</td>
<td>97.83</td>
<td>0.87</td>
</tr>
<tr>
<td>Ab (%)</td>
<td>0.31</td>
<td>6.91</td>
<td>1.95</td>
<td>1.14</td>
</tr>
<tr>
<td>UCS (MPa)</td>
<td>29.94</td>
<td>143.00</td>
<td>72.41</td>
<td>25.67</td>
</tr>
<tr>
<td>E (GPa)</td>
<td>5.23</td>
<td>22.51</td>
<td>11.20</td>
<td>3.77</td>
</tr>
</tbody>
</table>
The proposed ANFIS model was trained using 50 epochs. The most considerable step in the model is determining fuzzy membership function and corresponding amount. Due to their smoothness and brief notation, bell and Gaussian membership functions are the most common functions for identifying the fuzzy set. Both membership functions have benefits as they are smooth and non-zero at each point [48]. Therefore, after the training step, Figures 5a and 5b show the membership functions for the ANFIS model. In this study, UCS and E of sandstones were predicted using a Gaussian-type membership function.
Figures 6-9 indicate plots of the observed and estimated UCS for both training and testing stages by the ANFIS. Based on the mentioned figures, the constructed ANFIS model has a good performance for estimating the UCS for both data-sets. It should be noted that the conformity of the correlation coefficient line on the x=y line states the high performance of the developed ANFIS model. The prediction ability of the model is evaluated by the mean squares error (MSE) which takes more information about the error distribution. The histogram of error can be used to observe the error density. Figure 6a illustrates the ANFIS model results for UCS plotted versus the target data in the training stage. The $R^2$-value of the constructed ANFIS model in the training stage is obtained as 0.942. Figure 6b shows estimated (ANFIS outputs) and target data in the training stage. The changes of relative error between the observed and estimated values for normalized data in the training step are shown in Figure 6c. The MSE value of the
model was obtained as 0.0027 for training dataset. The error histogram is shown in Figure 6d for normalized data. Based on this figure, the proper performance of the proposed model is achieved because the error distribution focuses around zero. Figure 7a shows the outputs of the ANFIS model versus the target data for testing stage. The performance of this model was 0.910 for $R^2$-value. Figure 7b indicates the results of the ANFIS model with the target data for the testing data-set. Also, Figure 7c presents the variation of relative error between the target and estimated value for normalized data during the testing stage. For this model, MSE value is obtained 0.0050. The errors histogram is shown in Figure 7d for normalized data. As can be observed, the error distribution density is focused around zero, which confirms the high performance of the proposed estimated model.

Figure 6. The results of ANFIS for estimating of UCS in the training step: a relationship between observed (target) and estimated data (output), b comparision of predicted UCS by the ANFIS with the target data, c the variations of MSE and RMSE, d the histogram of errors
Figure 7. The results of ANFIS for estimating of UCS in the testing step: a relationship between observed (target) and estimated data (output), b comparison of predicted UCS by the ANFIS with the target data, c the variations of MSE and RMSE, d the histogram of errors.

Figures 8 and 9 depict the results of the proposed model for the training and testing datasets of E. Fig. 8a shows a good correlation between target and estimated values of E in the training step ($R^2=0.912$). Figures 8b, 8c, and 8d indicate the suitable potential of the ANFIS model for the E value estimation in the training stage. Figures 9a presents the ANFIS outputs (estimated data) versus the targets for the testing stage. The performance of this ANFIS, $R^2$-value, was 0.866. Figure 7b presents the ANFIS model results (outputs) with the targets for the testing data-set. Also, Figure 7c shows the relative error variation between the target and estimated for normalized data for testing stage. The MSE of this model is obtained 0.0074. The histogram of errors is shown in Figure 7d for normalized data. As can be observed, the error distribution density is focused around zero, which confirms the high performance of the proposed ANFIS.
Figure 8. The results of ANFIS for estimating of E in the training step: a relationship between observed (target) and estimated data (output), b comparison of predicted E by the ANFIS with the target data, c the variations of MSE and RMSE, d the histogram of errors.
Figure 9. The results of ANFIS for estimating of E in the testing step: a relationship between observed (target) and estimated data (output), b comparison of predicted E by the ANFIS with the target data, c the variations of MSE and RMSE, d the histogram of errors.

The performance of the estimative model was evaluated using different standard statistical performance evaluation criteria; the coefficient of determination ($R^2$), variance accounts (VAF) and root mean square error (RMSE) which were calculated as following equations:

$$R^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2(Y_i - \bar{Y})^2}}$$  \hspace{1cm} (8)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_i - Y_i)^2}{n}}$$  \hspace{1cm} (9)

$$VAF = \left[1 - \frac{\text{var}(X_i - Y_i)}{\text{var}(X_i)}\right] \times 100$$  \hspace{1cm} (10)

where $X_i$ and $Y_i$ are the observed and estimated data, respectively, and $n$ is the number of the dataset.

In theory, an estimation model is excellent when $R$ is more than 0.90, RMSE is equal to 0, and VAF is 100%. Values of statistical indices calculated for UCS and E of selected sandstones are listed in Table 4. As can be seen in this table, the ANFIS model is able to estimate UCS and E with high accuracy.
### Table 4. Performance indices of the ANFIS model

<table>
<thead>
<tr>
<th>Analysis</th>
<th>UCS (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>RMSE</td>
</tr>
<tr>
<td>ANFIS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>train</td>
<td>0.942</td>
<td>0.052</td>
</tr>
<tr>
<td>test</td>
<td>0.910</td>
<td>0.070</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

In the present research, the ANFIS model was constructed for estimating the UCS and E parameters of sandstones using five geo-engineering parameters that can be determined in the laboratory easily and with low cost. The main results of this study are presented as follows:

On the basis of the findings, the $R^2$-values for the training and testing stages of UCS prediction are 0.942 and 0.910, respectively, while these values for E are 0.912 and 0.866, respectively. The obtained results confirmed that the ANFIS technique can forecast UCS and E with high precision. The performance of the ANFIS model was assessed using MSE, RMSE, and VAF indices. According to the results, MSE, RMSE, and VAF in the training stage were 0.0027, 0.052, and 98 for UCS and 0.0084, 0.071, and 91 for E, respectively. In the testing stage, the values of MSE, RMSE, and VAF were 0.0050, 0.070, and 97 for UCS and 0.0074, 0.086, and 89 for E, respectively. On the basis of the results, it was found that the ANFIS model is a powerful technique for estimation of the observed UCS and E values.

Considering the above results, it is concluded that ANFIS as a smart, low cost and easy technique can be applied to estimate the UCS and E of different rocks. However, it should be highlighted once again, that the developed predictive models works good enough when the new data are in the range of the presented data in Table 2. Further research with much larger dataset is recommended for overcoming the aforementioned limitation.

ACKNOWLEDGEMENTS

The authors would like to appreciate the invaluable support of Lorestan University in this study, and also extend their gratitude to those individuals who were kind enough to review this paper, scrutinize the whole text and adduce their helpful comments for the improvement of this paper.

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