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SHIFTED THIRD KIND CHEBYSHEV OPERATIONAL MATRIX TO SOLVE BVPS OVER INFINITE INTERVAL

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مصفوفة العمليات لشبشيف من النوع الثالث المتحولة لحل مسائل القيم الحدودية على الفترة اللامنتهية

ملخص :-

الغرض الرئيسي من هذا البحث هو حل المسائل الحدودية ذات العدد اللامنتهي من الشروط الحدودية، عن طريق اختزال الفتره اللامنتهيه الى فتره منتهيه وتقريب المتغير باستخدام طريقه الفروق المنتهيه،المعادلات الحدوديه التي تستنج تختزل الى نظام خطي من المعادلات الجبريه بمعاملات غير معروفه من معاملات شبشيف من النوع الثالث المتحول تطبيق هذه الطريقه أثبتت بواسطه امثلة الاختبار .

Abstract

The main purpose of this research is to solve boundary value problems (BVPs) with an infinite number of boundary conditions. By reducing the infinite interval to finite interval that is large and approximating the variable using finite difference method, the resulting boundary value problem is reduced to linear system of algebraic equations with unknown shifted third kind chebychev coefficients. The applications are demonstrated via test examples.

Keywords:- Shifted third Chebyshev, Operational Matrix, Boundary Value Problem, Infinite interval.

1. Introduction:-

Boundary Value Problems on finite intervals appear often in applied mathematics and physics. More examples and collection of works on the existence of solution of boundary value problems on infinite interval for differential difference and integral equations may be found in monographs [1,2,3,4]. For some works and various techniques dealing with such boundary value problems see [5,6,7,8,9,10] and the references therein.

Consider the Tow Point BVPs on the form

$$\ddot{y} = f(x, y, \dot{y}) \qquad (a, b) \ldots(1)$$

$$y(a) = A \qquad , y(\infty) = B$$

Where $f(x, y, \dot{y})$ are continuous functions

before computing the solution , we plummet the infinite interval to finite, then we introduce a collocation method with shifted third chebyshev polynomials for solving (1).

2. Some Properties of Third Chebyshev Polynomials and Their Shifted Ones:- [11,12,13]

The chebyshev polynomial $V_r(x)$ of the third kind has trigonometric definitions involving the half angle $\theta/_2$ (where x= cosθ). It is the polynomials of degree r in x defined by:-

$$V_r(x) = \frac{\cos(r+\frac{1}{2})\theta}{\cos(\frac{1}{2})\theta} \text{ where x= } \cos\theta \text{ ...(2)}$$

the recurrence relation of $V_r(x)$ with initial condition for r-=1

$$V_{r+1}(x) = 2xV_r(x) - V_{r-1}(x) \text{ …(3)}$$

with initial conditions $V_0(x) = 1$,

$V_1(x) = 2x - 1$, r=1,2 ,…

Generally, we define shifted third kind chebyshev polynomials $V_r^*(x)$ appropriate to any given finite range a ≤ x ≤ b of x by making the interval correspond to the interval -1 ≤ x ≤ 1 of a new variable s under the linear transformation :-

$$S = \frac{2(x-a)}{b-a} - 1 \text{ …(4)}$$

The third kind polynomials appropriate to

a ≤ x ≤ b are thus given by $V_r(s)$, where s is given in eq.(4).using this in conjunction with (3) yields

$$V_0^*(x) = 1$$

$$V_1^*(x) = 2\left[\frac{2(x-a)}{b-a} - 1\right] - 1$$

$$V_2^*(x) = 4\left[\frac{2(x-a)}{b-a}\right]^2 - 2\left[\frac{2(x-a)}{b-a} - 1\right] - 1$$

$$V_3^*(x) = 8[\frac{2(x-a)}{b-a} - 1]^3 - 4\left[\frac{2(x-a)}{b-a} - 1\right]^2 - 4\left[\frac{2(x-a)}{b-a} - 1\right] + 1$$

with the recursive formula given as:-

$$V_{r+1}^*(x) = 2\left[\frac{2(x-a)}{b-a} - 1\right]V_r^*(x) - V_{r-1}^*(x) \quad \text{... (5)}$$

let $\emptyset(x) = [V_0^*(x), V_1^*(x), \dots, V_N^*(x)]^T$,then the operational matrix of the first derivative of shifted third kind chebyshev polynomial (SC3OMD) may be defined by:-

$$\frac{d\emptyset(x)}{dx} = E\,\emptyset(x) \text{ …(6)}$$

where E=$(e_{ij})_{0 \le i,j \le N}$ is the square operational matrix of order (N+1) whose nonzero elements are given explicitly by:-

$$e_{i,j} = \frac{2}{b-a}\begin{cases} i+j+1 & i > j, (i+j) odd \\ i-j & i > j, (i+j) even \end{cases}$$

For example if N=6, then the operational matrix E is given explicitly by:-

$$E = \frac{2}{b-a}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 6 & 0 & 0 & 0 & 0 \\ 4 & 6 & 2 & 8 & 0 & 0 & 0 \\ 6 & 4 & 8 & 2 & 10 & 0 & 0 \\ 6 & 8 & 4 & 12 & 2 & 12 & 0 \end{bmatrix}_{7*7} \quad \text{... (7)}$$

It can be easily shown that for any $n \in \mathbb{Z}^+$

$$\frac{d^n\emptyset(x)}{dx^n} = E^n\emptyset(x) \text{ ... (8)}$$

n=0,1,2,…

3.SC3OMD Collocation Method

Let us consider BVPs (1), first stage we replaced the boundary condition at infinite with the same conditions at a finite value b.[12]

$$y(\infty) \to B \ , \ b \to \infty, \quad then \quad y(b^N) = B$$

$$b^N = a + (N+1)h \text{ …(9)}$$

let ε be an a small arbitrary value, then we use the finite difference method as let

$$f_n = f_n(x), \qquad g_n = g_n(x)$$

$$\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} = f_n y_n + g_n$$

$$y_{n+1} = (2 + h^2 f_n)y_n - y_{n-1} + h^2 g_n \text{ ... (10)}$$

by substitute n=1,2,3,..,N+1 in eq. (9) we get

$$y_2 = (2 + h^2 f_1)y_1 - y_0 + h^2 g_1$$

and $y_0 = A$, $y_{n+1} = B$

$$B = \frac{B + A - h^2 g_1}{2 + h^2 f_1}$$

If $\left|y_{(n)}^{N+1} - y_{(n)}^{N}\right| < \varepsilon$ ... (11)

is satisfied we stop, else, take the values which obtained in eq.(11) and substituted in eq.(10).

Then approximate y(x) over (a,b) by the shifted third kind chebyshev polynomials as:-

$y_N(x) \cong \sum_{i=0}^{N} a_r V_r^*(x)$ …(12)

y(a)=A , y(b)=B

on the other hand, a different technique of selecting the collocation points is also explored, this is the use of equally spaced points defined by :-

$x_i = a + \frac{(b-a)i}{D+2}$ i=1,2,…,D+1

where D is the difference between the degree of the trial solution and the order n of the equation. The collocation equations produced through these points are equally solved alongside system of N+1equations arrived, is then solved to obtain the numerical values for a_i , i=0(1)N. the values of a_i are there after subsisted into the trial solution to obtain the required approximate solution.

4-Numrical Illustration

Problem 1

Consider BVPs $\ddot{y} = 9y - 9$ with infinite boundary conditions $y(0) = 2$, $y(\infty) = 1$

exact solution $y(x) = e^{-3x+1}$

we take h=0.5 and $\varepsilon = 10^{-18}$

$$\frac{-y_{n+1} + 2y_n - y_{n-1}}{h^2} = 9y_n - 9$$

$y_{n+1} = 4.25y_n - y_{n-1} - 2.25$ …(13)

Substituted n=1,2,3, … in (13) to obtain

$y_2 = 4.25y_1 - y_0 - 2.25$ by substituted $y_3 = 0$ to obtain $y_3 = 1.19047619$

but $|1.19047619 - 1| > \varepsilon$

so continue to n=7 the second condition becomes as follows

$b^{n+1} = a + (n+1)h \xrightarrow{yields} b^{10} = 2 + 8(0.5) \xrightarrow{yields} b^{10} = 4$

The boundary condition becomes y(0)=2 , y(4)=1

Then by approximate y(x) as

$y_N(x) = \sum_{i=0}^{6} a_r V_r^*(x)$

Table 1and Figure1 represent comparison between the exact and approximated which on least square error

x	**approximate**	**Exact**
0	2	2
0.8	1.09065	1.09072
1.6	1.00821	1.00823
2.4	1.00059	1.00075
3.2	1.00007	1.00007
4	1.00001	1
L.S.E	3.1e-008	

Table 1

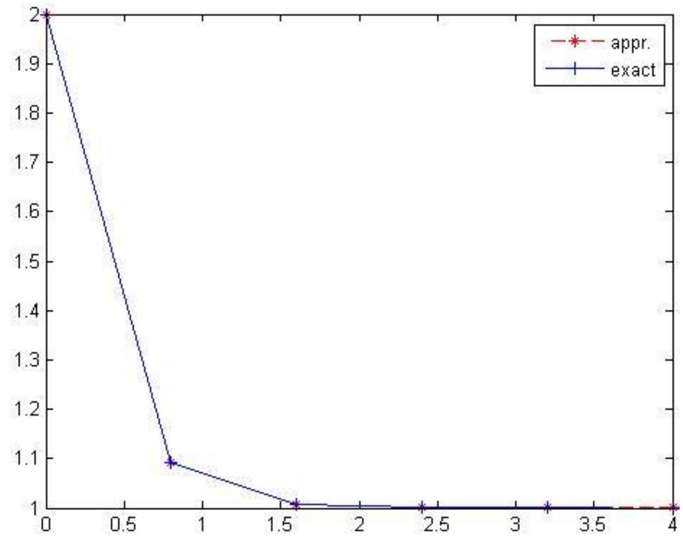

Fig1

Problem 2

$\ddot{y} = 2y - e^{-x}$, $y(0) = 1, y(\infty) = 0,$

$exact\ solution y(x) = e^{-x}$

we take $\varepsilon = 10^{-18}$ by solving in the same ways on examples we obtain n=19, b^{20}=10 so the boundary condition becomes y(0)=1, y(10)=0

and by assuming the approximate solution

$y_N(x) = \sum_{i=0}^{8} a_r . V_r^*(x)$

Table 2 and Figure 2 represent comparisons between the exact and approximated which on least square error.

x	Approximate	Exact
0	1	1
2	0.13548	0.13534
4	0.01821	0.01832
6	0.00239	0.00248
8	0.00026	0.00034
10	0.00001	0
L.S.E	$4.63e^{-008}$	

Table2

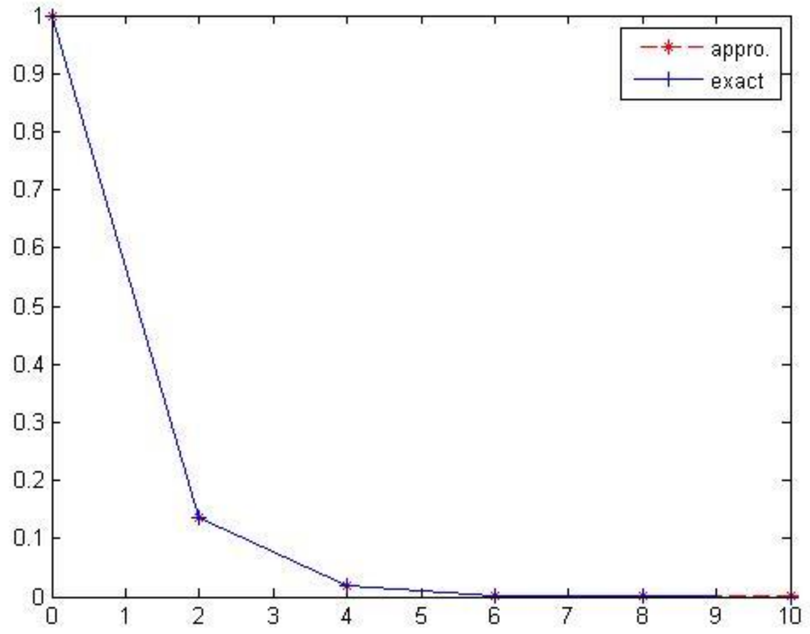

Fig2

Conclusions:-

The aim of present work is develop an efficient and accurate method for solving boundary value problem over infinite interval first the infinite interval changed by finite difference method to finite interval, then the shifted third kind chebyshev operational matrix of drivitive together with collocation method are used to reduce the problem into system of nonlinear equation in unknown approximate coefficient. Illustrative examples are include to demon state the validity and applicability of the technique.

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