Emirates Journal for Engineering Research

Volume 24 | Issue 3 Article 4

7-7-2019

Solution of Fuzzy Fredholm Integral Equation via Modified Homotopy Method

Raghad I. Sabri
University of Technology, Baghdad, raghadasm@yahoo.com

Rasha Jalal Mitlif
University of Technology, Baghdad, rasha.jalal83@yahoo.com

Fatema Ahmed Sadeq
University of Technology, Baghdad, 10122@uotechnology.edu.iq

SOLUTION OF FUZZY FREDHOLM INTEGRAL EQUATION VIA MODIFIED HOMOTOPY METHOD

Raghad I. Sabri

Rasha Jalal Mitlif

Fatema Ahmed Sadeq

Department of Applied Science, University of Technology, Baghdad, Iraq, raghadasm@yahoo.com

Department of Applied Science, University of Technology, Baghdad, Iraq, rasha.jalal83@yahoo.com

Department of Applied Science, University of Technology, Baghdad, Iraq, 10122@uotechnology.edu.iq

(Received on 30/03/2019 and Accepted 07/07/19)

في هذا البحث ، اقترحنا تطويرا لطريقة Homotopy من خلال إدخال معلمات تسريع لحل المعادلات التكاملية الضبابية. تم استخدام الطريقة المطورة لإيجاد حلول دقيقة لمعادلات Fredholm التكاملية الضبابية. النتائج تبين أن الطريقة المعدلة بسيطة جدا وفعالة.

In this paper, we proposed a modification to the Homotopy method by introducing accelerating parameters for solving fuzzy integral equations.The modified method is employed to find exact solutions for fuzzy Fredholm integral equations . The results imply that the modified method is very simple and effective.

1. INTRODUCTION

Numerical methods for solving fuzzy integral equations have been rapidly mounting in recent years and have been studied by M. Friedman, M. Ma, A. Kandel[1]. In 1992, Liao [2] proposed homotopy perturbation method (HPM) . In this method, the solution is taken into consideration as the summation of an infinite series, which usually converges rapidly to the exact solution. In addition to this, HPM has been successfully utilized to solve numerous types of nonlinear problems [3-14]. In this paper an efficient modification of the HPM was proposed for solving nonlinear fuzzy Fredholm integral equation. . The paper is organized as follows: In Section 2, the basic definitions and fuzzy background are briefly presented. In Section 3, fuzzy integral equation is introduced. HPM for solving fuzzy Fredholm integral equations is presented in section 4. Section 5 describes how to find the exact solution of fuzzy Fredholm integral equation by using the proposed method. Two illustrative examples are given in Section 6. Finally, conclusions are specified.

2-Preliminaries

Definition 2.1[15] : A fuzzy number is a fuzzy set $\tilde{u} : R \to I = [0,1]$ which satisfies

i. $\tilde{u}$ is upper semi continuous.

ii. $\tilde{u}$ (x) =0 outside some interval [a,d].

iii. There are real numbers b,c : a ≤b ≤c ≤d for which

1. $\tilde{u}$ (x) is monotonic increasing on $[a, b]$.

2. $\tilde{u}$ (x) is monotonic decreasing on $[c, d]$

3. $\tilde{u}$ (x) =1 , $b \le x \le c$.

The set of all fuzzy numbers (as given by Definition 2.1) is denoted by E^1.

Definition 2.2 [16]: A fuzzy number $\tilde{u}$ is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}$ (r), $\overline{u}$ (r) $0 \le r \le 1$ which satisfying the following properties:

i. $\underline{u}$ (r), is bounded monotonic increasing left continuous function,

ii. $\overline{u}$ (r) is bounded monotonic decreasing left continuous function,

iii. $\underline{u}(r) \le \overline{u}(r)$, $0 \le r \le 1$.

For arbitrary $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ and $\tilde{v} = (\underline{v}(r), \overline{v}(r))$, $0 \le r \le 1$, and scalar k, we define addition, subtraction, scalar product by k are respectively as following:

- addition: $\underline{(u+v)}(r) = \underline{u}(r) + \underline{v}(r)$, $\overline{(u+v)}(r) = \overline{u}(r) + \overline{v}(r)$.
- subtraction: $\underline{(u-v)}(r) = \underline{u}(r) - \underline{v}(r)$, $\overline{(u-v)}(r) = \overline{u}(r) - \overline{v}(r)$

• scalar product: $ku(r) =$

$$\begin{cases} \left(k\underline{u}(r), k\overline{u}(r)\right) & k \ge o \\ \left(k\overline{u}(r), k\underline{u}(r)\right) & k < o \end{cases}$$

Definition 2.3 [17]: For arbitrary fuzzy numbers $\tilde{u} = (\underline{u}(r), \overline{u}(r))$ and $\tilde{v} = (\underline{v}(r), \overline{v}(r))$ the quantity

$$D(\tilde{u}, \tilde{v}) = \max\{ \sup_{0\le r\le 1} |\underline{u}(r) - \underline{v}(r)|, \sup_{0\le r\le 1} |\overline{u}(r) - \overline{v}(r)| \}.$$

is the distance between $\tilde{u}$ and $\tilde{v}$.

3.Fuzzy Integral Equations

There are three main types of fuzzy integral equations .The integral equations which are discussed in this section are the Fredholm equations. A standared form of the Fredholm integral equation of second kind is given by[1]

$$\gamma(x) = g(x) + \lambda \int_a^b k(x,t)\, \gamma(t) dt \quad \ldots(1)$$

Where λ is a positive parameter and $k(x,t)$ is an arbitrary kernel function over the square $a \le x, t \le b$ and g (x) is a function of $x : a \le x \le b$. If g(x)is a crisp function then the solutions of Eq. (1) are crisp as well. However, if g(x)is a fuzzy function these equations may only possess fuzzy solutions.

Now with respect to definition (2.2) , we introduce parametric form of a Fuzzy Fredholm Integral Equation of the second kind (FFIE-2). Let $(\underline{g}(x,r), \overline{g}(x,r))$ and $(\underline{\gamma}(x,r), \overline{\gamma}(x,r))$,$0 \le r \le 1$ and $x \in [a,b]$ are parametric form of g(x)and γ(x) respectively then, parametric form of FFIE-2 is as follows:

$$\underline{\gamma}(x,r) = \underline{g}(x,r) + \lambda \int_a^b \underline{U}(t,r) dt$$

$$\ldots \quad (2)$$

$$\overline{\gamma}(x,r) = \overline{g}(x,r) + \lambda \int_a^b \overline{U}(t,r) dt$$

where

$$\underline{U}(t,r) = \begin{cases} k(x,t)\, \underline{\gamma}(t,r), & k(x,t) \ge 0 \\ k(x,t)\, \overline{\gamma}(t,r), & k(x,t) < 0 \end{cases}$$

and

$$\overline{U}(t,r) = \begin{cases} k(x,t)\, \overline{\gamma}(t,r), & k(x,t) \ge 0 \\ k(x,t)\, \underline{\gamma}(t,r), & k(x,t) < 0 \end{cases}$$

for each $0 \le r \le 1$ and $0 \le t, x \le b$. We can see that eq. (2) is a system of linear Fredholm integral equations in the crisp case for each $0 \le r \le 1$ and $a \le x \le b$.

4. Homotopy Perturbation Method for Solving Fuzzy Fredholm Integral Equations

Consider fuzzy Fredholm integral equation:

$$\underline{\gamma}(x,r) = \underline{g}(x,r) + \int_a^b k(x,t) [\underline{\gamma}(t,r)]^q dt$$

$$\ldots(3)$$

$$\overline{\gamma}(x,r) = \overline{g}(x,r) + \int_a^b k(x,t) [\overline{\gamma}(t,r)]^q dt$$

To explain the HPM, we reconstitute (3) as

$$L(\underline{u}) = \underline{u}(x,r) - \underline{g}(x,r) - \int_a^b k(x,t) [\underline{u}(t,r)]^q dt = 0 \quad .$$

$$\ldots(4)$$

$$L(\overline{u}) = \overline{u}(x,r) - \overline{g}(x,r) - \int_a^b k(x,t) [\overline{u}(t,r)]^q dt = 0$$

with solution $\underline{u}(x,r) = \underline{\gamma}(x,r)$, $\overline{u}(x,r) = \overline{\gamma}(x,r)$

Homotopy $H(\underline{u},p), H(\overline{u},p)$ is defined as follows:

$$\begin{cases} H(\underline{u},0) = F(\underline{u}), & H(\underline{u},1) = L(\underline{u}) \\ H(\overline{u},0) = F(\overline{u}), & H(\overline{u},1) = L(\overline{u}) \end{cases}$$

where $F(\underline{u}), F(\overline{u})$ are functional operators with solutions, say $\underline{u}_0, \overline{u}_0$ which can be

obtained easily. We may choose a convex homotopy

$$\begin{cases} H(\underline{u},p) = (1-p)F(\underline{u}) + p\, L(\underline{u}) = 0 \\ H(\overline{u},p) = (1-p)F(\overline{u}) + p\, L(\overline{u}) = 0 \end{cases} \quad \ldots(5)$$

In fact HPM uses the homotopy parameter p as an expanding parameter to obtain

$$\begin{cases} \underline{u} = \underline{u}_0 + p\underline{u}_1 + p^2\underline{u}_2 + \cdots \\ \overline{u} = \overline{u}_0 + p\overline{u}_1 + p^2\overline{u}_2 + \cdots \end{cases}$$

The embedding parameter $p \in (0,1]$ can be considered as an expanding parameter :

$$\underline{u} = \sum_{n=0}^{\infty} p^n \underline{u}_n$$

$$\ldots(6)$$

$$\overline{u} = \sum_{n=0}^{\infty} p^n \overline{u}_n$$

when $p \to 1$ Eq. (5) corresponds to Eq. (4) and Eq. (6) becomes the approximate solution of Eq. (4), i.e.,

$$\underline{\gamma}(x,r) = \lim_{p\to 1} \underline{u} = \sum_{n=0}^{\infty} \underline{u}_n$$

$$\overline{\gamma}(x,r) = \lim_{p\to 1} \overline{u} = \sum_{n=0}^{\infty} \overline{u}_n$$

Taking $F(\underline{u}) = \underline{u}(x,r) - \underline{g}(x,r)$ and $F(\overline{u}) = \overline{u}(x,r) - \overline{g}(x,r)$, we substitute (6) into (5) and equate the terms with identical powers of p, obtaining

$$\begin{cases} \underline{u}_0(x,r) = \underline{g}(x,r) \\ \overline{u}_0(x,r) = \overline{g}(x,r) \end{cases}$$

$$\underline{u}_{n+1}(x,r) = \int_a^b k(x,t)H_{n1}(t,r)dt \quad \ldots(7)$$

$$\overline{u}_{n+1}(x,r) = \int_a^b k(x,t)H_{n2}(t,r)dt$$

Where the Hn's are the so-called He's polynomials [18] which can be calculated by using the formula

$$H_{n1}(t.r) = H_n(\underline{u}_0, \underline{u}_1, \underline{u}_2, \ldots) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}((\sum_{k=0}^{n} p^k \underline{u}_k)^q)_{p=0}$$

$$H_{n2}(t.r) = H_n(\overline{u}_0, \overline{u}_1, \overline{u}_2, \ldots) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}((\sum_{k=0}^{n} p^k \overline{u}_k)^q)_{p=0}$$

5.Description of the New Method

We first reconstitute Eq. (3) as follows:

$$\underline{\gamma}(x,r) = \sum_{m=0}^{N} \beta_m \underline{v}_m(x,r) - \sum_{m=0}^{N} \beta_m \underline{v}_m(x,r) + \underline{g}(x,r) + \int_a^b k(x,t)[\underline{\gamma}(t,r)]^q dt \quad \ldots(8)$$

$$\overline{\gamma}(x,r) = \sum_{m=0}^{N} \beta_m \overline{v}_m(x,r) - \sum_{m=0}^{N} \beta_m \overline{v}_m(x,r) + \overline{g}(x,r) + \int_a^b k(x,t)[\overline{\gamma}(t,r)]^q dt$$

By using the HPM, we let

$$\begin{cases} F(\underline{u}) = \underline{u}(x,r) - \sum_{m=0}^{N} \beta_m \underline{v}_m(x,r) \\ F(\overline{u}) = \overline{u}(x,r) - \sum_{m=0}^{N} \beta_m \overline{v}_m(x,r) \end{cases}$$

Therefore, we define a new convex homotopy perturbation as:

$$H_\beta(\underline{u},p) = \underline{u}(x,r) - p\underline{g}(x,r) + (p-1)\left[\sum_{m=0}^{N} \beta_m \underline{v}_m(x,r)\right] - p\int_a^b k(x,t)[\underline{u}(t.r)]^q dt = 0 \quad \ldots(9)$$

$$H_\beta(\overline{u},p) = \overline{u}(x,r) - p\overline{g}(x,r) + (p-1)\left[\sum_{m=0}^{N} \beta_m \overline{v}_m(x,r)\right] - p\int_a^b k(x,t)[\overline{u}(t.r)]^q dt = 0 \quad \ldots(10)$$

where $\beta = [\beta_m]$ and $\beta_m . m = 0,1,2,\ldots N$ are called the accelerating components of the parameter, and

$$\begin{cases} \underline{v}(x,r) = [\underline{v}_m(x,r)] \\ \overline{v}(x,r) = [\overline{v}_m(x,r)] \end{cases} \quad m = 0,1,2,\ldots N$$

are selective functions .

6. Numerical Examples

Here, we consider two examples to illustrate the new method for solving Fredholm fuzzy integral equations.

Example 1: Consider the fuzzy Fredholm integral equation with

$$\underline{g}(x,r) = 3(r^2-2))(9x^2-10) + x^3(r^5+2r) - r(r^4+2)(3x^2+2)$$

$$\overline{g}(x,r) = 3(r^2-2)(3x^2+2) - x^3(3r^2-6) - r(r^4+2)(9x^2-10)$$

and kernel $k(x,t) = 3(2-t^2+x^2) \quad 0 \le t \ , x \le 2$ $a = 0 \ , b = 2.$.

The exact solution in this case is given by

$$\underline{\gamma}(x,r) = x^3(r^5+2r)$$

$$\overline{\gamma}(x,r) = x^3(6-3r^2)$$

We apply $H_\beta(u,p)$ method for upper and lower case respectively to approximate the solutions. In this example, we choose, $\underline{v}(x,r) = rx^3$ for lower case.

The homotopy equation, Eq. (9), becomes

$$H_\beta(\underline{u},p) = \underline{u}(x,r) - p\underline{g}(x,r) + (p-1)\beta r x^3 - p\int_0^2 3(2-t^2+x^2)\ [\underline{u}(t,r)]dt = 0 \quad \ldots(11)$$

now we substitute (6) into (11), and equate the terms with identical powers of p, we obtain

p°: $\underline{u}_\circ(x,r) - \beta r x^3 = 0 \qquad \Rightarrow \underline{u}_\circ(x,r) = \beta r x^3$

$$p^1: \underline{u}_1(x,r) - \underline{g}(x,r) + \beta r x^3 - \int_0^2 3(2-t^2+x^2)\ \underline{u}_\circ(t,r)dt = 0$$

$$\Rightarrow \underline{u}_1(x,r) = (60-30r^2-2r^5-4r-8\beta r) + (27r^2-54-3r^5-6r+12\beta r)x^2 + (r^5+2r-\beta r)x^3$$

$$p^2: \underline{u}_2(x,r) - \int_0^2 3(2-t^2+x^2)\ \underline{u}_1(t,r)dt = 0$$

$$\Rightarrow \underline{u}_2(x,r) = \int_0^2 3(2-t^2+x^2)\ \underline{u}_1(t,r)dt$$

and in general

$$\underline{u}_{n+1}(x,r) = \int_a^b H_{n1}(t,r)dt$$

So, to find β such that $\underline{u}_1 = 0$, we should have

$$\begin{cases} 60-30r^2-2r^5-4r-8\beta r = 0 \\ 27r^2-54-3r^5-6r+12\beta r = 0 \\ r^5+2r-\beta r = 0 \end{cases}$$

thus $\beta = \frac{r^5+2r}{r}$.

Therefore, we obtain

$\underline{\gamma}(x,r) = \underline{u}_\circ(x,r) = \beta r x^3 = \left(\frac{r^5+2r}{r}\right) r x^3 = (r^5+2r)x^3$, which is the same as the exact solutions for lower case.

Now , we choose $\overline{v}(x,r)=r^2x^3$ for upper case. Therefore, Eq. (10) can be written in the form:
$H_\beta(\overline{u},p)=\overline{u}(x,r)-p\overline{g}(x,r)+(p-1)\beta r^2x^3-p\int_0^2 3(2-t^2+x^2)\,[\overline{u}(t,r)]dt=0$ …(12)

collecting coefficients of like powers of p, and setting them equal to zero, we have

p°: $\overline{u}_\circ(x,r)-\beta r^2x^3=0 \quad \Rightarrow \overline{u}_\circ(x,r)=\beta r^2x^3$

p^1: $\overline{u}_1(x,r)-\overline{g}(x,r)+\beta r^2x^3 - \int_0^2 3(2-t^2+x^2)\,\overline{u}_\circ(t,r)dt=0$

$$\overline{u}_1(x,r)=(6r^2-12+10r^5+20r-8\beta r^2)+(9r^2-18-9r^5-18r+12\beta r^2)x^2+(6-3r^2-\beta r^2)x^3$$

$$p^2:\ \overline{u}_2(x,r)-\int_0^2 3(2-t^2+x^2)\ \overline{u}_1(t,r)dt=0$$

$$\overline{u}_2=\int_0^2 3(2-t^2+x^2)\ \overline{u}_1(t,r)dt$$

and in general

$$\overline{u}_{n+1}(x,r)=\int_a^b H_{n2}(t,r)dt$$

Now we find β in such a way that $\overline{u}_1=0$, if $\overline{u}_1=0$ then $\overline{u}_2=\overline{u}_3=\cdots=0$, and the exact solution will be obtained as $\overline{\gamma}(x,r)=\overline{u}_\circ(x,r)$; hence for all values of x we should have

$$\begin{cases}6r^2-12+10r^5+20r-8\beta r^2=0\\ 9r^2-18-9r^5-18r+12\beta r^2=0\\ 6-3r^2-\beta r^2=0\end{cases}$$

thus $\beta=\frac{6-3r^2}{r^2}$

Hence, the solutions will be as follows $\overline{\gamma}(x,r)=\overline{u}_\circ(x,r)=\beta r^2x^3=\left(\frac{6-3r^2}{r^2}\right)r^2x^3=(6-3r^2)x^3$, which is the same as the exact solutions for upper case.

Example 2: Consider the fuzzy Fredholm integral equation with

$\underline{g}(x.r)=r(\frac{1}{2}x-\frac{1}{3})$

$\overline{g}(x,r)=(2-r)(\frac{1}{2}x-\frac{1}{3})$

and kernel $k(x,t)=x+t \quad 0\le x\ ,t\le 1$ and $a=0\ ,b=1$. The exact solution in this case is given by

$$\underline{\gamma}(x,r)=rx$$
$$\overline{\gamma}(x,r)=(2-r)x$$

We apply the $H_\beta(\underline{u},p)$ method to approximate the solution. In this example, we choose $\underline{v}_1(x,r)=r$ and $\underline{v}_2(x,r)=rx$ for lower case.

Therefore, Eq. (9) can be written in the form:
$H_\beta(\underline{u},p)=\underline{u}(x,r)-pr\underline{g}(x,r)+(p-1)(\beta_\circ r+\beta_1 rx)-p\int_0^1(x+t)[\underline{u}(t,r)]dt=0$ …(13)

By the same manipulation as illustrated in the above example, we have

p°: $\underline{u}_\circ(x,r)-(\beta_\circ r+\beta_1 rx)=0 \Rightarrow \underline{u}_\circ(x,r)=\beta_\circ r+\beta_1 rx$

$$p^1:\ \underline{u}_1(x,r)-r\left(\frac{1}{2}x-\frac{1}{3}\right)+(\beta_\circ r+\beta_1 rx)-\int_0^1(x+t)\underline{u}_\circ(t,r)dt=0$$

$$\Rightarrow \underline{u}_1(x,r)=r\left(\frac{1}{2}x-\frac{1}{3}\right)-(\beta_\circ r+\beta_1 rx)+(\beta_\circ rx+\frac{1}{2}\beta_1 rx+\frac{1}{2}\beta_\circ r+\frac{1}{3}\beta_1 r)$$

$$\Rightarrow \underline{u}_1(x,r)=\left(\frac{1}{3}\beta_1 r-\frac{1}{2}\beta_\circ r-\frac{1}{3}r\right)+\left(\frac{1}{2}r-\beta_1 r+\beta_\circ r+\frac{1}{2}\beta_1 r\right)x$$

$$p^2:\ \underline{u}_2(x,r)-\int_0^1(x+t)\underline{u}_1(t,r)dt=0 \quad \Rightarrow \underline{u}_2(x,r)=\frac{1}{12}\beta_\circ r+(\frac{1}{12}\beta_1 r-\frac{1}{12}r)x$$

and in general

$$\underline{u}_{n+1}(x,r)=\int_a^b H_{n1}(t,r)dt$$

Now we find β in such a way that $\underline{u}_1=0$, if $\underline{u}_1=0$ then $\underline{u}_2=\underline{u}_3=\cdots=0$, and the exact solution will be obtained as $\underline{\gamma}(x,r)=\underline{u}_\circ(x,r)$; hence for all values of x we should have

$$\begin{cases}\frac{1}{3}\beta_1 r-\frac{1}{2}\beta_\circ r-\frac{1}{3}r=0\\ \frac{1}{2}r-\beta_1 r+\beta_\circ r+\frac{1}{2}\beta_1 r=0\end{cases}$$

thus $\beta_\circ=0$ and $\beta_1=1$

Hence, the solutions will be as follows $\underline{\gamma}(x,r)=\underline{u}_\circ(x,r)=\beta_\circ r+\beta_1 rx=rx$, which is the same as the exact solutions for lower case.

Now , we choose $\overline{v}_1(x,r)=r$ and $\overline{v}_2(x,r)=r^2x$ for upper case.

Therefore, Eq. (10) can be written in the form:
$H_\beta(\overline{u},p)=\overline{u}(x,r)-p\overline{g}(x,r)+(p-1)(\beta_\circ r+\beta_1 r^2x)-p\int_0^1(x+t)[\overline{u}(t,r)]dt=0$ …(14)

now we substitute (6) into (14), and equate the terms with identical powers of p, we obtain

p°: $\overline{u}_\circ(x,r)-(\beta_\circ r+\beta_1 r^2x)=0 \Rightarrow \overline{u}_\circ(x,r)=\beta_\circ r+\beta_1 r^2x$

$$p^1:\ \overline{u}_1(x,r)-(2-r)\left(\frac{1}{2}x-\frac{1}{3}\right)+(\beta_\circ r+\beta_1 r^2x)-\int_0^1(x+t)\,\overline{u}_\circ(t,r)dt=0$$

$$\Rightarrow \overline{u}_1(x,r)=\left(\frac{1}{3}r-\frac{1}{2}\beta_\circ r+\frac{1}{3}\beta_1 r^2-\frac{2}{3}\right)+\left(1-\frac{1}{2}r-\frac{1}{2}\beta_1 r^2+\beta_\circ r\right)$$

$p^2: \bar{u}_2(x,r) - \int_0^1 (x+t)\,\bar{u}_1(t,r)dt = 0$
$\Rightarrow \bar{u}_2(x,r) = \frac{1}{12}\beta_\circ r + \left(\frac{1}{12}r + \frac{1}{12}\beta_1 r^2 - \frac{1}{6}\right)x$
and in general
$\bar{u}_{n+1}(x,r) = \int_a^b H_{n2}(t,r)dt$

Now we find β in such a way that $\bar{u}_1$ = 0, if $\bar{u}_1$ = 0 then $\bar{u}_2 = \bar{u}_3 = \cdots = 0$, and the exact solution will be obtained as $\bar{\gamma}(x,r) = \bar{u}_\circ(x,r)$; hence for all values of x we should have

$$\begin{cases} \frac{1}{3}r - \frac{1}{2}\beta_\circ r + \frac{1}{3}\beta_1 r^2 - \frac{2}{3} = 0 \\ 1 - \frac{1}{2}r - \frac{1}{2}\beta_1 r^2 + \beta_\circ r = 0 \end{cases}$$

thus $\beta_\circ = 0$ and $\beta_1 = \frac{2-r}{r^2}$
Hence, the solutions will be as follows
$\bar{\gamma}(x,r) = \bar{u}_\circ(x,r) = \beta_\circ r + \beta_1 r^2 x = \left(\frac{2-r}{r^2}\right) r^2 x = (2-r)x$, which is the same as the exact solutions for upper case.

Conclusions

In this paper, we proposed a modification to the Homotopy perturbation method for solving fuzzy Fredholm integral equations. In this modification, a new homotopy, $H_\beta(u,p)$, was constructed where $\beta = [\beta_m]$ is called the accelerating parameter. The accelerating parameter gives fast convergent rate, since only one iteration leads to exact solutions. The examples reveal that this method is very simple and effective tool for computing the exact solutions.

REFERENCES

[1] M. Friedman, M. Ma, A. Kandel.1999. Numerical Solutions of fuzzy differential equations and integral equations, Fuzzy Sets and Systems 106 ,35-48.

[2] S.J. Liao.1992.The proposed homotopy analysis technique for the solution of nonlinear

problems, Ph.D. Thesis, Shanghai Jiao Tong University.

[3] M. Hasan and M. A. Matin, .2017.Approximate Solution of Nonlinear Integral Equations of the Second Kind by Using Homotopy Perturbation Method, Dhaka Univ. J. Sci. 65,151-155.

[4] S. Abbasbandy, E. Magyari, E. Shivanian, .2009.The homotopy analysis method for multiple solutions of nonlinear boundary value problems, Commun Nonlinear Sci. Numer. Simulat. 14 3530–3536.

[5] Mohsen Rabbani.2015. Modified homotopy method to solve non-linear integral equations,Int. J. Nonlinear Anal. Appl. 6 ,no.2, 133-136 ISSN: 2008-6822.

[6] S. Abbasbandy.2008.Homotopy analysis method for generalized Benjamin–Bona–Mahony equation, Z. Angew. Math. Phys. 59 , 51–62.

[7] Zainidin K. Eshkuvatov , Fatimah Samihah .2018. Homotopy perturbation method for the hypersingular integral equations of the first kind, Ain Shams Engineering Journal ,9 ,3359–3363.

[8] Seyyed Mahmood Mirzaei.2010. Homotopy Perturbation Method for Solving the Second Kind of Non-Linear Integral Equations, International Mathematical Forum , no. 23, 1149 – 1154.

[9] S. Abbasbandy, T. Hayat.2011. On series solution for unsteady boundary layer equations in a special third grade fluid, Commun. Nonlinear Sci. Numer. Simul.16 ,3140–3146.

[10] S. Nadeem, T. Hayat, S. Abbasbandy, M. Ali.2010. Effects of partial slip on a fourth-grade fluid with variable viscosity: an analytic solution, Nonlinear Anal.RWA 11 ,856–868.

[11] T. Hayat, R. Naz, S. Abbasbandy.2011. Poiseuille flow of a third grade fluid in a porous medium, Transp. Porous Media 87 ,355–366.

[12] A. Shidfar, A. Molabahrami.2011. Solving a system of integral equations by an analytic method, Math. Comput. Modelling 54 ,828–835.

[13] A. Shidfar, A. Babaei, A. Molabahrami, M. Alinejadmofrad.2011.Approximate analytical solutions of the nonlinear reaction–diffusion–convection problems,Math. Comput. Modelling 53 ,261–268.

[14] J. Rad, S. Ghaderi, K. Parand.2011. Numerical and analytical solution of gas flow through a micro–nano porous media: a comparison, J. Comput. Theor.Nanosci. 8 ,2033–2041.

[15] G.J. Klir, U.S. Clair and B. Yuan. 1997. “Fuzzy set theory: foundations and applications, ” Prentice-Hall Inc.

[16] S. Abbasbandy, E. Babolian, and M. Alavi, .2007 .“Numerical method forsolving linear Fredholm fuzzy integral equations of the second kind,” Chaos, Solitons and Fractals, Vol. 31, pp. 138-146.

[17] M.L. Puri and D. Ralescu.1986. Fuzzy random variables, J. Math. Anal. Appl.114, 409-22.

[18]A. Ghorbani.2007.Beyond Adomian polynomials: He polynomials, Chaos Solitons Fractals, doi:10.1016/j.chaos.2007.06.034.