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طريقة هرمت الموجبة لتخمين الحل لمسألة براتو

ملخص

في هذا البحث وضعت خوارزمية جديده اساسها استخدام متعددة حدود هيرمت الموجبة لايجاد الحل العددي لمسألة براتو ذات البعد الواحد. العمل الحسابي هو داعم بشكل كامل من التوافق بين الخورازميه المقترحة وبالتالي هي نفسها قد تمتد الى غيرها من المسائل الماديه ايضا. تم عرض الامثلة لتوضيح بساطه وفعالية الطريقه.

Abstract

In this paper, the construction of hermit wavelets function and their operational matrix of integration is presented. The matrix together with the collection method are then utilized to transform the differential equations to a system of algebraic equation. Avery high level of accuracy explicitly reflected by the proposed examples.

1. INTRODUCTION

Bratu´s problem is one of the important topics in applied mathematics such as the fuel ignition model of the thermal combustion theory, questions in geometry and relativity about the Chandrasekhar model, chemical reaction theory, radiative heat transfer and nanotechnology [1,2,3,4]

Wavelets permit the accurate representations of a variety of functions and operator. Special attention has been given to application of the Chebyshev wavelet [5], the SAC wavelet [6] and the legender wavelets [7].

In this paper, we consider the boundary- value problem and initial value problem of Bratu´s boundary value problem in one- dimensional planar coordinates is of the form:

$$\ddot{y} + \lambda e^{y} = 0 \qquad 0 < x < 1 \qquad \dots(1)$$

with the boundary conditions y (0) =y (1) =0. For $\lambda>0$ is a constant.

2. Hermite wavelets and their properties: [8]

Wavelets constitute a family of functions constructed from dilation and translation of a single function $\psi(x)$ called the mother wavelet.

The Hermite wavelets $\psi_{n,m}(x)=\psi(k,n,m,t)$ involve four arguments $n=1,2,\dots,2^{k-1}$, k is assumed any positive integer, m is the degree of the Hermite polynomials and t is the normalized time.

They are defined on the interval [0,1] as:

$$\psi_{n,m}(t)=\begin{cases}\sqrt{m+\frac{1}{2}}\,2^{k/2}\,H_m(2kt-n) & \frac{n-1}{2^k}\le t\le\frac{n+1}{2^k}\\ 0 & \end{cases} \qquad \dots(2)$$

Where m=0,1, 2, …, M-1. Here $H_m(x)$ is Hermite polynomials of degree m with respect to weight function= $\sqrt{1-x^2}$ on the real R and satisfies the following recurrence formula $H_0(x)=1$, $H_1(x)=2x$, $H_{m+2}=2x\, H_{m+1} -2\,(m+1)\, H_m(x)$, m=0,1,2,…

3. Operational Matrix of Integration (OMI): [9]

In this section, we get the structure of OMI for Hermite wavelet, particularly at k=1 and M=6, here we consider six basis functions on [0,1], are given by: -

$$\psi_{1,0}(x) = \frac{2}{\sqrt{\pi}} \quad , \psi_{1,1}(x) = \frac{2}{\pi}\,(4x-2),$$

$$\psi_{1,2}(x) = \frac{2}{\pi}\,(16x^2-16x+2),$$

$$\psi_{1,3}(x) = \frac{2}{\pi}\,(64x^3-96x^2-16x+36x-2),$$

$$\psi_{1,4}(x) = \frac{2}{\pi}\,(256x^4-512x^3+320x^2-64x+2)\,,$$

$$\psi_{1,5}(x) = \frac{2}{\pi}\,(1024x^5-2560x^4+2240x^3-800x^2 + 100x-2)\,,$$

$$let\ \psi_6(x) = \left(\left(\psi_{1,0}(x), \psi_{1,1}(x), \psi_{1,2}(x), \psi_{1,3}(x), \psi_{1,4}(x), \psi_{1,5}(x)\right)\right)^T$$

by integrating above basis functions with respect to x from 0 to x and expressing in matrix form, we obtain:-

$$\int_0^x \psi_6(x)dx = P_{6\times 6}\psi_6(x) + \overline{\psi_6}\,(x) \dots (3)$$

where

$$P_{6\times6}=\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{-1}{4} & 0 & \frac{1}{8} & 0 & 0 & 0 \\ \frac{-1}{3} & \frac{-1}{4} & 0 & \frac{1}{12} & 0 & 0 \\ \frac{1}{8} & 0 & \frac{-1}{8} & 0 & \frac{1}{16} & 0 \\ \frac{-1}{15} & 0 & 0 & \frac{-1}{12} & 0 & \frac{1}{20} \\ \frac{1}{24} & 0 & 0 & 0 & \frac{-1}{16} & 0 \end{bmatrix} \dots (4)$$

$$\overline{\psi_6}(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{24}\psi_{1,6}(x) \end{bmatrix} \quad \dots (5)$$

again double integration of above six basis is given by:-

$$\int_0^x \int_0^x \psi_6(x)\,dx\,dx = \acute{p}_6\,(x) + \overline{\acute{\psi}_6}(x) \quad \dots (6)$$

where

$$P'_{6\times6} = \begin{bmatrix} \frac{3}{16} & \frac{1}{8} & \frac{1}{32} & 0 & 0 & 0 \\ \frac{-1}{6} & \frac{-3}{32} & 0 & \frac{1}{96} & 0 & 0 \\ \frac{-3}{32} & \frac{-1}{12} & \frac{-1}{24} & 0 & \frac{1}{192} & 0 \\ \frac{1}{10} & \frac{1}{16} & 0 & \frac{-1}{64} & 0 & \frac{1}{320} \\ \frac{-1}{24} & \frac{-1}{60} & \frac{1}{96} & 0 & \frac{-1}{120} & 0 \\ \frac{1}{42} & \frac{1}{96} & 0 & \frac{1}{192} & 0 & \frac{-1}{192} \end{bmatrix} \dots (7)$$

and

$$\overline{\acute{\psi}_6}(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{480}\psi_{1,6}(x) \\ \frac{1}{672}\psi_{1,7}(x) \end{bmatrix} \quad \dots (8)$$

In this way we can generate the other matrix by consuming more number of basis functions.

4. Solution of Bratu's Problem: -

Consider Bratu's problem given in (1). In order use Hermite wavelets, we first approximate y(x) as: -

$$\ddot{y}(x) \simeq \sum_{n=1}^{2^k}\sum_{m=0}^{m} C_{nm}\,\psi_{nm} = C^T\psi(x) \quad \dots(9)$$

$$Y(0) = Y(1) = 0 \quad \dots (10)$$

where C & $\psi(x)$ are $2^k M\times 1$ Matrices given by : -

$$C = \left[c_{1,0}, c_{1,1}, c_{1,2}, \dots, c_{1,M-1}\ \right]^T$$

and $\psi(x) = [\psi_{1,0}, \psi_{1,1}, \psi_{1,2}, \dots, \psi_{1,M-1}\ ]^T$

integrate (9) with respect 0 to x tighter with boundary conditions given in (10), we get: -

$$\dot{y}(x) + \dot{y}(0) + 2\int_0^x e^{C^T\psi(x)} = 0$$

$$y(x) + 2\int_0^x\int_0^x e^{C^T\psi(x)} = 0 \qquad \dots (11)$$

then collect (11) at $2^{k-1}M$-2 points at x_i as

$$y(x) = -2\int_0^x\int_0^x e^{C^T\psi(x)} \qquad \dots (12)$$

suitable collection points are

$$x_i = \frac{1}{2}\ [\ 1 + \cos(\frac{(i-1)\pi}{2^k M - 1})]$$

$$i = 1,2,3, \dots, 2^{k-1}M - 2$$

Thus with the boundary conditions y (0) =y (1) =0 we have: -

$$C^T\psi(0) = 0\,,\ C^T\psi(1) = 0 \qquad \dots (13)$$

Equations 12,13 generate 2^{k-1} M set of nonlinear equations. The approximate solution of the vector C is obtained by solving nonlinear system using Gauss-Newton method.

5. Numerical Examples: -

For showing efficiency of our approximate method we consider the following examples.

Example (1): - [10,11]

Consider the first case for bratu's equation as follows: -

$\ddot{u} + 2e^u$=0 0<x< 1

u (0) = u (1) =0

we solve the equation by using the hermite wavelet method with k=2, M=4,6. First we assume that

$$\ddot{u} = C^T\ \psi(x)$$

Applying (11) we get

$$u(x) = -2\ \int_0^x\int_0^x e^{C^T}\ \psi(x) \qquad \dots (14)$$

using the boundary condition

$C^T\ \psi\ (0) =0$, $C^T\ \psi\ (1) =0$ …(15)

Equations (14) and (15) generate a system of nonlinear equations. These equations can be solved for unknown coefficients of the vector C. the numerical results obtained are presented in table1. Table 1 show the comparison between the absolute error of approximate solution for various value of M (with k=2).

Table 1. Computed absolute errors for example 1.

X	**K=2, M=4**	**K=2, M=6**
0.1	1.33351×10^{-3}	1.18934×10^{-5}
0.2	2.58731×10^{-3}	2.62388×10^{-5}
0.3	5.88772×10^{-3}	2.26721×10^{-5}
0.4	3.36292×10^{-3}	5.20350×10^{-5}
0.5	5.98507×10^{-3}	4.44306×10^{-5}
0.6	3.59812×10^{-3}	2.53272×10^{-5}
0.7	5.14784×10^{-3}	4.17100×10^{-5}
0.8	1.43561×10^{-3}	3.26316×10^{-5}
0.9	1.26861×10^{-3}	1.72589×10^{-5}

Example (2): - [12,13]

Consider the initial value problem: -

$\ddot{u} - 2e^u$=0 0<x< 1

u (0) =0 , $\dot{u}(0)$ =0

the exact solution is u(x)= -2 ln(cos(x)). Table2 shows the comparison of the absolute error between exact solution and approximate solution for k=1 M=6 by hermite wavelet method.

Table 2. comparison the results with exact solution

X	K=1, M=6	Exact Solution
0.1	1.00167×10^{-2}	1.00167×10^{-2}
0.2	4.02696×10^{-2}	4.02695×10^{-2}
0.3	9.13832×10^{-2}	9.13833×10^{-2}
0.4	1.64449×10^{-1}	1.64458×10^{-1}
0.5	2.61161×10^{-1}	2.61168×10^{-1}
0.6	3.83393×10^{-1}	3.83930×10^{-1}
0.7	5.36172×10^{-1}	5.36172×10^{-1}
0.8	7.22890×10^{-1}	7.22781×10^{-1}
0.9	9.50884×10^{-1}	9.50885×10^{-1}

6. Conclusions: -

The aim of present work is to develop an efficient and accurate method for solving Bratu's problem. The Hermite wavelet operational matrix of integral are used to reduce the problem into a system of nonlinear equation in unknown approximate coefficient illustrative examples are included to demo n state the validity and applicability of the technique.

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